NETWORK ANALYSIS

QUESTION & ANSWER

YAAKUB BIN OMAR
WAN MOHD ZAMRI BIN WAN AB RAHMAN

NETWORK ANALYSIS
Question and Answer

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NETWORK ANALYSIS
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Preface

NETWORK ANALYSIS is a fundamental course that involves the process of finding the voltages across, and the currents through, every component in the network with different techniques for calculating these values. It provides exposure and knowledge to the student in solving alternating current (AC) electrical network circuit problem. The emphasis of this course is on using basic circuit theorems, Laplace Transform, Fourier Series and Two Port Network Parameters.

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Chapter 1

1.0 Circuit Analysis

1.1 The application of network analysis theorems to AC circuits:

- 1.1.1 Mesh Analysis
- 1.1.2 Nodal Analysis
- 1.1.3 Superposition Theorem
- 1.1.4 Thevenin’s Theorems
- 1.1.5 Norton’s Theorems

1.1.1 Mesh Analysis

Question 1

Analyze alternating current (AC) circuit for the Diagram 1. Determine the current through the resistor *8*Ω using the Mesh Analysis.

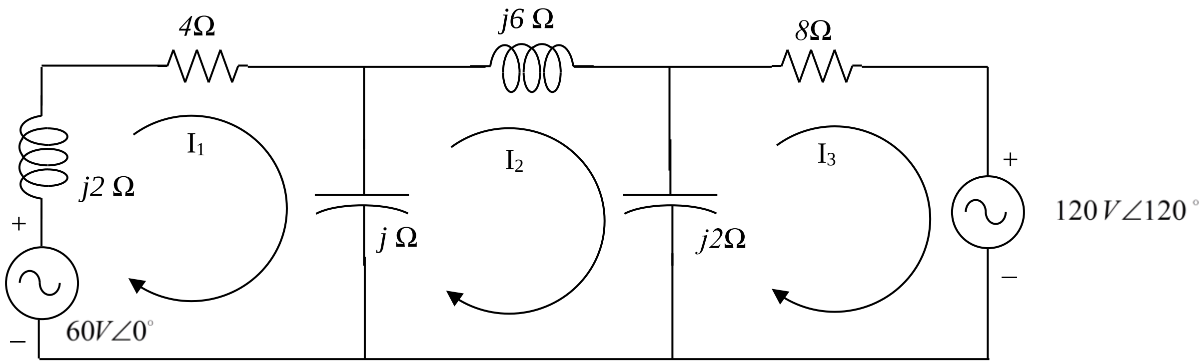


Diagram 1

Answer:

Loop 1

$$(4+j2)I_1-(-j)I_2+0I_3=60\angle 0^\circ$$

Loop 2

$$-(-j)I_1+(j6-j-j2)I_2-(-j2)I_3=0$$

Loop 3

$$0I_1-(-j2)I_2+(8-j2)I_3=-120\angle 120^\circ$$

Equations in the form of Matrix :

$$\begin{bmatrix} 4+j2 & j & 0 \\ j & j3 & j2 \\ 0 & j2 & 8-j2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 60\angle 0^\circ \\ 0 \\ 120\angle 300^\circ \end{bmatrix}$$

Value of Delta :

$$\Delta=\begin{bmatrix} 4+j2 & j & 0 \\ j & j3 & j2 \\ 0 & j2 & 8-j2 \end{bmatrix}\begin{vmatrix} 4+j2 & j \\ j & j3 \\ 0 & j2 \end{vmatrix}$$

$$=[(4+j2)(j3)(8-j2)+(j)(j2)(0)+(0)(j)(j2)]-[(0)(j3)(0)+(j2)(j2)(4+j2)+(8-j2)(j)(j)]$$
$$=(24+j108)-(-24-j6)$$

$$=0+j114$$

Value of Delta I3:

$$\Delta I_3=\begin{bmatrix} 4+j2 & j & 60\angle 0^\circ \\ j & j3 & 0 \\ 0 & j2 & 120\angle 300^\circ \end{bmatrix}\begin{vmatrix} 4+j2 & j \\ j & j3 \\ 0 & j2 \end{vmatrix}$$

$$=[(4+j2)(j3)(120\angle 300^\circ)+(j)(0)(0)+(60\angle 0^\circ)(j)(j2)]-$$

$$[(0)(j3)(60\angle 0^\circ)+(j2)(0)(4+j2)+(120\angle 300^\circ)(j)(j)]$$

$$=(767.07658+j1343.53829)-(-60+j103.923)$$

$$=827.07658+j1239.615242$$

Value of current I3

$$I_3=\frac{\Delta I_3}{\Delta}=\frac{827.07658+j1239.6152}{0+j114}$$

$$=13.0719A\angle -33.7114^\circ$$

Question 2

Diagram 2 shows an alternating current (AC) circuit which consists of several components such as resistors, capacitors and inductor. The circuit has two voltage source of 100Volt with a phase angle of 90º and 220V with a phase angle of 0º. By using **mesh analysis** and **Maxwell's equation**, calculate the value of the current $\boldsymbol{I_3}$.

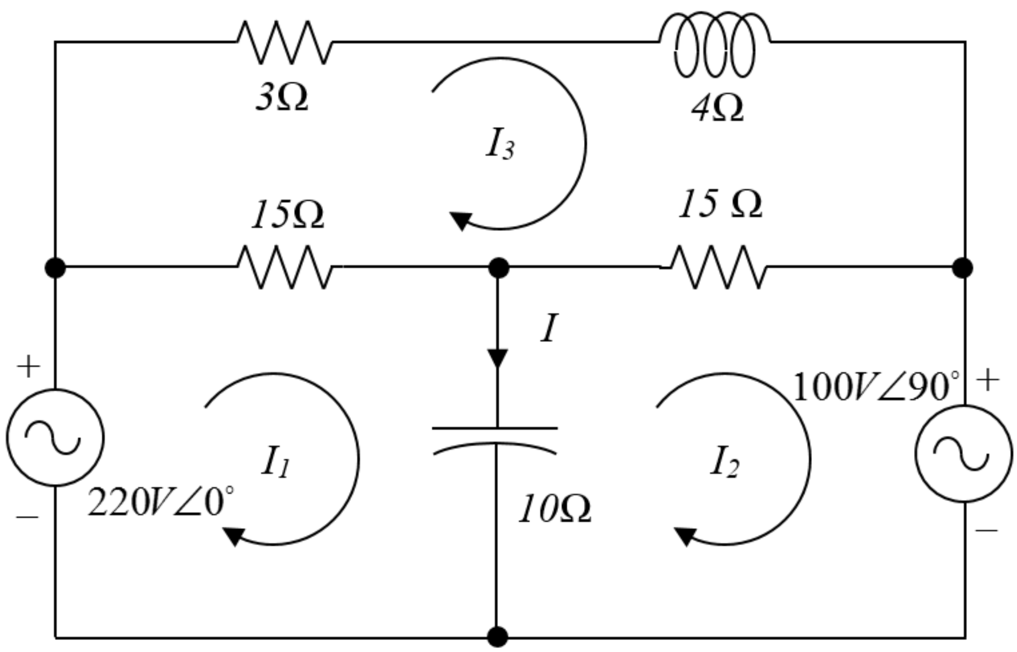


Diagram 2

Answer:

Applying KVL to mesh 1, we obtain:

$$(5 - j10)I_1 - (-j10)I_2 - (15)I_3 = 220\angle 0^\circ$$

Applying KVL to mesh 2, we obtain:

$$-(-j10)I_1 + (15 - j10)I_2 - (15)I_3 = -100\angle 90^\circ$$

Applying KVL to mesh 3, we obtain:

$$-(15)I_1 - (15)I_2 + (33 + j4)I_3 = 0$$

From equation 1, 2 and 3, we use Maxwell's equation to get current I_1, I_2 and I_3.

$$\begin{bmatrix} 15-j10 & j10 & -15 \\ j10 & 15-j10 & -15 \\ -15 & -15 & 33+j4 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 220\angle 0^\circ \\ -100\angle 90^\circ \\ 0 \end{bmatrix}$$

$$\Delta = \left|\begin{matrix} 15-j10 & j10 & -15 \\ j10 & 15-j10 & -15 \\ -15 & -15 & 33+j4 \end{matrix}\right| \begin{matrix} 15-j10 & j10 \\ j10 & 15-j10 \\ -15 & -15 \end{matrix}\Bigg|$$

$$= (15-j10)(15-j10)(33+j4)+(j10)(-15)(-15)+(-15)(j10)(-15)$$
$$-[(-15)(15-j10)(-15)+(-15)(-15)(15-j10)+(33+j4)(j10)(j10)]$$

$$= (5325-j4900)-(3450-j4900)$$

$$= 1875+j0 \quad atau \quad 1875\angle 0^\circ$$

$$\Delta I_3 = \left|\begin{matrix} 15-j10 & j10 & 220\angle 0^\circ \\ j10 & 15-j10 & 100\angle 270^\circ \\ -15 & -15 & 0 \end{matrix}\right| \begin{matrix} 15-j10 & j10 \\ j10 & 15-j10 \\ -15 & -15 \end{matrix}\Bigg|$$

$$= (15-j10)(15-j10)(0)+(j10)(100\angle 270^\circ)(-15)+(220\angle 0^\circ)(j10)(-15)$$
$$-[(-15)(15-j10)(220\angle 0^\circ)+(-15)(100\angle 270^\circ)(15-j10)+(0)(j10)(j10)]$$

$$= (-15000-j33000)-(-34500+j55500)$$

$$= 19500-j88500 \quad atau \quad 90622.8448\angle -77.5741^\circ$$

$$I_3 = \frac{\Delta I_3}{\Delta} = \frac{19500-j88500}{1875\angle 0^\circ}$$

$$I_3 = 10.4-j47.2 \quad atau \quad 48.33\angle -77.57^\circ$$

Question 3

Diagram 3, shows an alternating current circuit (AC). This circuit has two voltage source $E_1 = 20V\angle 0^\circ$ and $E_2 = 40V\angle 60^\circ$. By using **Mesh analysis,** determine the current through the resistor ***R₁***.

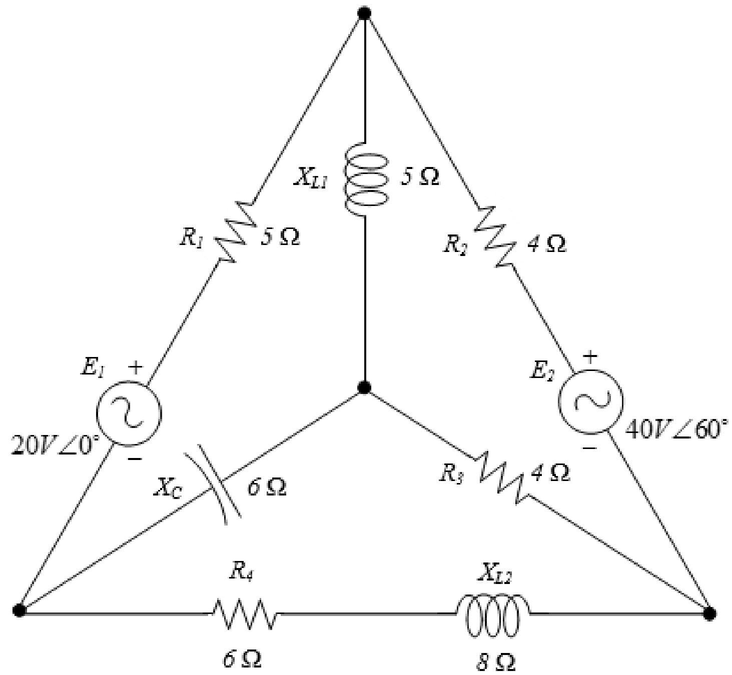
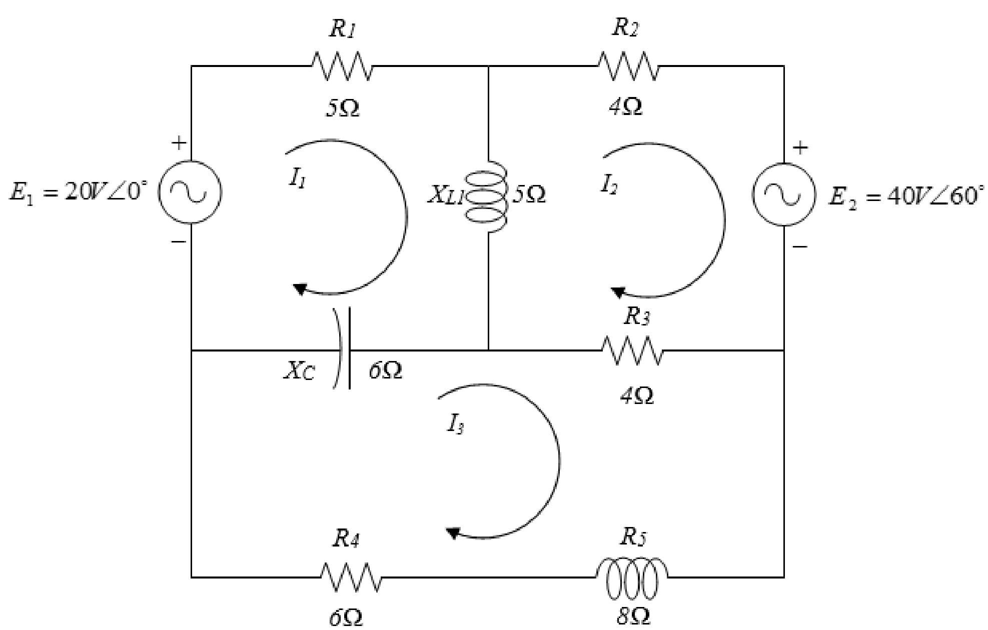


Diagram 3

Answer:



Applying KVL to mesh 1, we obtain:

$$(5-j)I_1-(j5)I_2+(j6)I_3=20\angle 0^\circ$$

Applying KVL to mesh 2, we obtain:

$$-(j5)I_1+(8+j5)I_2-(4)I_3=40\angle 240^\circ$$

Applying KVL to mesh 3, we obtain:

$$(j6)I_1-(4)I_2+(10+j2)I_3=0$$

From equation 1, 2 and 3, we use Maxwell's equation to get current I_1, I_2 and I_3.

$$\begin{bmatrix} 5-j & -j5 & j6 \\ -j5 & 8+j5 & -4 \\ j6 & -4 & 10+j2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}=\begin{bmatrix} 20\angle 0^\circ \\ 40\angle 240^\circ \\ 0 \end{bmatrix}$$

$$\Delta=\left|\begin{bmatrix} 5-j & -j5 & j6 \\ -j5 & 8+j5 & -4 \\ j6 & -4 & 10+j2 \end{bmatrix}\begin{matrix} 5-j & -j5 \\ -j5 & 8+j5 \\ j6 & -4 \end{matrix}\right|$$

$$=\left[(5-j)(8+j5)(10+j2)+(-j5)(-4)(j6)+(j6)(-j5)(-4)\right]$$
$$-[(j6)(8+j5)(j6)+(-4)(-4)(5-j)+(10+j2)(-j5)(-j5)]$$

$$=(176-j260)-(-458-j246)$$

$$=634+j506$$

$$\Delta I_1=\left|\begin{matrix} 20\angle 0^\circ & -j5 & j6 \\ 40\angle 240^\circ & 8+j5 & -4 \\ 0 & -4 & 10+j2 \end{matrix}\right|\begin{matrix} 20\angle 0^\circ & -j5 \\ 40\angle 240^\circ & 8+j5 \\ 0 & -4 \end{matrix}\Bigg|$$

$$=\left[(20\angle 0^\circ)(8+j5)(10+j2)+(-j5)(-4)(0)+(j6)(40\angle 240^\circ)(-4)\right]$$
$$-[(0)(8+j5)(j6)+(-4)(-4)(20\angle 0^\circ)+(10+j2)(40\angle 240^\circ)(-j5)]$$

$$=(568.616+j1800)-(-1612.0508+j653.5898)$$

$$=2180.6668+j1146.4102$$

$$I_1 = \frac{\Delta I_1}{\Delta} = \frac{2180.6668 + j1146.4102}{634 + j506}$$

$$= (568.616 + j1800) - (-1612.0508 + j653.5898)$$

$$= 2180.6668 + j1146.4102$$

Question 4

Diagram 4 shows an alternating current circuit (AC). This circuit has two voltage source $E_1 = 40V\angle 60^\circ$ and $E_2 = 20V\angle 0^\circ$. By using **Mesh analysis, evaluate** the current through the resistor ***R₁***.

