SULIT

BAHAGIAN PEPERIKSAAN DAN PENILAIAN
JABATAN PENDIDIKAN POLITEKNIK
KEMENTERIAN PENDIDIKAN TINGGI

JABATAN MATEMATIK, SAINS DAN KOMPUTER

PEPERIKSAAN AKHIR

SESI JUN 2015

BA601: ENGINEERING MATHEMATICS 5

TARIKH : 26 OKTOBER 2015
MASA : 2.30 PM - 4.30 PM (2 JAM)

Kertas ini mengandungi **EMPAT BELAS (14)** halaman bercetak.
Bahagian A: Struktur (2 soalan)
Bahagian B: Struktur (2 soalan)
Bahagian C: Struktur (2 soalan)
Jawab 1 soalan dari setiap Bahagian dan pilih 1 soalan dari mana-mana Bahagian
Dokumen sokongan yang disertakan : Formula

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIARAHKAN

(CLO yang tertera hanya sebagai rujukan)

INSTRUCTION:
Answer **ONE (1)** question from each section (A, B and C) and answer **ONE (1)** question from any section that has not been answered.

ARAHAN :
*Jawab **SATU (1)** soalan daripada setiap bahagian (A, B dan C) dan Jawab **SATU (1)** lagi soalan yang belum dijawab dari mana-mana bahagian.*

SECTION A
BAHAGIAN A

QUESTION 1
SOALAN 1

CLO1 C2

(a) By using definition of hyperbolic functions, find the value of:

Dengan menggunakan definisi fungsi hiperbola, cari nilai bagi:

i. $\sinh(-3.254)$

[2 marks]
[2 markah]

ii. $4\sec h(\sqrt{8})$

[2 marks]
[2 markah]

iii. $\dfrac{8\coth(2.48)}{\cos ech\left(\dfrac{3}{4}\right)}$

[3 marks]
[3 markah]

CLO1 C2

(b) If $\sinh\frac{3x}{5} + k^3 = 2z^2\coth(x+z^2)$, find the value of k when $x = 2$ and $z = 4$.

Jika $sinh\frac{3x}{5} + k^3 = 2z^2\, coth(x+z^2)$, *cari nilai bagi k bila x = 2 dan z = 4.*

[5 marks]
[5 markah]

CLO1 C2

(c) Prove that $coth^2 x - 1 = cosech^2 x$ by using definition of hyperbolic functions.

Buktikan bahawa $coth^2 x - 1 = cosech^2 x$ dengan menggunakan definisi fungsi hiperbolik.

[7 marks]
[*7 markah*]

CLO1 C2

(d) Complete the table below for equation $y = \frac{\cosh x}{3}$. Then sketch the graph in the range of $-3 \le x \le 3$.

Lengkapkan jadual di bawah bagi persamaan $y = \frac{\cosh x}{3}$. Seterusnya, lakarkan graf pada julat $-3 \le x \le 3$.

[6 marks]
[*6 markah*]

x	-3	-2	-1	0	1	2	3
y							

QUESTION 2
SOALAN 2

CLO1 C2

(a) Find the value of the following functions by using the definition of hyperbolic functions.

Cari nilai bagi fungsi-fungsi yang berikut dengan menggunakan definisi fungsi hiperbolik.

i. $\sinh(\ln 2.7)$

[2 marks]
[*2 markah*]

ii. $cosech\sqrt{2.5}$

[2 marks]
[*2 markah*]

iii. $\frac{\sinh 2.5}{\cosh(-0.54)}$

[3 marks]
[*3 markah*]

CLO1 C2

(b) Find the value for $sinh\, x$ if $2\, cosh\, x = 3$.

Cari nilai bagi $sinh\, x$ jika2 $cosh\, x = 3$.

[5 marks]
[5 markah]

CLO1 C2

(c) Complete the table below for the equation of $y = \operatorname{sech}\left(\frac{x}{2}\right)$ and sketch the graph.

Lengkapkan jadual di bawah bagi persamaan $y = \operatorname{sech}\left(\frac{x}{2}\right)$ dan lakarkan graf.

x	-4	-2	0	2	4
y					

[5 marks]
[*5 markah*]

CLO1 C3

(d) Sketch a quadrant graph and determine the principal value for each of the following functions given:

Lakarkan graf sukuan dan dapatkan nilai prinsip bagi setiap fungsi yang diberikan:

i. $\cos^{-1}(0.8557)$

[4 marks]
[4 markah]

ii. $\tan^{-1}(-1.75)$

[4 marks]
[4 markah]

SECTION B
BAHAGIAN B

QUESTION 3
SOALAN 3

CLO2 C3

(a) Differentiate the following functions in respect to x.