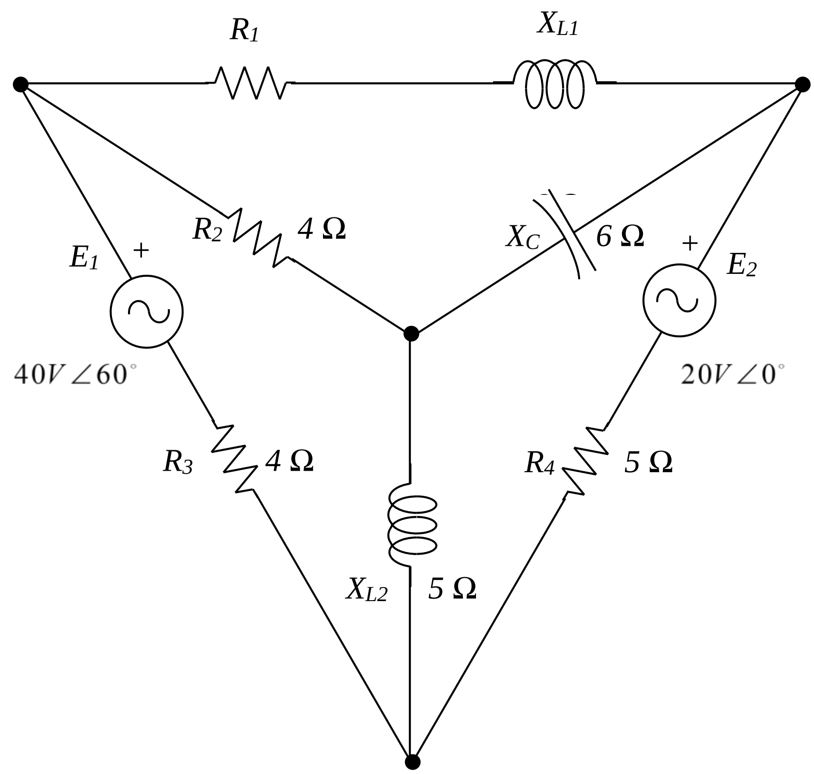
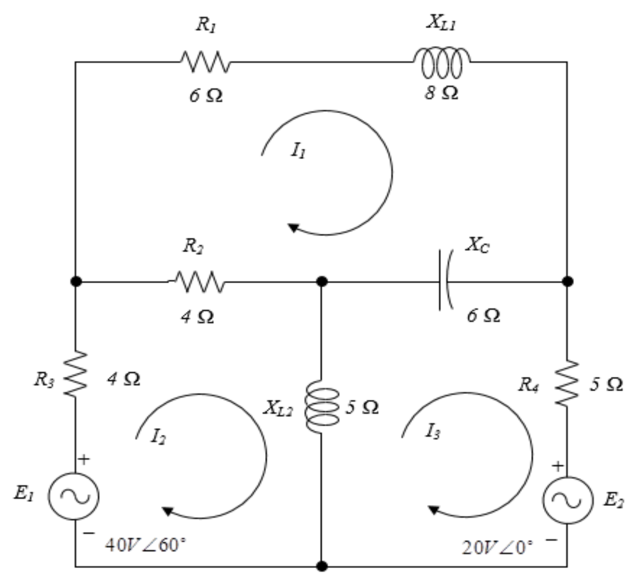


Diagram 4

Answer:



Applying KVL to mesh 1, we obtain:

$$(6+4+j8-j6)I_1-(4)I_2-(-j6)I_3=0$$

$$(10+j2)I_1-(4)I_2+(j6)I_3=0$$

Applying KVL to mesh 2, we obtain:

$$-(4)I_1+(4+4+j5)I_2-(j5)I_3-40\angle 60^\circ=0$$

$$-(4)I_1+(8+j5)I_2-(j5)I_3=40\angle 60^\circ$$

Applying KVL to mesh 3, we obtain:

$$-(-j6)I_1-(j5)I_2-(5+j5-j6)I_3+20\angle 0^\circ=0$$

$$(j6)I_1-(j5)I_2+(5-j)I_3=20\angle 180^\circ$$

From equation 1, 2 and 3, we use Maxwell's equation to get current I_1, I_2 and I_3.

$$\begin{bmatrix} 10+j2 & -4 & j6 \\ -4 & 8+j5 & -j5 \\ j6 & -j5 & 5-j \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 40\angle 60^\circ \\ 20\angle 180^\circ \end{bmatrix}$$

Value Δ

$$\Delta = \begin{vmatrix} 10+j2 & -4 & j6 & 10+j2 & -4 \\ -4 & 8+j5 & -j5 & -4 & 8+j5 \\ j6 & -j5 & 5-j & j6 & -j5 \end{vmatrix}$$

$$= \left[(10+j2)(8+j5)(5-j) + (-4)(-j5)(j6) + (j6)(-4)(-j5)\right]$$
$$- \left[(j6)(8+j5)(j6) + (-j5)(-j5)(10+j2) + (5-j)(-4)(-4)\right]$$

$$= (176 - j260) - (-458 - j246)$$

$$= 634 + j506$$

Value ΔI_1

$$\Delta I_1 = \begin{vmatrix} 0 & -4 & j6 & 0 & -4 \\ 40\angle 60^\circ & 8+j5 & -j5 & 40\angle 60^\circ & 8+j5 \\ 20\angle 180^\circ & -j5 & 5-j & 20\angle 180^\circ & -j5 \end{vmatrix}$$

$$= \left[(0)(8+j5)(5-j) + (-4)(-j5)(20\angle 180^\circ) + (j6)(40\angle 60^\circ)(-j5)\right]$$
$$- \left[(20\angle 180^\circ)(8+j5)(j6) + (-j5)(-j5)(0) + (5-j)(40\angle 60^\circ)(-4)\right]$$

$$= (600 + j639.2304845) - (61.4359 - j1572.8203)$$

or

$$= (876.7073\angle 46.8132^\circ) - (1574.01974\angle -87.76311^\circ)$$

$$= 538.5641 + j2212.0508$$

or

$$= 2276.668625\angle 76.31652^\circ$$

$$I_1 = \frac{538.5641 + j2212.0508}{634 + j506}$$

$$I_1 = 2.22001 + j1.717234$$

or

$$= 2.8066A\angle 37.7229^\circ$$

Question 5

Diagram 5 shows an alternating current circuit having voltage source $E_1 = 12V\angle 64^\circ$, is connected with a number of electronic passive components. Evaluate alternating current (AC) circuit, evaluate the current through the resistor $R_2 = 10\ \Omega$ using the Mesh Analysis.

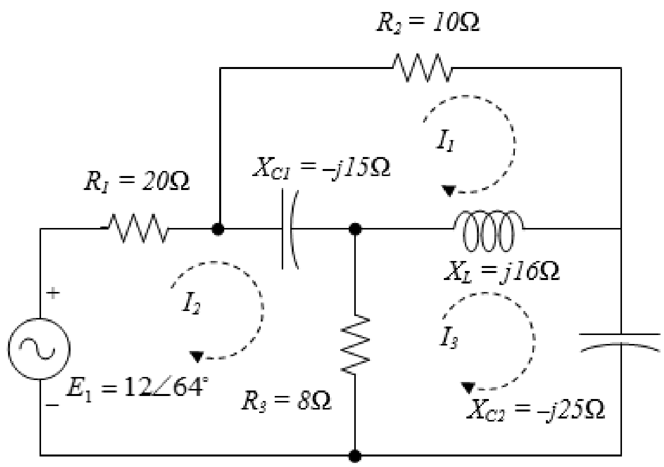


Diagram 5

Answer:

Applying KVL to mesh 1, we obtain:

$$(28 - j15)I_1 - (8)I_2 + (j15)I_3 = 12\angle 64^\circ$$

Applying KVL to mesh 2, we obtain:

$$-(8)I_1 + (8 - j9)I_2 - (j16)I_3 = 0$$

Applying KVL to mesh 3, we obtain:

$$(j15)I_1 - (j16)I_2 + (10 + j)I_3 = 0$$

From equation 1, 2 and 3, we use Maxwell's equation to get current I_1, I_2 and I_3.

$$\begin{bmatrix} 28-j15 & -8 & j15 \\ -8 & 8-j9 & -j16 \\ j15 & -j16 & 10+j \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 12\angle 64^\circ \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = \left| \begin{matrix} 28-j15 & -8 & j15 \\ -8 & 8-j9 & -j16 \\ j15 & -j16 & 10+j \end{matrix} \right| \begin{matrix} 28-j15 & -8 \\ -8 & 8-j9 \\ j15 & -j16 \end{matrix} \Bigg|$$

$$= \left[(28-j15)(8-j9)(10+j)+(-8)(-j16)(j15)+(j15)(-8)(-j16)\right]$$
$$-[(j15)(8-j9)(j15)+(-j16)(-j16)(28-j15)+(10+j)(-8)(-8)]$$

$$\Delta = (-2578-j3631)-(-8328-j5929)$$

atau

$$\Delta = (4453.1163\angle -125.3746)-(10222.946\angle 144.5516)$$

$$\Delta = 5750-j9560$$

atau

$$\Delta = 11.1550k\angle -58.9746^\circ$$

$$\Delta I_3 = \left| \begin{matrix} 12\angle 64^\circ & -8 & j15 \\ 0 & 8-j9 & -j16 \\ 0 & -j16 & 10+j \end{matrix} \right| \begin{matrix} 12\angle 64^\circ & -8 \\ 0 & 8-j9 \\ 0 & -j16 \end{matrix} \Bigg|$$

$$= \left[(12\angle 64^\circ)(8-j9)(10+j)+(-8)(-j16)(0)+(j15)(0)(-j16)\right]$$
$$-[(0)(8-j9)(j15)+(-j16)(-j16)(12\angle 64^\circ)+(10+j)(0)(-8)]$$

$$\Delta I_1 = (1352.5937+j528.5548)-(-1346.6762-j2761.0953)$$

atau

$$\Delta I_1 = (1452.1983\angle 21.3441^\circ)-(3072\angle -116^\circ)$$

$$\Delta I_1 = 2699.2699 + j3289.2699$$

atau

$$\Delta I_1 = 4255.3327\angle 50.6299^\circ$$

$$I_1 = \frac{\Delta I_1}{\Delta} = \frac{2699.2699 + j3289.2699}{5750 - j9560}$$

$$I_1 = 1 - 0.127982 + j0.3593$$

atau

$$I_1 = 0.3814\angle 109.6045^\circ$$

1.1.2 Nodal Analysis

Question1

Analyze the alternating current (AC) circuit shown in Diagram 1. By using **Nodal analysis**, **determine** the nodal voltage V_1.

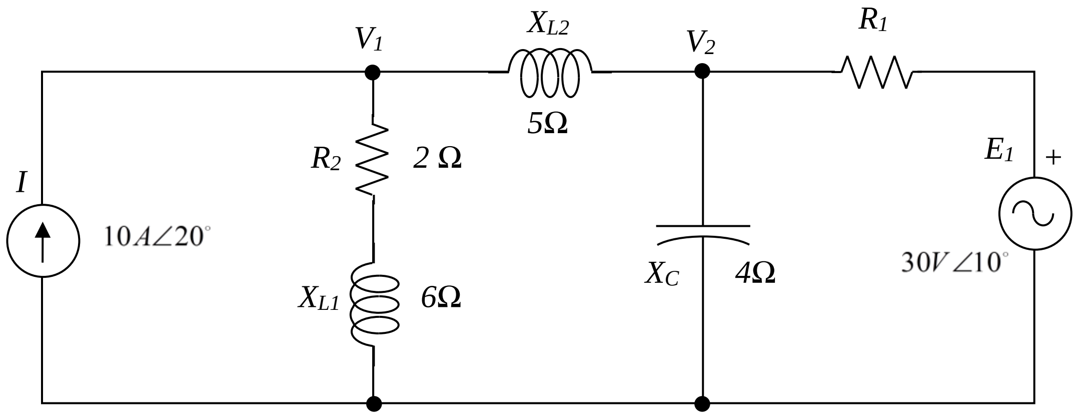
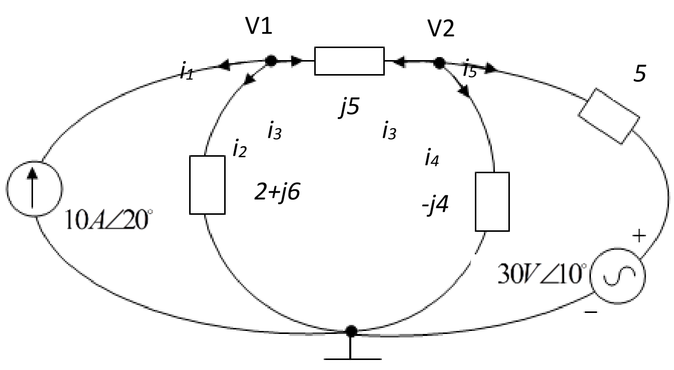


Diagram 1

Answer:



For nood V1

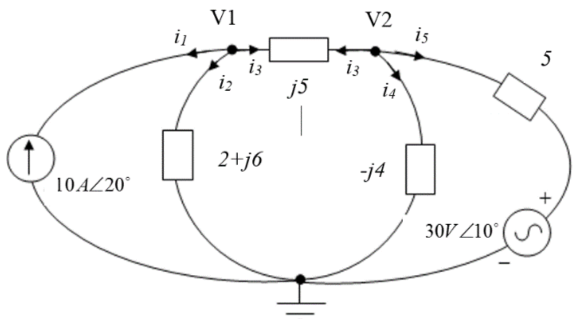
$$i_1 + i_2 + i_3 = 0$$

$$-10\angle 20^{\circ} + \frac{V_1}{2 + j6} + \frac{V_1 - V_2}{j5} = 0$$

$$\left(\frac{1}{2+j6}+\frac{1}{j5}\right)V1-\left(\frac{1}{j5}\right)V2=10\angle 20^{\circ}$$

$$(0.05-j0.35)V1+(0+j0.2)V2=10\angle 20^{\circ}$$

$$(0.3536\angle -81.8699^{\circ})V1+(0.2\angle 90^{\circ})V2=10\angle 20^{\circ}$$



For nood V2

$$i_3+i_4+i_5=0$$

$$\frac{V_2-V_1}{j5}+\frac{V_2}{-j4}+\frac{V_2-(30\angle 10^{\circ})}{5}=0$$

$$-\left(\frac{1}{j5}\right)V_1+\left(\frac{1}{j5}-\frac{1}{j4}+\frac{1}{5}\right)V_2=\frac{30\angle 10^{\circ}}{5}$$

$$(0+j0.2)V_1+(0.2+j0.05)V_2=6\angle 10^{\circ}$$

$$(0.2\angle 90^{\circ})V_1+(0.2062\angle 14.0362^{\circ})V_2=6\angle 10^{\circ}$$

Using cramer rule :

$$\begin{bmatrix}0.05-j0.35 & 0+j0.2\\ 0+j0.2 & 0.2+j0.05\end{bmatrix}\begin{bmatrix}V_1\\ V_2\end{bmatrix}=\begin{bmatrix}10\angle 20^{\circ}\\ 6\angle 10^{\circ}\end{bmatrix}$$

$$\Delta=\begin{bmatrix}0.05-j0.35 & 0+j0.2\\ 0+j0.2 & 0.2+j0.05\end{bmatrix}$$

$$= \left[(0.05 - j0.35)(0.2 + j0.05) \right] - \left[(0 + j0.2)(0 + j0.2) \right]$$

$$= 0.0675 - j0.0675$$

$$= 0.0955 \angle -45^{\circ}$$

$$\Delta V_1 = \begin{bmatrix} 10\angle 20^{\circ} & 0 + j0.2 \\ 6\angle 10^{\circ} & 0.2 + j0.05 \end{bmatrix}$$

$$= \left[(10\angle 20^{\circ})(0.2 + j0.05) \right] - \left[(6\angle 10^{\circ})(0 + j0.2) \right]$$

$$= 1.9168 - j0.0279$$

$$= 1.9170 \angle -0.8334^{\circ}$$

$$V_1 = \frac{\Delta V_1}{\Delta} = \frac{1.9170\angle -0.8334^{\circ}}{0.0955\angle -45^{\circ}}$$

$$V_1 = 14.3986 - j13.9857$$

atau

$$V_1 = 20.0733\angle 44.1666^{\circ}$$

Question2

Diagram 2 shows an alternating current circuit with the connection of several components resistors, capacitors and inductors. This circuit having a voltage source $E_1 = 30V\angle 50^{\circ}$ and current source $I = 0.04V\angle 20^{\circ}$. By using N**ode Analysis, evaluate** the value of the voltage at node V_1.

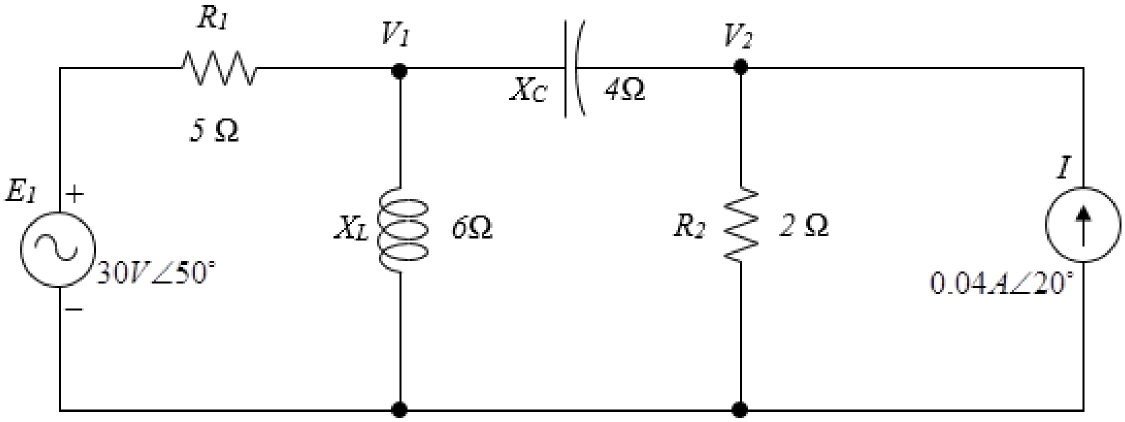
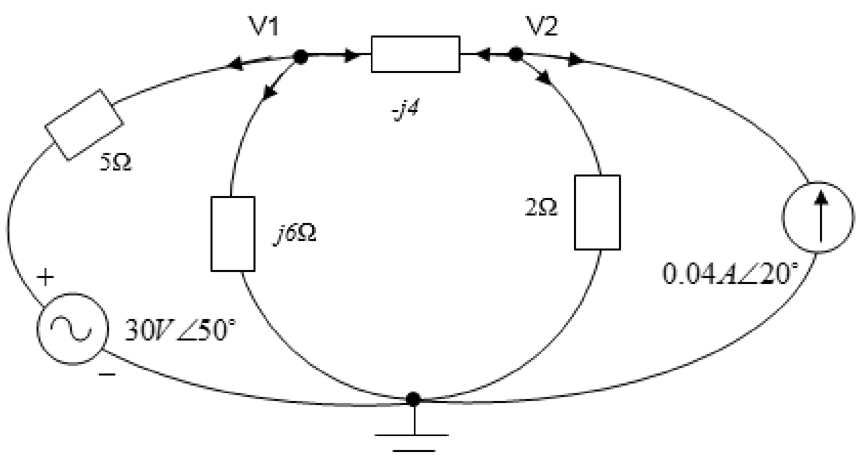


Diagram 2

Answer:



Nilai Voltan V_1

For nood V1

$$i_1 + i_2 + i_3 = 0$$

$$\frac{V_1 - E}{5} + \frac{1}{j6} + \frac{V_1 - V_2}{-j4} = 0$$

$$\left(\frac{1}{5}+\frac{1}{j6}+\frac{1}{-j4}\right)V1-\left(\frac{1}{-j4}\right)V2=\frac{30\angle 50^\circ}{5}$$

$$(0.2+j0.08333)V1+(0-j0.25)V2=6\angle 50^\circ$$

$$(0.2167\angle 22.61986^\circ)V1+(0.25\angle -90^\circ)V2=6\angle 50^\circ$$

For nood V2

$$i_3+i_4+i_5=0$$

$$\frac{V_2-V_1}{-j4}+\frac{V_2}{2}-0.04\angle 20^\circ=0$$

$$\left(\frac{1}{j4}\right)V_1+\left(\frac{1}{2}-\frac{1}{j4}\right)V_2=0.04\angle 20^\circ$$

$$(0-j0.25)V_1+(0.5+j0.25)V_2=0.04\angle 20^\circ$$

$$(0.25\angle -90^\circ)V_1+(0.5590\angle 26.5651^\circ)V_2=0.04\angle 20^\circ$$

Using cramer rule :

$$\begin{bmatrix}0.2+j0.0833 & 0-j0.25\\ 0-j0.25 & 0.5+j0.25\end{bmatrix}\begin{bmatrix}V_1\\ V_2\end{bmatrix}=\begin{bmatrix}6\angle 50^\circ\\ 0.04\angle 20^\circ\end{bmatrix}$$

$$\Delta=\begin{bmatrix}0.2+j0.0833 & 0-j0.25\\ 0-j0.25 & 0.5+j0.25\end{bmatrix}$$

$$=[(0.2+j0.0833)(0.5-j0.25)]-[(0-j0.25)(0-j0.25)]$$

$$=(0.079175+j0.091665)-(-0.0625+j0)$$

$$=0.1416675+j0.091665$$

$$=0.1687369\angle 32.90461407^\circ$$

$$\Delta V_1 = \begin{bmatrix} 6\angle 50^\circ & 0 - j0.25 \\ 0.04\angle 20^\circ & 0.5 + j0.25 \end{bmatrix}$$

$$= \left[(6\angle 50^\circ)(0.5 + j0.25)\right] - \left[(0.04\angle 20^\circ)(0 - j0.25)\right]$$

$$= (0.779296 + j3.2623) - (0.00342 - j0.0093969)$$

$$= \left(16.77051\angle 76.5651^\circ\right) - (0.01\angle -70^\circ)$$

$$= 0.77588 + j3.27171$$

$$= 3.36245\angle 76.6589^\circ$$

$$V_1 = \frac{\Delta V_1}{\Delta} = \frac{3.36245\angle 76.6589^\circ}{0.1687\angle 32.9046^\circ}$$

$$V_1 = 14.396789 + j13.78401$$

atau

$$V_1 = 19.93154\angle 43.7543^\circ$$

Question 3

Diagram 3 shows an alternating current circuit with the connection of several components resistors, capacitors and inductors. This circuit has a 40V voltage source and a source current of 5A with a phase angle of 30º. By using **node analysis,** calculate the value of the voltage at node V_1.

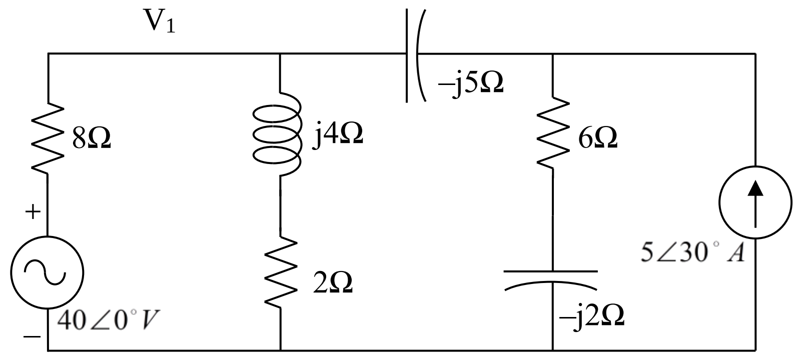
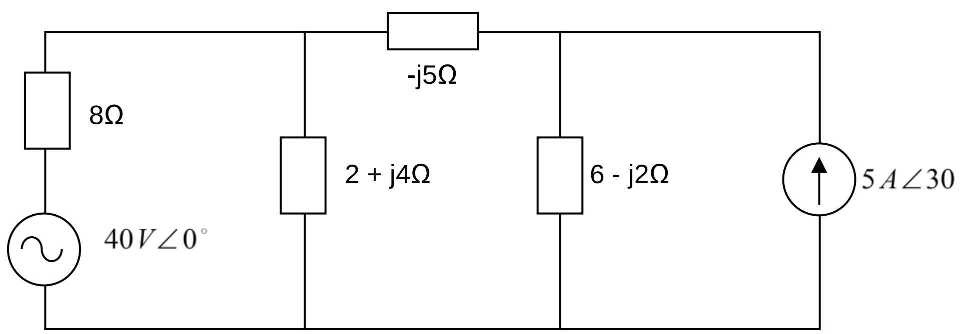
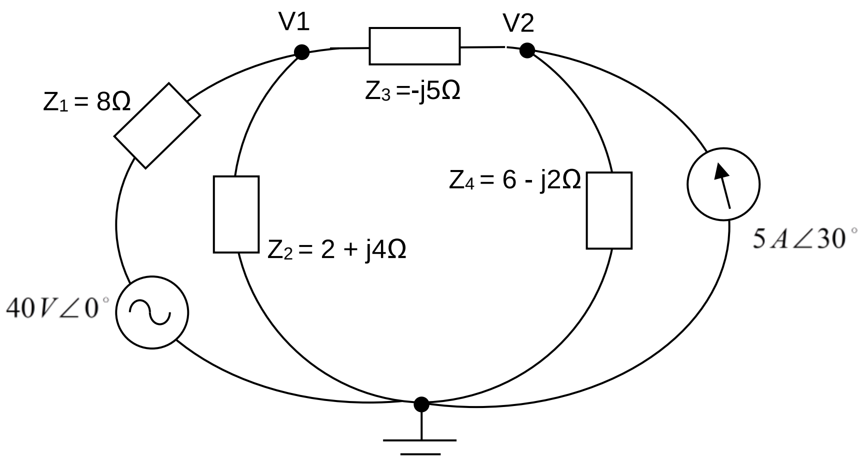


Diagram 3

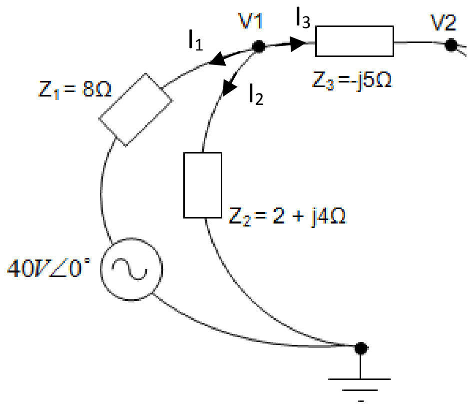
Answer:





Apply KCL at point V1 :

$I_1 + I_2 + I_3 = 0$



$$\frac{V1-\left(40\angle 0^\circ\right)}{Z1}+\frac{V1}{Z2}+\frac{V1-V2}{Z3}=0$$

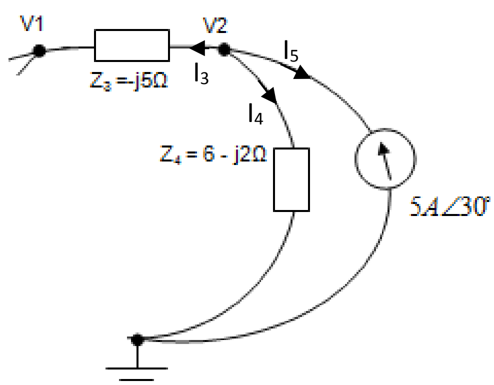
$$\frac{V1-\left(40\angle 0^\circ\right)}{8}+\frac{V1}{2+j4}+\frac{V1-V2}{(-j5)}=0$$

$$\frac{V1}{8}-\frac{40\angle 0}{8}+\frac{V1}{2+j4}-\frac{V1}{j5}+\frac{V2}{j5}=0$$

$$V1\left(\frac{1}{8}+\frac{1}{2+j4}+\frac{1}{(-j5)}\right)+V2\left(\frac{1}{j5}\right)=\frac{40\angle 0}{8}$$

$$V1\left(0.225\angle 0^\circ\right)+V2\left(0.2\angle -90^\circ\right)=5\angle 0^\circ$$

$$V1(0.255+j0)+V2(0-j0.2)=5+j0$$



Apply KCL at point V2 :