Bezakan setiap fungsi yang berikut terhadap x.

i. $y = \cot^{-1}(e^x)$

[3 marks]
[3 markah]

ii. $y = \operatorname{cosech}^{-1}(2x)$

[3 marks]
[3 markah]

iii. $y = x^2 \ln(\sinh x)$

[5 marks]
[5 markah]

iv. $y = \cosh^{-1} \sec(x)$

[5marks]
[5 markah]

v. $y = (\sinh x)(\coth x)$

[3 marks]
[3 markah]

CLO2 C3

(b) Prove that $\frac{d}{dx}(\cosh^{-1}(\cosh x)) = 1$

Buktikan bahawa $\frac{d}{dx}(\cosh^{-1}(\cosh x)) = 1$

[6 marks]
[6 markah]

QUESTION 4
SOALAN 4

CLO2 C3

(a) Determine the following integrals :

Tentukan setiap kamiran yang berikut:

i. $\int \frac{2x}{x^2+4} dx$

[4 marks]
[*4 markah*]

ii. $\int \frac{1}{(x+1)\sqrt{x^2+2x}} dx$

[6 marks]
[*6 markah*]

iii. $\int \sinh(2x) dx$

[3 marks]
[*3 markah*]

CLO2 C3

(b) Solve the following integrals given :

Selesaikan kamiran berikut:

i. $\int x^2 \ln(3x) dx$

[6 marks]
[*6 markah*]

ii. $\int \frac{x-1}{2x^2-x-3} dx$

[6 marks]
[*6 markah*]

SECTION C
BAHAGIAN C

QUESTION 5
SOALAN 5

CLO3 C3

(a) Form a differential equation for each of the following functions:

Bentukkan persamaan pembezaan bagi setiap fungsi yang berikut :

i. $y = Ax^3 + x^4$

[5 marks]
[*5 markah*]

ii. $y = Acos(3x + B)$

[3 marks]
[*3 markah*]

CLO3 C3

(b) Solve the following differential equations:

Selesaikan persamaan pembezaan berikut:

i. $y\sqrt{(1+x^2)}dy = -x\sqrt{(1+y^2)}\, dx$

[7 marks]
[*7 markah*]

ii. $x\frac{dy}{dx} - 5y = x^7$

[6marks]
[*6 markah*]

CLO3 C3

(c) Solve the second order differential equation below.

Selesaikan persamaan pembezaan peringkat kedua di bawah.

$$4\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$$

[4 marks]
[*4 markah*]

QUESTION 6
SOALAN 6

CLO3
C3

(a) Form a differential equation for $y = Ax + \frac{B}{x}$.

Bentukkan persamaan pembezaan bagi $y = Ax + \frac{B}{x}$.

[7 marks]
[*7 markah*]

CLO3
C3

(b) Solve the differential equation for $\frac{dy}{dx} = \frac{2y^3 - x^3}{3xy^2}$.

Selesaikan persamaan pembezaan bagi $\frac{dy}{dx} = \frac{2y^3 - x^3}{3xy^2}$.

[8 marks]
[*8 markah*]

CLO3
C3

(c) Solve the following second order of differential equations:
Selesaikan persamaan pembezaan peringkat kedua berikut:

i. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

[3 marks]
[*3 markah*]

ii. $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 41y = 0$

[7 marks]
[*7 markah*]

SOALAN TAMAT

HYPERBOLIC FUNCTIONS	**INVERSE HYPERBOLIC FUNCTIONS**
$\sinh x = \frac{e^x - e^{-x}}{2}$	$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right); -\infty < x < \infty$
$\cosh x = \frac{e^x + e^{-x}}{2}$	$\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right); x \geq 1$
$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$	$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right); \lvert x\rvert < 1$
$\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}}; x \neq 0$	$\coth^{-1} x = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right); \lvert x\rvert > 1$
$\text{sec}h\, x = \frac{2}{e^x + e^{-x}}$	$\text{sech}^{-1} x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right); 0 < x \leq 1$
$\text{cos}ech\, x = \frac{2}{e^x - e^{-x}}; x \neq 0$	$\text{cosech}^{-1} = \ln\left(\frac{1}{x} + \frac{\sqrt{1+x^2}}{\lvert x\rvert}\right); x \neq 0$

RECIPROCAL TRIGONOMETRIC IDENTITIES	**RECIPROCAL HYPERBOLIC IDENTITIES**
$\text{cos}ec\, x = \frac{1}{\sin x}$	$\text{cos}ech\, x = \frac{1}{\sinh x}$
$\sec x = \frac{1}{\cos x}$	$\text{sec}h\, x = \frac{1}{\cosh x}$
$\cot x = \frac{1}{\tan x}$	$\coth x = \frac{1}{\tanh x}$