$I_3 + I_4 - I_5 = 0$

$$\frac{V2-V1}{Z3}+\frac{V2}{Z4}-5A\angle 30=0$$

$$\frac{V2-V1}{-j5}+\frac{V2}{6-j2}-5A\angle 30^\circ=0$$

$$\frac{V2}{-j5}+\frac{V1}{j5}+\frac{V2}{6-j2}=5A\angle 30^\circ$$

$$V1\left(\frac{1}{j5}\right)+V2\left(\frac{-1}{j5}+\frac{1}{6-j2}\right)=5A\angle 30^\circ$$

Equation 2

$$V1\left(0.2\angle -90^\circ\right)+V2\left(0.291\angle 59.036^\circ\right)=5\angle 30^\circ$$

$$V1(0-j0.2)+V2(0.15+j0.25)=4.33+j2.5$$

Using Cramer Rule

$$\begin{bmatrix} 0.225\angle 0^\circ & 0.2\angle -90^\circ \\ 0.2\angle -90^\circ & 0.291\angle 59.036^\circ \end{bmatrix}=\begin{bmatrix} 5\angle 0^\circ \\ 5\angle 30^\circ \end{bmatrix}$$

$$\Delta = \begin{matrix} 0.225\angle 0^\circ & 0.2\angle -90^\circ \\ 0.2\angle -90^\circ & 0.291\angle 59.036^\circ \end{matrix}$$

$$\left(0.225\angle 0^\circ\right)\left(0.291\angle 59.036^\circ\right)-\left(0.2\angle -90^\circ\right)\left(0.2\angle -90^\circ\right)$$

$$\left(0.0655\angle 59.036^\circ\right)-\left(0.04\angle 180^\circ\right)$$

$$0.0926\angle 37.311^\circ$$

$$\Delta V1\begin{bmatrix} 5\angle 0^\circ & 0.2\angle -90^\circ \\ 5\angle 30^\circ & 0.291\angle 59.036^\circ \end{bmatrix}$$

$$\left(5\angle 0^\circ\right)\left(0.291\angle 59.036^\circ\right)-\left(5\angle 30^\circ\right)\left(0.2\angle -90^\circ\right)$$

$$\left(1.455\angle 59.036^\circ\right)-\left(1\angle -60^\circ\right)$$

$$(0.749+j1.247)-(0.5-j0.866)$$

$$2.128\angle 83.292^\circ$$

$$0.2485+j2.1136^\circ$$

$$=15.9696+j16.5274$$

$$V1=\frac{\Delta V1}{\Delta}$$

$$=\frac{0.2485+j2.1136^\circ}{0.0926\angle 37.311^\circ}$$

$$=22.9822\angle 45.9834^\circ$$

Question 4

Diagram A2(d) shows an alternating current (AC) circuit with the connection of components resistors, capacitors and inductors. This circuit having a current source $I_1 = 2A\angle 0^\circ$ and $I_2 = 3A\angle 45^\circ$. By using N**ode Analysis,** evaluate the value of the voltage at node V_1.

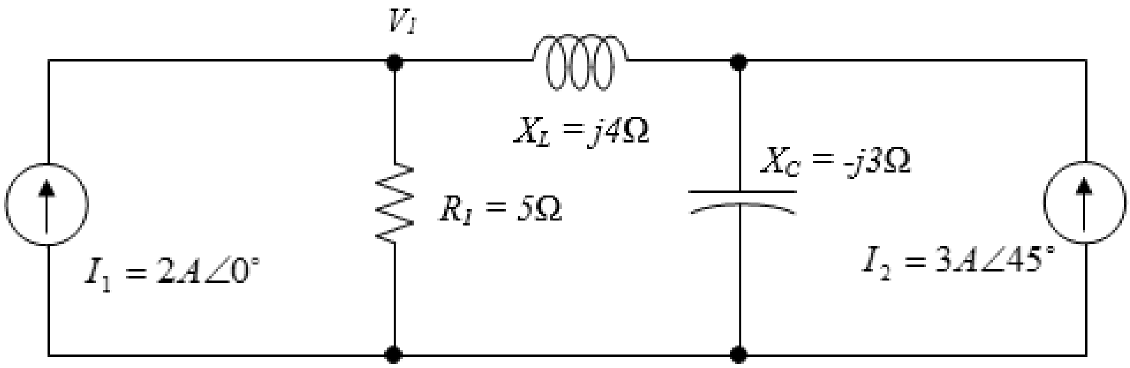
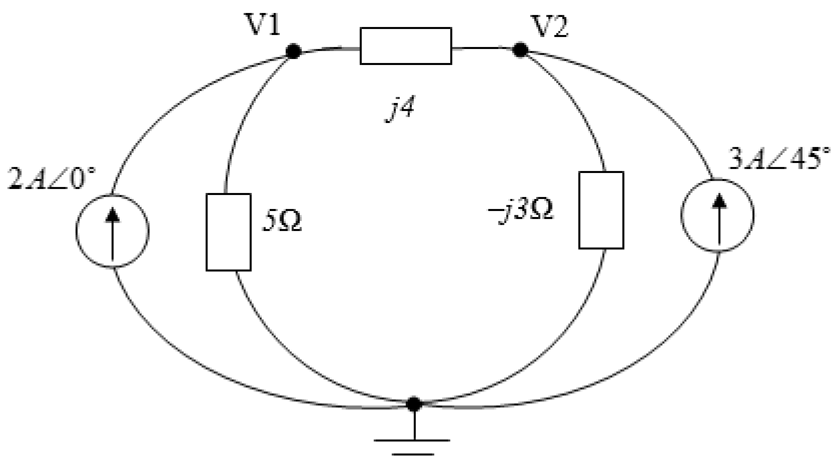
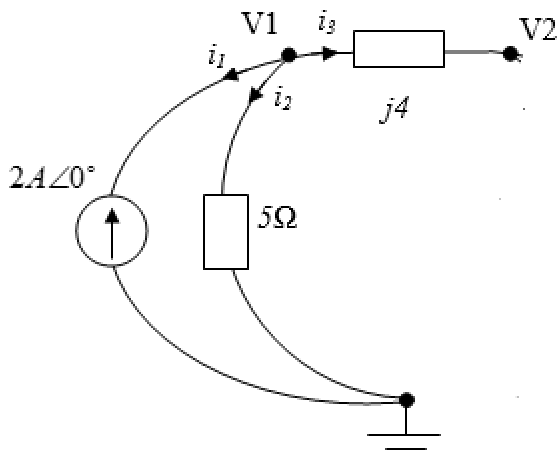


Diagram A2(d)

Answer:



For Nood V1



$-i_1 + i_2 + i_3 = 0$

$i_1 = i_2 + i_3$

$$-2\angle 0^\circ + \frac{V_1}{5} + \frac{V_1 - V_2}{j4} = 0$$

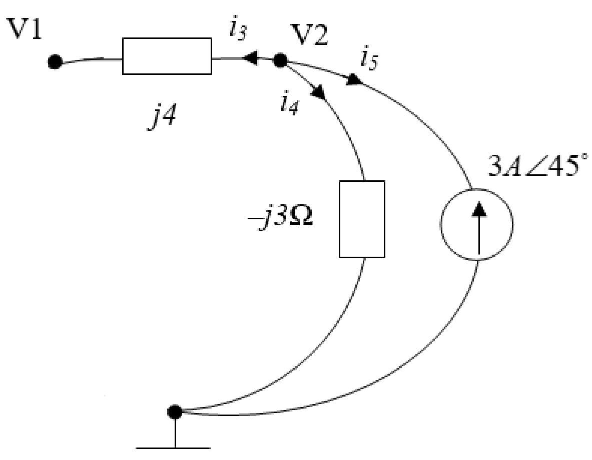
$$V_1\left(\frac{1}{5} + \frac{1}{j4}\right) - V_2\left(\frac{1}{j4}\right) = -2\angle 0^\circ$$

$$(0.2 - j0.25)V1 - (0 - j0.25)V2 = 2\angle 0^\circ$$

or

$$(0.3202\angle -51.3402))V1 - (0.25\angle -90)V2 = 2\angle 0^\circ$$

For nood V2



$i_3 + i_4 + i_5 = 0$

$$\frac{V_2 - V_1}{j4} + \left(\frac{V_2}{-j3}\right) - 3\angle 45^\circ = 0$$

$$-V_1\left(\frac{1}{j4}\right) + V_2\left(\frac{1}{j4} - \frac{1}{j3}\right) = 3\angle 45^\circ$$

$$-(0 - j0.25)V1 + (0 - j0.0833)V2 = 3\angle 45^\circ$$

or

$$(0.25\angle 90))V1 + (0.0833\angle 90)V2 = 3\angle 45^\circ$$

In matrix form

$$\begin{bmatrix} 0.2 - j0.25 & 0 + j25 \\ 0 + j0.25 & 0 + j0.0833 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2\angle 0^\circ \\ 3\angle 45^\circ \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 0.2 - j0.25 & 0 + j25 \\ 0 + j0.25 & 0 + j0.0833 \end{vmatrix}$$

$$= [(0.2 - j0.25)(0 + j0.0833)] - [(0 + j25)(0 + j0.25)]$$

$$= (0.020833 - j0.016667) - (-0.0625 + j0)$$

or

$$= (0.02668\angle 38.6598) - (0.0625\angle 180^\circ)$$

$$\Delta = 0.083325 + j0.0166667$$

or

$$\Delta = 0.08497\angle 11.3098^\circ$$

$$\Delta V_1 = \begin{vmatrix} 2\angle 0 & 0 + j25 \\ 3\angle 45 & 0 + j0.0833 \end{vmatrix}$$

$$= (2\angle 0)(0 + j0.0833) - (0 + j25)(3\angle 45)$$

$$= (0 + j0.16667) - (-0.5303 + j0.5303)$$

or

$$= (0.1667\angle -90^\circ) - (0.75\angle 135^\circ)$$

$$\Delta V_1 = 0.5303 + j0.3637$$

or

$$\Delta V_1 = 0.6431\angle -34.4445^\circ$$

Nilai V_1

$$V_1 = \frac{\Delta V_1}{\Delta} = \frac{0.8758\angle -52.7346}{0.08497\angle 11.3098}$$

$$V_1 = 5.2806 + j5.4216$$

or

$$V_1 = 7.5693\angle -45.7543^\circ$$

1.1.3 Superposition Theorem

Question 1

Analyze the alternating current (AC) circuit shown in Diagram 1, using **Superposition Theorem.** Calculate the current I passing through the 10Ω resistor. Given that the current passing through the *10*Ω resistor, considering current source $2A\angle 0^{\circ}$ is $0.83A\angle 148^{\circ}$.

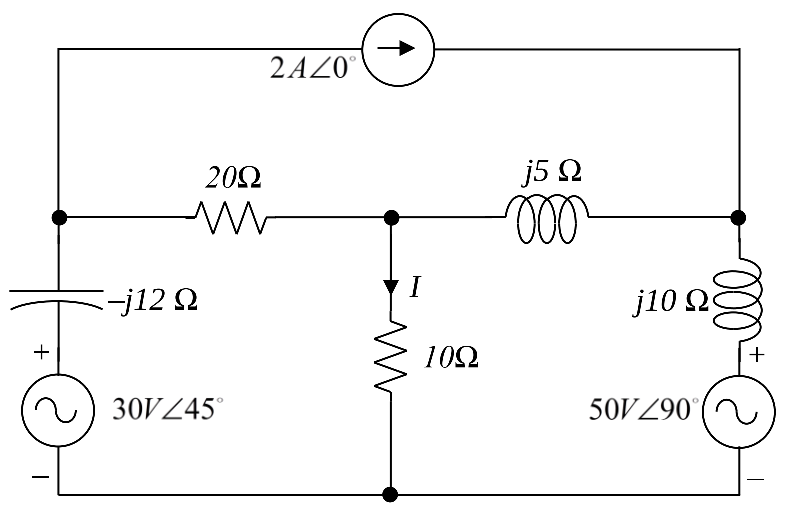
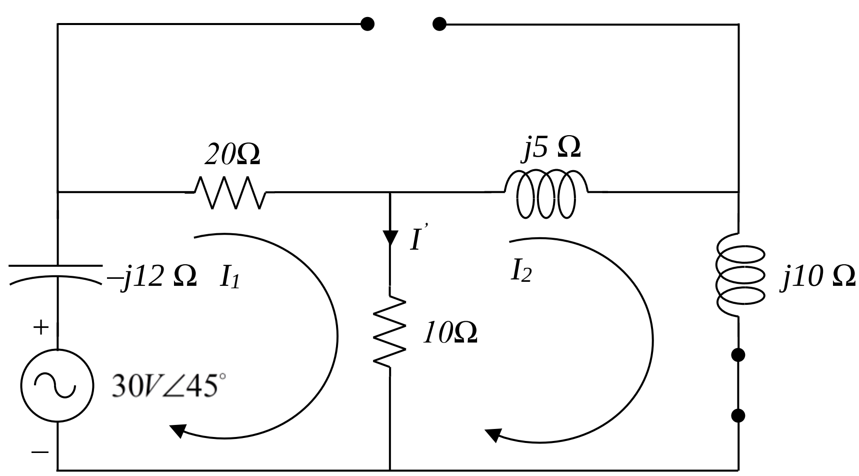


Diagram 1

Answer:

Step 1

Considering the effect of the 30V source:



Find total resistance R_T

$$R_T = [10//(j5 + j10)] + (20 - j12)$$

$$= \left[\frac{10 \times j15}{10 + j15}\right] + (20 - j12)$$

$$R_T = 26.92 - j7.38$$

$$R_T = 27.92\Omega\angle -15.34^\circ$$

$$I_T = \frac{30\angle 45^\circ}{27.92\angle -15.34^\circ}$$

$$I_T = 0.53 + j0.93$$

$$I_T = 1.07A\angle 60.34^\circ$$

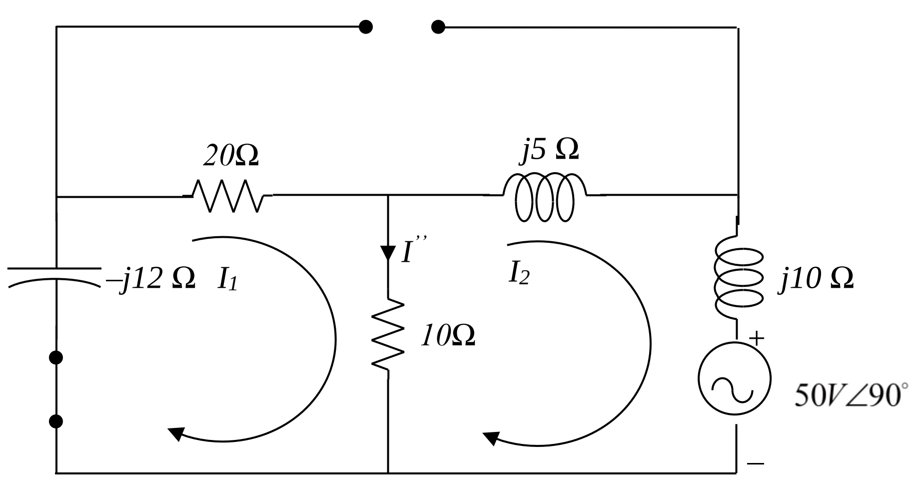
$$= \frac{1.07A\angle 60.34^\circ \times j15}{10 + j15}$$

$$I^{'} = 0.89\angle 94.03^\circ$$

$$I' = -0.062 + j0.89$$

Step 2

Considering the effect of the 50V source:



Find total resistance R_T

$$R_T = [10//(20 - j12)] + (j5 + j10)$$

$$= \left[\frac{10 \times (20 - j12)}{10 + (20 - j12)}\right] + (j15)$$

$$R_T = 7.13 + j13.85$$

$$R_T = 15.57\Omega\angle 62.77^\circ$$

$$I_T = \frac{50\angle 90^\circ}{15.57\Omega\angle 62.77^\circ}$$

$$I_T = 2.86 - j1.47$$

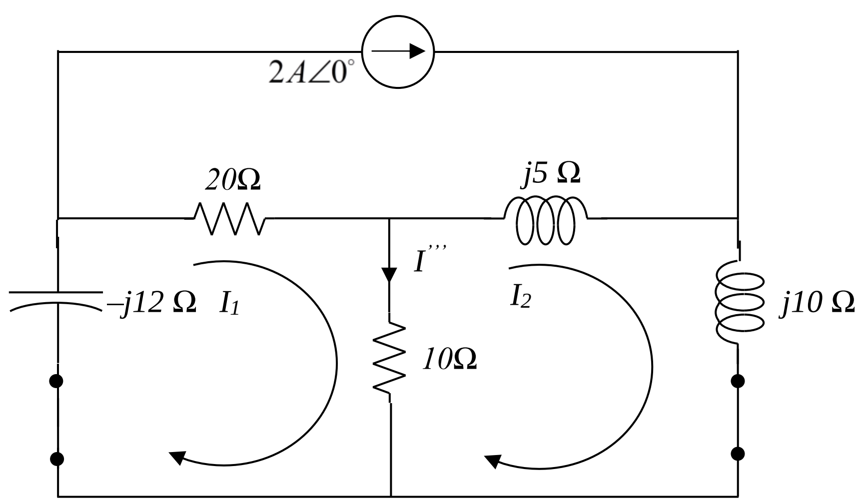
$$I_T = 3.21A\angle 27.23^\circ$$

$$= \frac{3.21A\angle 27.23^\circ \times (20 - j12)}{20 + 10 - j12}$$

$$I'' = 2.32A\angle 18.07^\circ$$

$$I'' = 2.2 + j0.72$$

Given the current through the resistance *10*Ω with source $2A\angle 0^\circ$ is $0.83A\angle 148^\circ$.



$$I''' = 0.83A\angle 148^\circ$$

$$I = I' + I'' + I'''$$

$$I = (-0.062 + j0.89) + (2.2 + j0.72) + (0.83A\angle 148^\circ)$$

$$I = 2.5\angle 54.98^\circ$$

Question 1

Diagram 1 shows an alternating current circuit having a current source $I_1 = 0.5A\angle 60^\circ$ and voltage source $E_1 = 120V\angle 0^\circ$, is connected with a number of electronic passive components. Analysis the circuit using Thevenin Theorem to calculate the voltage Thevenin (V_{TH}), when the load resistance is opened at terminal a-b.

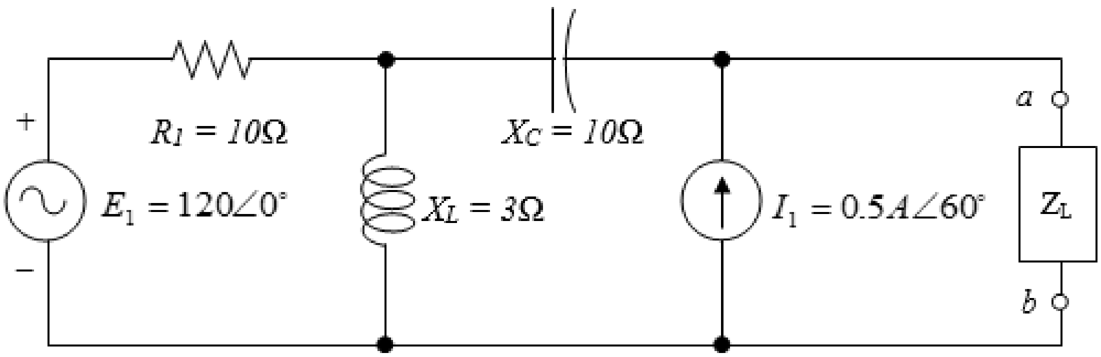
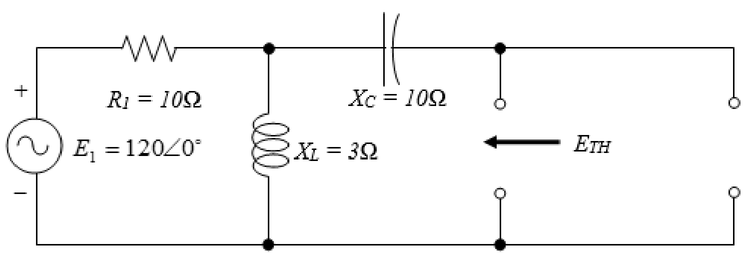


Diagram 1

Answer:

Using Superposition Theorem



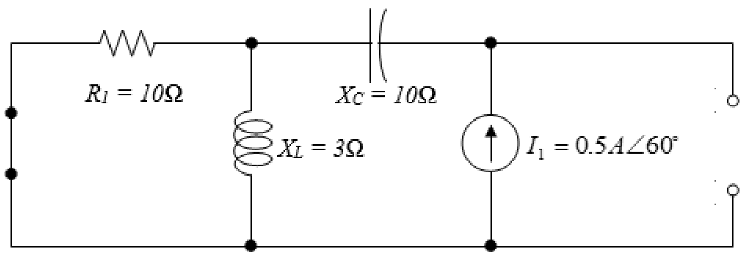
Value of E_{TH}':

$$E_{TH}' = \frac{E_1 \times X_L}{R_1 + X_L}$$

$$E_{TH}' = \frac{(120\angle 0) \times (j3)}{10 + j3}$$

$$E_{TH}{}' = 9.9083 + j33.0275$$
or
$$E_{TH}{}' = 34.4817\angle 73.3008^\circ$$

Value of $E_{TH}{}''$:



$$E_{TH}{}'' = V_{XL} + V_{XC}$$

$$E_{TH}{}'' = (I_1 \times X_C) + \left[I_1 \times \left(\frac{R_1 \times X_L}{R_1 + X_L} \right) \right]$$

$$E_{TH}{}'' = I_1 \times \left[X_C + \left(\frac{R_1 \times X_L}{R_1 + X_L} \right) \right]$$

$$E_{TH}{}'' = 0.5\angle 60^\circ \times \left[-j10 + \left(\frac{10 \times j3}{10 + j3} \right) \right]$$

$$E_{TH}{}'' = 3.3448 - j1.4544$$
or
$$E_{TH}{}'' = 3.6473\angle -23.5006^\circ$$

Value of E_{TH}:

$$E_{TH} = E_{TH}{}' + E_{TH}{}''$$

$$E_{TH} = (9.9083 + j33.0275) + (3.3448 - j1.4544)$$
or
$$E_{TH} = (34.4817\angle 73.3008^\circ) + (3.6473\angle -23.5006^\circ)$$

$$E_{TH} = 13.2531 - j31.5731$$
or
$$E_{TH} = 34.2418\angle 67.2294^\circ$$

Question 2

Diagram 2 shows an alternating current circuit having a voltage source $E_1 = 120A\angle 30^\circ$ and current source $I = 0.4V\angle 20^\circ$, is connected with a number of electronic passive components. Analysis the circuit using **Thevenin Theorem** to get the current I_z flowing through the load Z_L.

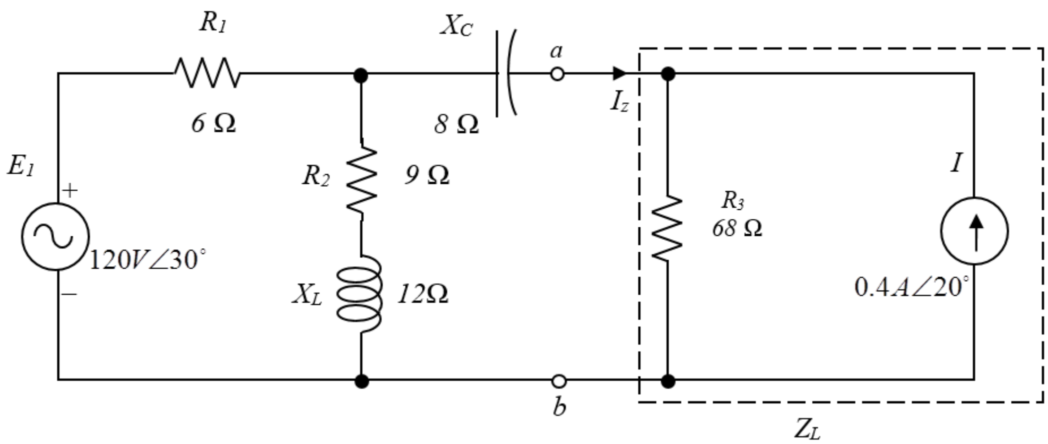
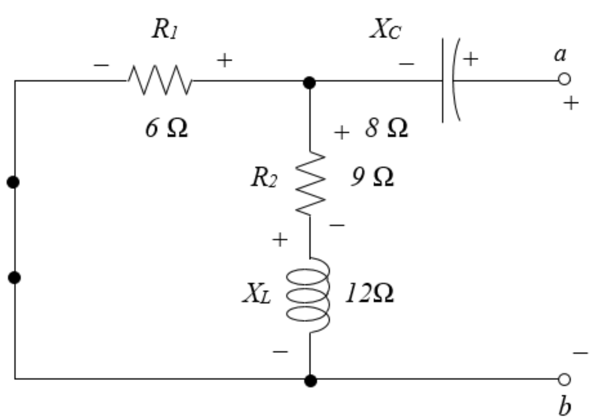
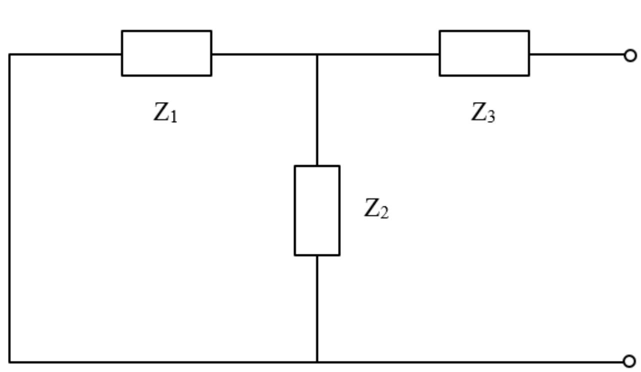


Diagram 2

Answer:





$Z_1 = 6\Omega$

$Z_2 = 9 + j12$

$Z_3 = -j8$

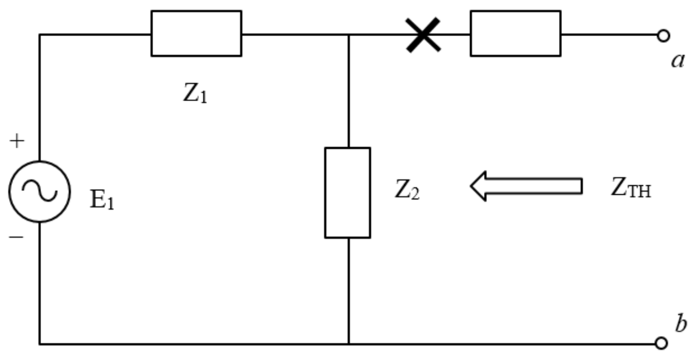
Value Z_{TH} :

$$Z_{TH} = (Z_1 // Z_2) + Z_3$$

$$Z_{TH} = \left(\frac{6 \times (9 + j12)}{6 + 9 + j12} \right) + (-j8)$$

$$Z_{TH} = 4.5366 + j1.1707 - j8$$
atau
$$Z_{TH} = 4.6852\angle 14.4703 + 8\angle 90^\circ$$

$$Z_{TH} = 4.53659 - j6.8293$$
atau
$$Z_{TH} = 8.19875\angle -56.40448^\circ$$



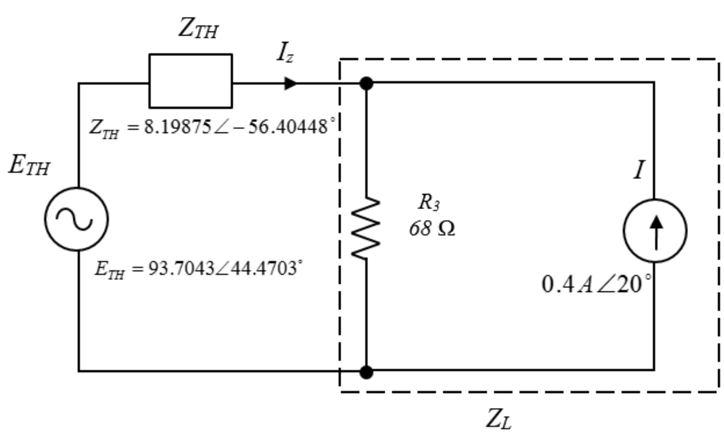
Value V_{TH} :

KVL

$$V_{TH} = \frac{E_1 \times Z_2}{Z_1 + Z_2}$$

$$V_{TH} = \frac{120\angle 30^\circ \times 9 + j12}{6 + 9 + j12}$$

$$V_{TH} = 66.8686 + j65.6435$$
atau
$$V_{TH} = 93.7043\angle 44.4703^\circ$$



$$I_Z = \frac{E_{TH}}{Z_{TH}}$$

$$I_Z = \frac{93.7043\angle 44.4703^\circ}{8.19785\angle -56.40448}$$

$$I_Z = -2.1562 + j11.22385$$

Atau

$$I_Z = 11.42909\angle 100.8748^\circ$$

1.1.5 Norton's Theorems

Question 1

Evaluate the total value of **Norton impedance and Norton current**, when the elements between points a and b are removed in Diagram 1. Using the **Superposition Theorem**, calculate Norton current (I_N).