TRIGONOMETRIC IDENTITIES		**HYPERBOLIC IDENTITIES**	
$\cos^2 x + \sin^2 x = 1$		$\cosh^2 x - \sinh^2 x = 1$	
$1 + \tan^2 x$	$= \sec^2 x$	$1 - \tanh^2 x$	$= \text{sech}^2 x$
$\cot^2 x + 1$	$= \text{cosec}^2 x$	$\coth^2 x - 1$	$= \text{cosech}^2 x$

$\sin 2x = 2\sin x\cos x$	$\sinh 2x = 2\sinh x\cosh x$
$\cos 2x = \cos^2 x - \sin^2 x$ $= 2\cos^2 x - 1$ $= 1 - 2\sin^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$ $= 2\cosh^2 x - 1$ $= 1 + 2\sinh^2 x$
$\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$	$\tanh 2x = \dfrac{2\tanh x}{1+\tanh^2 x}$
$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$	$\sinh(x \pm y) =$ $\sinh x\cosh y \pm \cosh x\sinh y$
$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$	$\cosh(x \pm y) = \cosh x\cosh y \pm \sinh x\sinh y$
$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x\tan y}$	$\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x\tanh y}$

BASIC OF DIFFERENTIATION	**BASIC OF INTEGRATION**
$\dfrac{d}{dx}(k) = 0$; k = constant	$\int k\,du = ku + C$; k = constant
$\dfrac{d}{dx}(u^n) = nu^{n-1}$	$\int u^n\,du = \dfrac{u^{n+1}}{n+1} + C$; $n \neq -1$
$\dfrac{d}{dx}(\ln\|u\|) = \dfrac{1}{u}\cdot\dfrac{du}{dx}$	$\int \dfrac{1}{u}\,du = \dfrac{\ln\|u\|}{u'} + C$
$\dfrac{d}{dx}(e^u) = e^u\cdot\dfrac{du}{dx}$	$\int e^u\,du = \dfrac{e^u}{u'} + C$

DIFFERENTIATION OF TRIGONOMETRIC FUNCTIONS	**INTEGRATION OF TRIGONOMETRIC FUNCTIONS**
$\dfrac{d}{dx}(\cos u) = -\sin u.\dfrac{du}{dx}$	$\int \sin u\,du = \dfrac{-\cos u}{u'} + C$
$\dfrac{d}{dx}(\sin u) = \cos u.\dfrac{du}{dx}$	$\int \cos u\,du = \dfrac{\sin u}{u'} + C$
$\dfrac{d}{dx}(\tan u) = \sec^2 u.\dfrac{du}{dx}$	$\int \sec^2 u\,du = \dfrac{\tan u}{u'} + C$
$\dfrac{d}{dx}(\cot u) = -\mathrm{cosec}^2 u.\dfrac{du}{dx}$	$\int \mathrm{cosec}^2 u\,du = \dfrac{-\cot u}{u'} + C$
$\dfrac{d}{dx}(\sec u) = \sec u.\tan u.\dfrac{du}{dx}$	$\int \sec u\tan u\,du = \dfrac{\sec u}{u'} + C$
$\dfrac{d}{dx}(\mathrm{cosec}\, u) = -\mathrm{cosec}\, u.\cot u.\dfrac{du}{dx}$	$\int \mathrm{cosec}\, u\cot u\,du = \dfrac{-\mathrm{cosec}\, u}{u'} + C$

DIFFERENTIATION OF HYPERBOLIC FUNCTIONS	**INTEGRATION OF HYPERBOLIC FUNCTIONS**
$\dfrac{d}{dx}(\cosh u) = \sinh u.\dfrac{du}{dx}$	$\int \sinh u\,du = \dfrac{\cosh u}{u'} + C$
$\dfrac{d}{dx}(\sinh u) = \cosh u.\dfrac{du}{dx}$	$\int \cosh u\,du = \dfrac{\sinh u}{u'} + C$
$\dfrac{d}{dx}(\tanh u) = \mathrm{sech}^2 u.\dfrac{du}{dx}$	$\int \mathrm{sech}^2 u\,du = \dfrac{\tanh u}{u'} + C$
$\dfrac{d}{dx}(\coth u) = -\mathrm{cosech}^2 u.\dfrac{du}{dx}$	$\int \mathrm{cosech}^2 u\,du = \dfrac{-\coth u}{u'} + C$
$\dfrac{d}{dx}(\mathrm{sech}\, u) = -\mathrm{sech}\, u.\tanh u.\dfrac{du}{dx}$	$\int \mathrm{sech}\, u\tanh u\,du = \dfrac{-\mathrm{sech}\, u}{u'} + C$
$\dfrac{d}{dx}(\mathrm{cosech}\, u) = -\mathrm{cosech}\, u.\coth u.\dfrac{du}{dx}$	$\int \mathrm{cosech}\, u\coth u\,du = \dfrac{-\mathrm{cosech}\, u}{u'} + C$

DIFFERENTIATION OF INVERSE TRYGONOMETRIC FUNCTIONS	**INTEGRATION OF INVERSE TRYGONOMETRIC FUNCTION**
$\dfrac{d}{dx}(\sin^{-1} u) = \dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}$, $\|u\| < 1$	$\int \dfrac{1}{\sqrt{a^2-u^2}}du = \sin^{-1}\dfrac{u}{a} + C$, $\|u\| < a$
$\dfrac{d}{dx}(\cos^{-1} u) = -\dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}$, $\|u\| < 1$	$\int -\dfrac{1}{\sqrt{a^2-u^2}}du = \cos^{-1}\dfrac{u}{a} + C$, $\|u\| < a$
$\dfrac{d}{dx}(\tan^{-1} u) = \dfrac{1}{1+u^2}\dfrac{du}{dx}$	$\int \dfrac{1}{a^2+u^2}du = \dfrac{1}{a}\tan^{-1}\dfrac{u}{a} + C$
$\dfrac{d}{dx}(\cot^{-1} u) = -\dfrac{1}{1+u^2}\dfrac{du}{dx}$	$\int -\dfrac{1}{a^2+u^2}du = \dfrac{1}{a}\cot^{-1}\dfrac{u}{a} + C$
$\dfrac{d}{dx}(\sec^{-1} u) = \dfrac{1}{\|u\|\sqrt{u^2-1}}\dfrac{du}{dx}$, $\|u\| > 1$	$\int \dfrac{1}{\|u\|\sqrt{u^2-a^2}}du = \dfrac{1}{a}\sec^{-1}\dfrac{u}{a} + C$, $\|u\| > a$

$\frac{d}{dx}\left(\operatorname{cosec}^{-1} u\right) = -\frac{1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx},\ \lvert u\rvert > 1$	$\int -\frac{1}{\lvert u\rvert\sqrt{u^2-a^2}}d = \frac{1}{a}\operatorname{cosec}^{-1}\frac{u}{a} + C,\ \lvert u\rvert > a$

DIFFERENTIATION OF INVERSE HYPERBOLIC FUNCTIONS	INTEGRATION OF INVERSE HYPERBOLIC FUNCTIONS
$\frac{d}{dx}\left(\sinh^{-1} u\right) = \frac{1}{\sqrt{u^2+1}}\frac{du}{dx}$	$\int \frac{1}{\sqrt{a^2+u^2}}d = \sinh^{-1}\frac{u}{a} + C,\ a > 0$
$\frac{d}{dx}\left(\cosh^{-1} u\right) = \frac{1}{\sqrt{u^2-1}}\frac{du}{dx},\ \lvert u\rvert > 1$	$\int \frac{1}{\sqrt{u^2-a^2}}d = \cosh^{-1}\frac{u}{a} + C,\ u > a$
$\frac{d}{dx}\left(\tanh^{-1} u\right) = \frac{1}{1-u^2}\frac{du}{dx},\ \lvert u\rvert < 1$	$\int \frac{1}{a^2-u^2}du = \frac{1}{a}\tanh^{-1}\frac{u}{a} + C;\ \lvert u\rvert < a$
$\frac{d}{dx}\left(\coth^{-1} u\right) = \frac{1}{1-u^2}\frac{du}{dx}, \lvert u\rvert > 1$	$\int \frac{1}{u^2-a^2}du = \frac{1}{a}\coth^{-1}\frac{u}{a} + C;\ \lvert u\rvert > a$
$\frac{d}{dx}\left(\operatorname{sech}^{-1} u\right) = -\frac{1}{u\sqrt{1-u^2}}\frac{du}{dx},\ 0 < u < 1$	$\int \frac{1}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\frac{u}{a} + C$
$\frac{d}{dx}\left(\operatorname{cosech}^{-1} u\right) = -\frac{1}{\lvert u\rvert\sqrt{1+u^2}}\frac{du}{dx},\ u \neq 0$	$\int \frac{1}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\operatorname{cosech}^{-1}\frac{u}{a} + C$

INTERGRALS INVOLVING QUADRATIC EXPRESSION

Completing the square

$$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

SOLUTION FOR 1st ORDER DIFFERENTIAL EQUATION

Homogeneous Equations	Linear Factors (Integrating Factors)
• Substitution $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$	$y \bullet IF = \int Q \bullet IF\, dx$ Where $IF = e^{\int P dx}$
	Logarithmic $a = e^{\ln a}$ $a^x = e^{x\ln a}$

GENERAL SOLUTION FOR 2nd ORDER DIFFERENTIAL EQUATION

Equation of the form $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$

1. Real & different roots: $y = Ae^{m_1 x} + Be^{m_2 x}$
2. Real & equal roots: $y = e^{mx}(A + Bx)$
3. Complex roots: $y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

ROOTS OF QUADRATIC EQUATIONS

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$