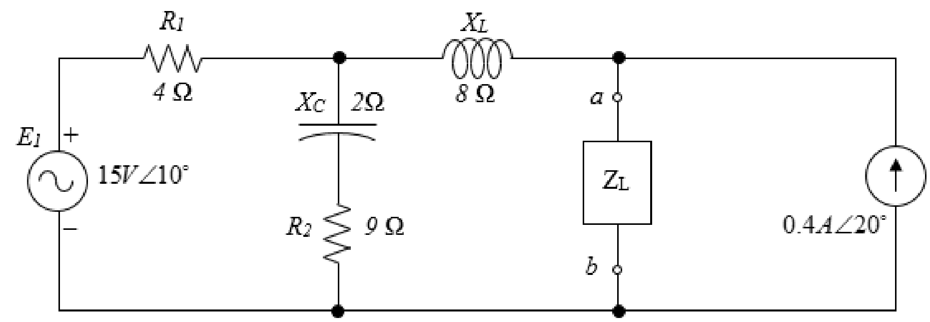
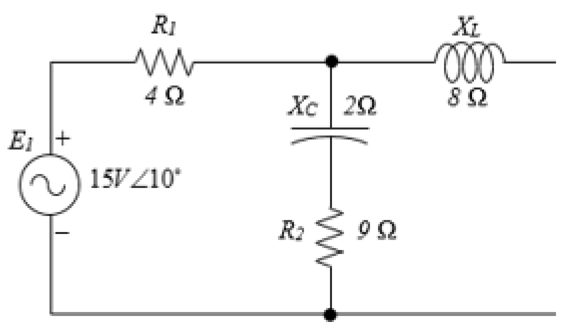
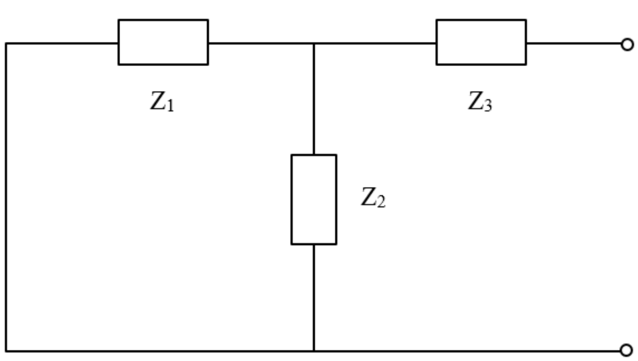


Diagram 1

Answer:





$Z_1 = 4\Omega$

$Z_2 = 9 - j2$

$Z_3 = j8$

Value Z_N :

$$Z_N = (Z_1 // Z_2) + Z_3$$

$$Z_N = \left(\frac{4 \times (9 - j2)}{4 + 9 - j2} \right) + (j8)$$

$$Z_N = 2.7977 - j0.1850 + j8$$

atau

$$Z_N = (2.8038\angle -3.7826) + 8\angle 90^\circ$$

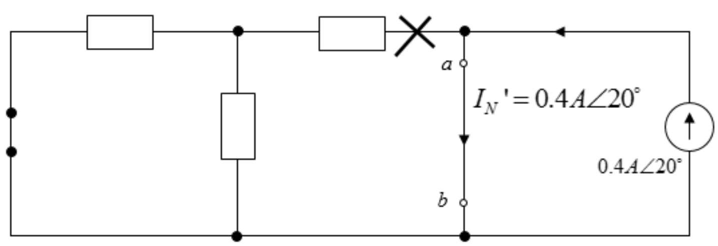
$$Z_N = 2.7977 - j7.8150$$

atau

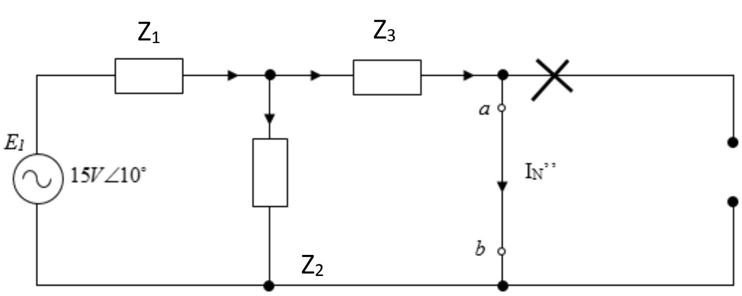
$$Z_N = 8.3007\angle 70.3032^\circ$$

Value I_N'

$$I_N' = 0.4A\angle 20^\circ$$



Value I_N''



$$Z_T = Z_1 + (Z_2 // Z_3)$$

$$Z_T = 4 + \left(\frac{(9 - j2) \times j8}{9 - j2 - j8} \right)$$

$$Z_T = 4 + (4.923077 - j7.717948)$$

atau

$$Z_T = 4\angle 0^\circ + (6.818777\angle 43.781125^\circ)$$

$$Z_T = 8.923076 - j4.717948$$

atau

$$Z_T = 10.093579\angle 27.8670^\circ$$

$$I_T = \frac{E_T}{Z_T}$$

$$I_T = \frac{15\angle 10^\circ}{10.093579\angle 27.8670^\circ}$$

$$I_T = 1.41442 - j0.455946$$

Atau

$$I_T = 1.48609\angle -17.8670^\circ$$

Question 2

Diagram 2 shows an alternating current circuit having a voltage source $E_1 = 20V\angle 0^\circ$ and current source $I = 4A\angle 20^\circ$. Analysis the circuit using **Norton's Theorem** to get the total Norton's impedance (Z_N) and Norton's current (I_N), when a terminal a-b of the load Z_L is opened.

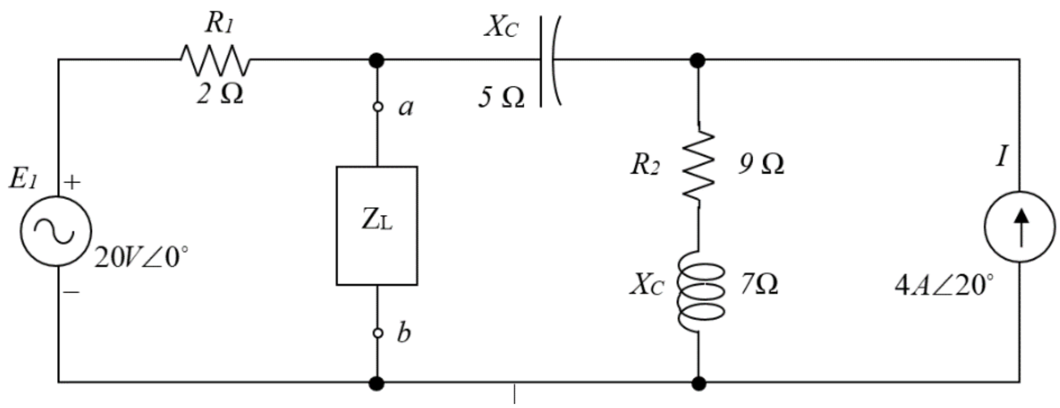
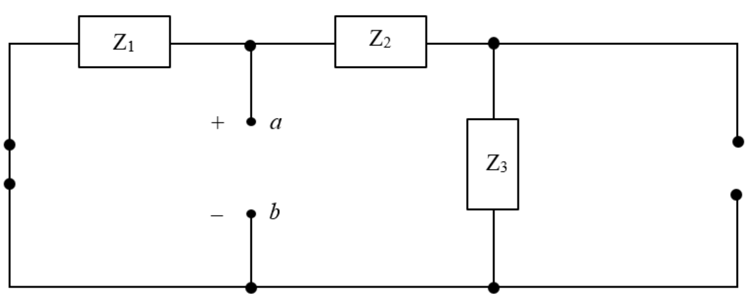


Diagram 2

Answer:



$Z_1 = 2\Omega$

$Z_2 = -j5$

$Z_3 = 9 + j7$

Value Z_N :

$Z_{TH} = Z_1 // (Z_2 + Z_3)$

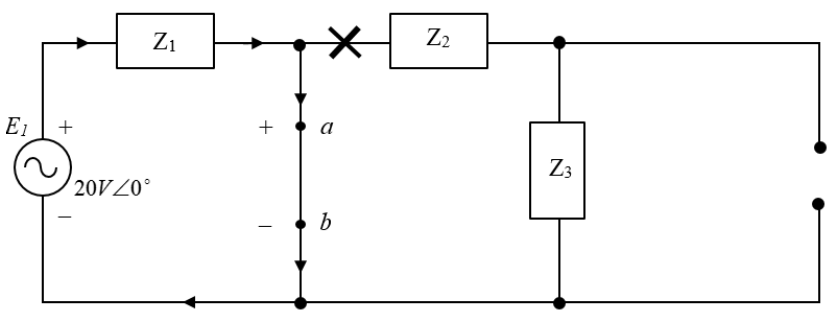
$$Z_{TH} = \left(\frac{2 \times [(-j5) + (9 + j7)]}{2 + (-j5 + 9 + j7)} \right)$$

$$Z_{TH} = 1.6480 - j0.0640$$

atau

$$Z_{TH} = 1.64924\angle 2.22396^{\circ}$$

Value I_N’

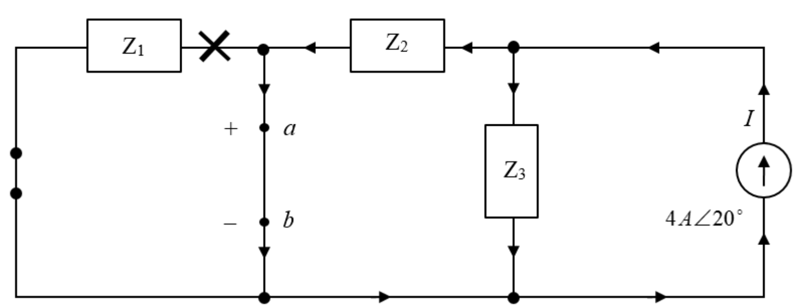


$$I_N{}' = \frac{E_1}{Z_1}$$

$$I_N{}' = \frac{20\angle 0}{5 + j0}$$

$$I_N{}' = 4A\angle 0^{\circ}$$

Value I_N’’



$$I_N{}'' = \frac{I \times Z_3}{Z_2 + Z_3}$$

$$I_N'' = \frac{4\angle 20^\circ \times (9 - j12)}{j8 + 9 - j12}$$

$$I_N'' = 6.0143 - j0.9706$$

atau

$$I_N'' = 6.0921\angle -9.1676^\circ$$

Nilai I_N

$$I_N = I_N' + I_N''$$

$$I_N = 4A\angle 0^\circ + 16.0921\angle -9.1676^\circ$$

$$I_N = 10.0143 - j0.9706$$

$$I_N = 10.0612A\angle -5.5359^\circ$$

Question 3

Analyze alternating current (AC) circuit for the Diagram 3. Evaluate the total value of Norton impedance and Norton current, when the elements between points a and b are removed. Using the Superposition Theorem, calculate Norton current (I_N).

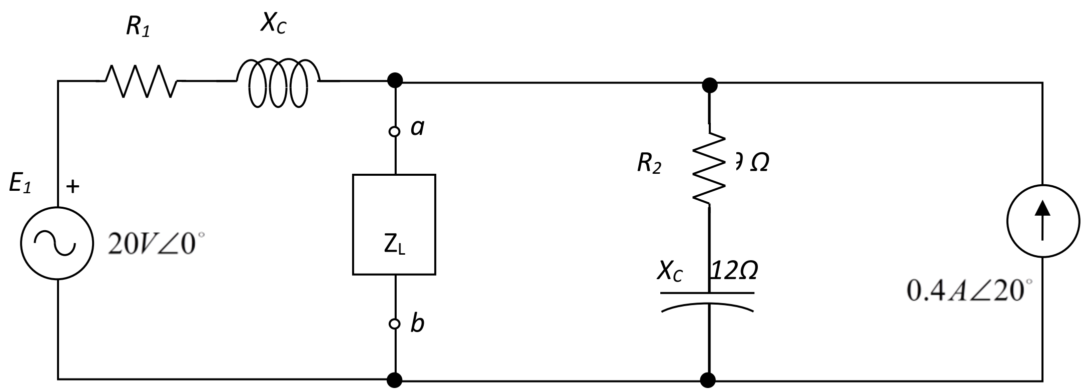
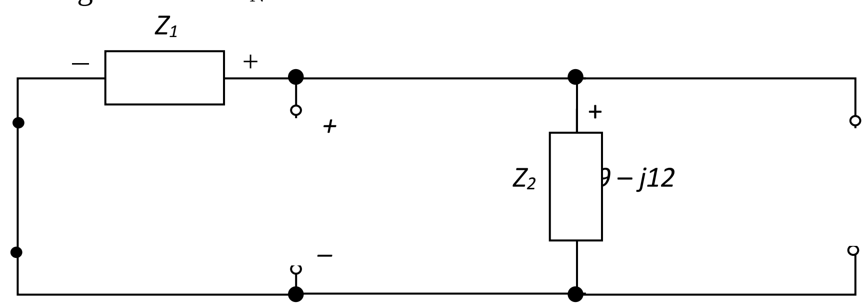


Diagram 1

Answer:

Nilai Galangan Norton Z_N

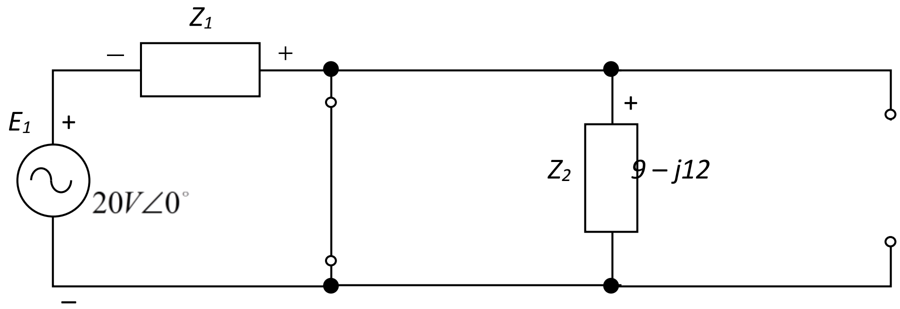


$$Z_N = Z_1 // Z_2$$
$$= \frac{(6+j8)(9-j12)}{(6+j8)+(9-j12)}$$
$$= 9.3361 + j2.4896$$
$$= 9.6623\angle 14.9314^{\circ}$$

Nilai Arus Norton Z_N

Guna terom tindihan :

Analisa terhadap punca bekalan E_1



Nilai I'

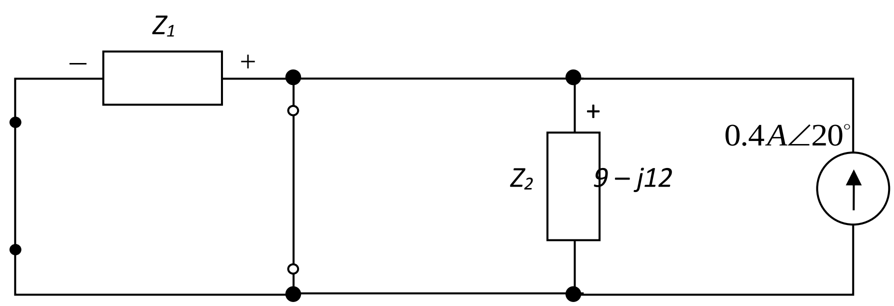
$$I'_N = \frac{E_1}{Z_1}$$

$$I'_N = \frac{20\angle 0^\circ}{6+j8}$$

$$= 1.2 - j1.6$$

$$I'_N = 2\angle -53.1301^\circ$$

Analisa terhadap punca arus



Nilai I''

$$I''_N = I = 0.4\angle 20^\circ$$

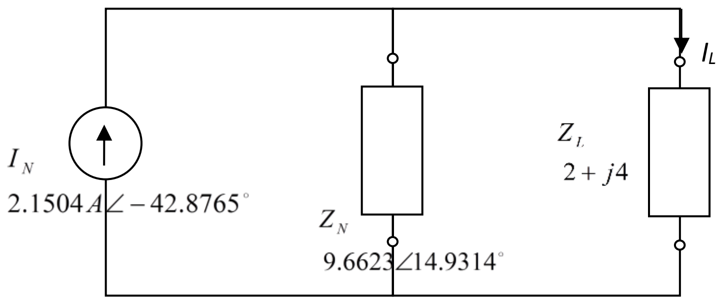
$$I_N = I'_N + I''_N$$

$$= (2\angle -53.1301^\circ) + (0.4\angle 20^\circ)$$

$$= 1.57587 - j1.46319$$

$$= 2.1504A\angle -42.8765^\circ$$

Litar Setara Norton :



Chapter 2

2.0 Laplace Transform

2.1 Laplace Transform using:
- 2.1.1 Integral definition
- 2.1.2 Table
- 2.1.3 Linearity property
- 2.1.4 First shift theorem
- 2.1.5 Derivatives and integrals

2.2 Inverse Laplace Transform using
- 2.2.1 Partial fractions
- 2.2.2 Completing the square
- 2.2.3 First and second derivatives

2.3 Laplace Transform in RLC circuit analysis

Question 1

$f(t) = 1 \, for \, t \geq 0.$

Answer:

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$$
$$= \int_0^{\infty} e^{-st} . 1 \, dt$$
$$= -\frac{1}{s} e^{-st} \Big|_0^{\infty}$$
$$= -\frac{1}{s}\left[e^{-s(\infty)} - e^{-s(0)}\right]$$
$$= -\frac{1}{s}[0 - 1]$$
$$= \frac{1}{s}$$

Question 2

$f(t) = e^t u(t)$

Answer:

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$$
$$\mathcal{L}\{e^t\} = \int_0^{\infty} e^{-st} . e^t \, dt$$
$$= \int_0^{\infty} e^{-(s-1)t} \, dt$$
$$= \frac{-1}{(s-1)}\left[\left(e^{-(s-1)(\infty)}\right) - \left(e^{-(s-1)(0)}\right)\right]$$
$$= \frac{-1}{(s-1)}[0 - 1]$$
$$= \frac{1}{s-1}$$

Question 3

$f(t) = e^{-t}u(t)$

Answer:

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t)dt$$

$$\mathcal{L}\{e^{-t}\} = \int_0^{\infty} e^{-st} . e^{-t}\, dt$$

$$= \int_0^{\infty} e^{-(s+1)t}\, dt$$

$$= \frac{-1}{(s+1)} e^{-(s+1)t} \Big|_0^{\infty}$$

$$= \frac{-1}{(s+1)} \left[\left(e^{-(s+1)(\infty)}\right) - \left(e^{-(s+1)(0)}\right)\right]$$

$$= \frac{-1}{(s+1)} [0 - 1]$$

$$= \frac{1}{s+1}$$

Question 4

$f(t) = e^{-at}u(t)$

Answer:

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t)dt$$

$$\mathcal{L}\{e^{-at}\} = \int_0^{\infty} e^{-st} . e^{-at} u(t)\, dt$$

$$= \int_0^{\infty} e^{-(s+a)t}\, dt$$

$$= \frac{-1}{(s+a)} e^{-(s+a)t} \Big|_0^{\infty}$$

$$= \frac{-1}{(s+a)} \left[\left(e^{-(s+a)(\infty)}\right) - \left(e^{-(s+a)(0)}\right)\right]$$

$$= \frac{-1}{(s+a)} [0 - 1]$$

$$= \frac{1}{s+a}$$

Question 5

$f(t) = e^{-2t}u(t)$

Answer:

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t)dt$$

$$\mathcal{L}\{e^{-2t}\} = \int_0^{\infty} e^{-st} . e^{-2t}\, dt$$

$$= \int_0^{\infty} e^{-(s+2)t}\, dt$$

$$= \frac{-1}{(s+2)} e^{-(s+2)t} \Big|_0^{\infty}$$

$$= \frac{-1}{(s+2)} \left[\left(e^{-(s+2)(\infty)}\right) - \left(e^{-(s+2)(0)}\right)\right]$$

$$= \frac{1}{s+2}$$

Question 6

$f(t) = 2e^{0.5t}u(t)$

Answer:

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t)dt$$

$$\mathcal{L}\{2e^{0.5t}\} = \int_0^{\infty} e^{-st} . 2e^{0.5t}\, dt$$

$$= 2\int_0^{\infty} e^{-(s-0.5)t}\, dt$$

$$= \frac{-2}{(s-0.5)} e^{-(s-0.5)t} \Big|_0^{\infty}$$

$$= \frac{-2}{(s-0.5)} \left[\left(e^{-(s-0.5)(\infty)}\right) - \left(e^{-(s-0.5)(0)}\right)\right]$$

$$= \frac{-2}{(s-0.5)} [0-1]$$

$$= \frac{2}{s-0.5}$$

Question 7

$f(t) = e^{-at}u(-t)$

Answer:

$$F(s) = \mathcal{L}\{f(t)\} = \int_{0}^{\infty} e^{-st} f(t)dt$$

$$\mathcal{L}\{e^{-at}\} = \int_{0}^{\infty} e^{-st} . e^{-at}u(-t)\, dt$$

$$= \int_{-\infty}^{0^-} e^{-(s+a)t}\ dt$$

$$= \frac{-1}{(s+a)} e^{-(s+a)t} \Big| \begin{matrix} 0- \\ -\infty \end{matrix}$$

$$= \frac{-1}{(s+a)} \left[\left(e^{-(s+a)(0)}\right) - \left(e^{-(s+a)(\infty)}\right)\right]$$

$$= \frac{-1}{(s+a)} [1 - 0]$$

$$= \frac{-1}{s+a}$$

Question 8

$f(t) = -e^{at}u(-t)$

Answer:

$$F(s) = \mathcal{L}\{f(t)\} = \int_{0}^{\infty} e^{-st} f(t)dt$$

$$\mathcal{L}\{e^{-at}\} = -\int_{0}^{\infty} e^{-st} . e^{at}u(-t)\, dt$$

$$= -\int_{-\infty}^{0^-} e^{-(s-a)t}\ dt$$

$$= \frac{1}{(s+a)} e^{-(s-a)t} \Big| \begin{matrix} 0- \\ -\infty \end{matrix}$$

$$= \frac{1}{(s+a)} \left[\left(e^{-(s-a)(0)}\right) - \left(e^{-(s-a)(\infty)}\right)\right]$$

$$= \frac{1}{(s+a)} [1 - 0]$$

$$= \frac{1}{s-a}$$

Question 9

$f(t) = 4t$ using the integral definition.

Answer

$f(t) = 4tu(t)$ $\qquad F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty f(t).e^{-st}\,dt$

$$\mathcal{L}\{t\} = 4\int_0^\infty t.e^{-st}\,dt$$

$$\int_a^b u\,dv = uv - \int_a^b v\,du$$

$$u = t$$

$$\frac{du}{dt} = 1$$

$$du = dt$$

$$\int v = e^{-st}$$

$$\int dv = \int_0^\infty e^{-st}\,dt$$

$$v = \frac{e^{-st}}{-s}$$

$$= (4)[t\frac{e^{-st}}{-s} - \int_a^b \frac{e^{-st}}{-s}\,dt]$$

$$= \int \frac{e^{-st}}{-s}\,dt = \frac{e^{-st}}{-s.-s} = \frac{e^{-st}}{s^2}$$

$$= 4\left[t\frac{e^{-st}}{-s} + \frac{e^{-st}}{s^2}\right]_0^\infty$$

$$= 4\left[(\infty)\frac{e^{-s(\infty)}}{-s} + \frac{e^{-s(\infty)}}{s^2}\right] - \left[(0)\frac{e^{-s(0)}}{-s} + \frac{e^{-s(0)}}{s^2}\right]$$

$$= 4.\frac{1}{s^2}$$

$$= \frac{4}{s^2}$$

2.1.2 Table

Question1

Determine the Laplace Transform of the following function

$f(t) = 0.25\sin\left(\sqrt{0.25}\right)t + 4t^3 e^{-\frac{t}{3}}$ by using Laplace Transform table.

Answer:

$$f(t) = 0.25\sin\sqrt{0.25}t + 4t^3 e^{-\frac{t}{3}}$$

$$F(s) = 0.25\ \mathcal{L}^{-1}\ \left\{\sin\sqrt{0.25}t\right\} + 4\ \mathcal{L}^{-1}\ \left\{t^3 e^{-\frac{t}{3}}\right\}$$

$$F(s) = \frac{1}{4}\left[\frac{0.25}{(s-0.5)^2 + (\sqrt{0.25})^2}\right] + 4\left[\frac{3!}{(s+\frac{1}{3})^4}\right]$$

Question2

Determine the Laplace Transform of the following functions by **using the table of Laplace Transform**.

$$f(t) = 0.25\sin\left(\sqrt{0.25}\right)t + 4t^3 e^{-\frac{t}{3}}$$

Answer:

$$f(t) = 0.25\sin\sqrt{0.25}t + 4t^3 e^{-\frac{t}{3}}$$

$$F(s) = 0.25\ \mathcal{L}^{-1}\ \left\{\sin\sqrt{0.25}t\right\} + 4\ \mathcal{L}^{-1}\ \left\{t^3 e^{-\frac{t}{3}}\right\}$$

$$F(s) = 0.25\left[\frac{\sqrt{0.25}}{(s-0.5)^2 + (\sqrt{0.25})^2}\right] + 4\left[\frac{3!}{(s+\frac{1}{3})^4}\right]$$

$$\left[\frac{\frac{1}{8}}{(s-0.5)^2+(\sqrt{0.25})^2}\right]+\left[\frac{24}{(s+\frac{1}{3})^4}\right]$$

$$\left[\frac{0.125}{(s-0.5)^2+(\sqrt{0.25})^2}\right]+\left[\frac{24}{(s+\frac{1}{3})^4}\right]$$

Question 3

Determine the Laplace Transform of the following functions by **using the table of Laplace Transform**.

$$f(t)=9\sin\frac{1}{9}t+0.3e^{-5t}t^3$$

Answer:

$$f(t)=9\sin\frac{1}{9}t+0.3e^{-5t}t^3$$

$$F(s)=9\,\mathcal{L}\left\{\sin\frac{1}{9}t\right\}+0.3\,\mathcal{L}\left\{e^{-5t}t^3\right\}$$

$$=9\left[\frac{\frac{1}{9}}{s^2+\left(\frac{1}{9}\right)^2}\right]+0.3\left[\frac{3!}{(s+5)^{3+1}}\right]$$

$$=\frac{1}{81s^2+1}+0.3\left[\frac{6}{(s+5)^4}\right]$$

$$F(s) = \frac{1}{s^2 + \left(\frac{1}{9}\right)^2} + \left[\frac{1.8}{(s+5)^4}\right]$$

$$F(s) = \frac{1}{(81s^2 + 1)} + \frac{1.8}{(s+5)^4}$$

Question 4

Determine the Laplace Transform of the following functions by **using the table of Laplace Transform**.

$$f(t) = e^{-2t}(4\cos 5t + 3\sin 5t)$$

Answer:

$$F(s) = \mathcal{L}\{x(t)\}$$

$$\mathcal{L}\{e^{-2t}(4cos5t + 3sin5t)\} = 4\mathcal{L}\{e^{-2t}cos5t\} + 3\mathcal{L}\{e^{-2t}sin5t\}$$

$$= 4\left(\frac{(s+2)}{(s+2)^2 + 5^2}\right) + 3\left(\frac{5}{(s+2)^2 + 5^2}\right)$$

$$= \frac{4(s+2)}{(s+2)^2 + 25} + \frac{15}{(s+2)^2 + 25}$$

2.1.3 Linearity property

Question1

Determine the Laplace Transform of the following function $f(t) = 3\sin 5t - 2\cosh 3t$, by using the linearity theorem.

Answer :

$$f(t) = 3\sin 5t - 2\cosh 3t$$

$$F(s) = 3\zeta\{\sin 5t\} - 2\zeta\{\cosh 5t\}$$

$$F(s) = 3\left\{\frac{5}{s^2 + 5^2}\right\} - 2\left\{\frac{s}{s^2 - 3^2}\right\}$$

$$F(s) = \frac{15}{s^2 + 25} - \frac{2s}{s^2 - 9}$$

Question2

Determine the Laplace Transform of the following function $f(t) = \delta(t) + 2u(t) - 3e^{-2t}$ by using the **linearity property theorem**.

Answer :

$$f(t) = \delta(t) + 2u(t) - 3e^{-2t}$$

$$F(s) = \mathcal{L}^{-1}\{\delta(t)\} + 2\mathcal{L}^{-1}\{u(t)\} - 3\mathcal{L}^{-1}\{e^{-2t}\}$$

$$F(s) = 1 + \frac{2}{s} - \frac{3}{(s+2)}$$

$$F(s) = \frac{s^2 + s + 4}{s(s+2)}$$

Question3

$f(t)=(1-e^{-5t})^2$, using **linearity property theorem**.

Answer:

$$f(t)=(1-e^{-5t})^2$$

$$=(1-e^{-5t})(1-e^{-5t})$$

$$=(1-2e^{-5t}+e^{-10t})$$

$$F(s)=\mathcal{L}\{1\}-2\,\mathcal{L}\{e^{-5t}\}+\mathcal{L}\{e^{-10t}\}$$

$$F(s)=\frac{1}{s}-\frac{2}{(s+5)}+\frac{1}{(s+10)}$$

Question 4

Determine the **Laplace Transform** of the following function

$$f(t)=2t^3e^{-0.5t}-10e^{\frac{1}{2}t}\sin\sqrt{5}t$$

by using **Linearity Property Theorem**.

Answer:

$$f(t)=2t^3e^{-0.5t}-10e^{\frac{1}{2}t}\sin\sqrt{5}t$$

$$F(s)=2\,\mathcal{L}\{t^3e^{-0.5t}\}-10\,\mathcal{L}\left\{e^{\frac{1}{2}t}\sin\sqrt{5}t\right\}$$

$$F(s)=2\left[\frac{3!}{(s+0.5)^{3+1}}\right]-10\left[\frac{\sqrt{5}}{\left(s-\frac{1}{2}\right)^2+\left(\sqrt{5}\right)^2}\right]$$

$$F(s)=\frac{12}{(s+0.5)^4}-\frac{10\sqrt{5}}{\left(s-\frac{1}{2}\right)^2+5}$$

Question 5

Determine the **Laplace Transform** of the following function

$$f(t) = 0.3t^4 e^{-0.2t} + \frac{1}{3} e^{-0.2t} \cosh t$$

by using **Linearity Property Theorem**.

Answer:

$$f(t) = 0.3t^4 e^{-0.2t} + \frac{1}{3} e^{-0.2t} \cosh t$$

$$F(s) = 0.3\zeta(t^4 e^{-0.2t}) + \frac{1}{3}\zeta(e^{-0.2t} \cosh t)$$

$$F(s) = 0.3\left[\frac{4!}{(s+0.2)^{4+1}}\right] + \frac{1}{3}\left[\frac{(s+0.2)}{(s+0.2)^2 - (1)^2}\right]$$

$$F(s) = \frac{7.2}{(s+0.2)^5} + \frac{(s+0.2)}{3[(s+0.2)^2 + 1]}$$

Question 6

Determine the **Laplace Transform** of the following function

$$f(t) = e^{-2t}(5\cos\sqrt{2}t - 3\sin t)$$

by using **Linearity Property Theorem**.

Answer:

$$f(t) = e^{-2t}(5\cos\sqrt{2}t - 3\sin t)$$

$$f(t) = e^{-2t}(5\cos\sqrt{2}t - 3\sin t)$$

$$f(t) = 5e^{-2t} \cos\sqrt{2}t - 3e^{-2t} \sin t)$$

$$f(t) = 5\zeta\left[e^{-2t} \cos\sqrt{2}t\right] - 3\zeta\left[e^{-2t} \sin t\right]$$

$$f(t) = 5\left[\frac{(s+2)}{(s+2)^2 + (\sqrt{2})^2}\right] - 3\left[\frac{1}{(s+2)^2 + 1^2}\right]$$

$$f(t) = \frac{5(s+2)}{(s+2)^2 + (\sqrt{2})^2} - \frac{3}{(s+2)^2 + 1^2}$$

Question 1

Determine the Laplace Transform of the following function $f(t)=2\sin 3t\sinh 7t$ by using the First Shift Theorem.

Answer :

$$f(t)=2\sin 3t\sinh 7t$$

According to Hyperbolic function :

$$\sinh x=\frac{1}{2}\left(e^{x}-e^{-x}\right)$$

Then :

$$\sinh 7t=\frac{1}{2}\left(e^{7t}-e^{-7t}\right)$$

by writing :

$$2\sin 3t\sinh 7t=\sinh\ 7t\ \ f(t)$$

Where :

$$f(t)=\sin 3t$$

$$F(s)=\zeta\{\sin 3t\}=\frac{3}{s^{2}+3^{2}}$$

by using the First Shift Theorem :

$$\zeta\{2\sin 3t\sinh 7t\}=\frac{1}{2}\left(e^{7t}-e^{-7t}\right)f(t)$$

$$=2\left[\frac{1}{2}\left(\frac{3}{(s-7)^{2}+3^{2}}-\frac{3}{(s+7)^{2}+3^{2}}\right)\right]$$

$$=\frac{3}{(s-7)^{2}+3^{2}}-\frac{3}{(s+7)^{2}+3^{2}}$$

Question 2

Solve the following Laplace Transform function $f(t) = \frac{1}{2}e^{-3t}\cos 6t$, using **first shift Theorem**

Answer :

$$f(t) = \frac{1}{2}e^{-3t}\cos 6t$$

We know that $\mathcal{L}\left\{\cos\ 6t\right\} = \left[\frac{s}{(s^2+6^2)}\right]$

With s replaced by $(s+3)$

$$f(t) = \frac{1}{2}e^{-3t}\cos 6t = \frac{1}{2}\ \mathcal{L}\left\{e^{-3t}\cos 6t\right\}$$

$$= \frac{1}{2}\left[\frac{(s+3)}{(s+3)^2+6^2}\right]$$

$$= \frac{s+3}{2s^2+12s+90}$$

Question 3

Determine the **Laplace Transform** of the following functions

$f(t) = 2\cos 4t\ \cosh 3t$

by using **First Shift Theorem.**

Answer:

$$f(t) = 2\cos 4t\ \cosh 3t$$

$$F(s) = \zeta\left\{2\cos 4t\ \cosh 3t\right\}$$

According to Hyperbolic function :

$$\cosh x = \frac{1}{2}\left(e^{x}+e^{-x}\right)$$

Then :

$$\cosh 3t = \frac{1}{2}\left(e^{3t} + e^{-3t}\right)$$

by writing :

$$2\cos 4t \cosh 3t = 2\cosh\ 3t\ \ f(t)$$

Where :

$$f(t) = \cos 4t$$

$$F(s) = \zeta\{\cos 4t\}$$

$$= \frac{s}{s^2 + 4^2}$$

by using the First Shift Theorem :

$$2\zeta\{\cos 4t \cosh 3t\} = \frac{1}{2}\left(e^{3t} + e^{-3t}\right) f(t)$$

$$= 2\left[\frac{1}{2}\left(\frac{(s-3)}{(s-3)^2 + 4^2} - \frac{(s+3)}{(s+3)^2 + 4^2}\right)\right]$$

$$= \frac{(s-3)}{(s-3)^2 + 4^2} - \frac{(s+3)}{(s+3)^2 + 4^2}$$

Question 4

Determine the **Laplace Transform** of the following functions

$$f(t) = 0.5\sin 2t\ \cosh 6t$$

by using **First Shift Theorem.**

Answer:

$f(t) = 0.5\sin 2t\ \cosh 6t$

$F(s) = \zeta\{0.5\sin 2t\ \cosh 6t\}$

According to Hyperbolic function :

$$\cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right)$$

Then :

$$\cosh 6t = \frac{1}{2}\left(e^{6t} + e^{-6t}\right)$$

by writing :

$$0.5\sin 2t \ \cosh 6t = 0.5\cosh \ 6t \ \ f(t)$$

Where :

$$f(t) = \sin 2t$$

$$F(s) = \zeta\left\{\cos 2t\right\}$$

$$= \frac{2}{s^{2} + 2^{2}}$$

by using the First Shift Theorem :

$$0.5\zeta\left\{\cos 2t \cosh 6t\right\} = \frac{1}{2}\left(e^{6t} + e^{-6t}\right) f(t)$$

$$= 0.5\left[\frac{1}{2}\left(\frac{2}{(s-6)^{2} + 2^{2}} - \frac{2}{(s+6)^{2} + 2^{2}}\right)\right]$$

$$= \frac{1}{2}\left[\frac{1}{(s-6)^{2} + 4} - \frac{1}{(s+6)^{2} + 4}\right]$$

Question 5

Determine the **Laplace Transform** of the following functions

$$f(t)=\frac{1}{4}\cos t\ \cosh 9t$$

by using **First Shift Theorem.**

Answer:

$$f(t)=\frac{1}{4}\cos t\ \cosh 9t$$

$$F(s)=\zeta\left\{\frac{1}{4}\cos t\ \cosh 9t\right\}$$

According to Hyperbolic function :

$$\cosh x=\frac{1}{2}\left(e^{x}+e^{-x}\right)$$

Then :

$$\cosh 9t=\frac{1}{2}\left(e^{9t}+e^{-9t}\right)$$

by writing :

$$\frac{1}{4}\cos t\ \cosh 9t\ =\frac{1}{4}\cosh 9t\ \ f(t)$$

Where :

$$f(t)=\cos t$$

$$F(s)=\zeta\left\{\cos t\right\}$$

$$=\frac{s}{s^{2}+1^{2}}$$

by using the First Shift Theorem :

$$\frac{1}{4}\zeta\left\{\cos t\ \cosh 9t\right\}=\frac{1}{2}\left(e^{9t}+e^{-9t}\right)f(t)$$

$$= \frac{1}{4}\left[\frac{1}{2}\left(\frac{(s-9)}{(s-9)^2+1^2}+\frac{(s+9)}{(s+6)^2+1^2}\right)\right]$$

$$= \frac{1}{8}\left(\frac{(s-9)}{(s-9)^2+1^2}+\frac{(s+9)}{(s+6)^2+1^2}\right)$$

Question 1

For the following functions, determine the Laplace Transform For $f(t) = t^2 \sin at$ using frequency differentiation.

Answer:

$$f(t) = t^2 \sin at$$

We know that $\mathcal{L}\ \{\sin at\} = \left[\frac{a}{(s^2 + a^2)}\right]$

$$\mathcal{L}\ \{t^2 \sin at\} = (-1)\frac{d}{ds}\left(\frac{a}{(s^2 + a^2)}\right)$$

$$\frac{U}{V} = \frac{V\frac{dU}{dt} - U\frac{dV}{dt}}{V^2}$$

$$V = s^2 + a^2$$

$$\frac{dV}{ds} = 2s$$

$$U = a$$

$$\frac{dU}{ds} = 0$$

$$\frac{U}{V} = (-1)\frac{(s^2 + a^2)(0) - a(2s)}{(s^2 + a^2)^2}$$

$$= (-1)\frac{-2as}{(s^2 + a^2)^2}$$

$$= \frac{2as}{(s^2 + a^2)^2}$$

Question 2

For $f(t) = \cos 3t$ using the integral definition.

Answer :

$$\cos 3t = \frac{1}{2}\left(e^{j3t} + e^{-j3t}\right)$$

$$\mathcal{L}\{cos\ 3t\} = \frac{1}{2}\left[\mathcal{L}\left(e^{j3t}\right) + \mathcal{L}\left(e^{-j3t}\right)\right]$$

$$= \frac{1}{2}\left(\frac{1}{s - j3} + \frac{1}{s + j3}\right)$$

$$= \frac{1}{2}\left(\frac{s + j3 + (s - j3)}{(s - j3)(s + j3}\right)$$

$$= \frac{s}{s^2 + 3^2}$$

2.2.1 Partial fractions

Question 1

Determine the following Inverse Laplace Transform function F(s), in the form of parctial fraction and then determine inverse laplace for following function.

$$F(s) = \frac{s^2 + 6s + 6}{(s+1)(s+2)^2}$$

Answer:

$$F(s) = \frac{s^2 + 6s + 6}{(s+1)(s+2)^2}$$

$$s^2 + 6s + 6 = \frac{A}{(s+1)} + \frac{B}{(s+2)} + \frac{C}{(s+2)^2}$$

$$s^2 + 6s + 6 = A(s+2)^2 + B(s+1)(s+2) + C(s+1)$$

$$\begin{aligned} &\textit{Gantikan} \quad s = -1, \\ &(-1)^2 + 6(-1) + 6 = A(-1+2)^2 \\ &A = 1 \end{aligned}$$

$$\begin{aligned} &\textit{Gantikan} \quad s = -2, \\ &(-2)^2 + 6(-2) + 6 = C(-2+1) \\ &C = \frac{-2}{-1} = 2 \end{aligned}$$

$$\begin{aligned} &\textit{Gantikan} \quad s = 0, \\ &(0)^2 + 6(0) + 6 = 1(0+2) + B(0+1)(0+2) + 2(0+1) \\ &B = \frac{6-6}{2} = 0 \end{aligned}$$

$$\frac{s^2 + 6s + 6}{(s+1)(s+2)^2} = \frac{1}{(s+1)} + \frac{2}{(s+2)^2}$$

$$F(s) = \mathcal{L}^{-1}\left\{\frac{1}{(s+1)}\right\} + \mathcal{L}^{-1}\left\{\frac{2}{(s+2)^2}\right\}$$

$$f(t) = e^{-t} + 2te^{-2t}$$

Question 2

$$F(s) = \frac{s^2+12}{s(s+2)(s+3)}$$

Answer:

$$F(s) = \frac{s^2+12}{s(s+2)(s+3)}$$

$$\frac{s^2+12}{s(s+2)(s+3)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{(s+3)}$$

$$A = sF(s)\Big|_{s=0} = \frac{s^2+12}{(s+2)(s+3)}\Big|_{s=0} = \frac{12}{(2)(3)} = 2$$

$$B = (s+2)F(s)\Big|_{s=-2} = \frac{s^2+12}{s(s+3)}\Big|_{s=-2} = \frac{4+12}{(-2)(1)} = -8$$

$$C = (s+3)F(s)\Big|_{s=-3} = \frac{s^2+12}{s(s+2)}\Big|_{s=-3} = \frac{9+12}{(-3)(-1)} = 7$$

$$\frac{s^2+12}{s(s+2)(s+3)} = \frac{2}{s} - \frac{8}{s+2} + \frac{7}{(s+3)}$$

$$f(t) = \mathcal{L}^{-1}\left\{\frac{2}{s}\right\} - 8\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} + 7\mathcal{L}^{-1}\left\{\frac{1}{(s+3)}\right\}$$

$$f(t) = 2 - 8e^{-2t} + 7e^{-3t}$$

Question 3

Use the method of partial fractions to find the given inverse laplace transforms.

$$\mathcal{L}^{-1}\left\{\frac{s+3}{s^2+4s-5}\right\}$$

Answer:

We can factor the denominator to obtain

$$s^2+4s-5=(s-1)(s+5)$$

The partial fractions decomposition is of the form

$$\frac{s+3}{(s-1)(s+5)}=\frac{A}{(s-1)}+\frac{B}{(s+5)}$$

Using the cover-up method we obtain A and B as follows

$$A=\frac{s+3}{(s-1)(s+5)}\bigg|_{s=1}=\frac{2}{3}$$

$$B=\frac{s+3}{(s-1)(s+5)}\bigg|_{s=-5}=\frac{1}{3}$$

$$\mathcal{L}^{-1}\left\{\frac{s+3}{s^2+4s-5}\right\}=\frac{2}{3}\mathcal{L}^{-1}\left\{\frac{1}{(s-1)}\right\}+\frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{(s+5)}\right\}$$

$$=\frac{2}{3}e^{t}+\frac{1}{3}e^{-5t}$$

Question 4

Use the method of partial fractions to find the given inverse laplace transforms.

$$\mathcal{L}^{-1}\left\{\frac{s+1}{(2s-1)(s+2)}\right\}$$

Answer:

The partial fractions decomposition is of the form

$$\frac{s+1}{(2s-1)(s+2)}=\frac{A}{(2s-1)}+\frac{B}{(s+2)}$$

Using the cover-up method we obtain A and B as follows

$$A = \frac{s+1}{(2s-1)(s+2)}\Bigg|_{s=\frac{1}{2}} = \frac{3}{5}$$

$$B = \frac{s+1}{(2s-1)(s+2)}\Bigg|_{s=-2} = \frac{1}{5}$$

$$\mathcal{L}^{-1}\left\{\frac{s+1}{(2s-1)(s+2)}\right\} = \frac{3}{5.2}\mathcal{L}^{-1}\left\{\frac{1}{\left(s-\frac{1}{2}\right)}\right\} + \frac{1}{5}\mathcal{L}^{-1}\left\{\frac{1}{(s+2)}\right\}$$

$$= \frac{3}{10}e^{\frac{1}{2}t} + \frac{1}{5}e^{-2t}$$

Question 5

Write the function in the form of **partial fractions** and then construct the **Laplace Laplace Inverse** for the functions.

$$F(s) = \frac{6s^2+8s+3}{s(s^2+2s+5)}$$

Answer:

$$F(s) = \frac{6s^2+8s+3}{s(s^2+2s+5)}$$

$$\frac{6s^2+8s+3}{s(s^2+2s+5)} = \frac{A}{s} + \frac{Bs+c}{s^2+2s+5}$$

$$6s^2+8s+3 = A(s^2+2s+5) + Bs^2 + Cs$$

: $3 = 5A$

Constant $A = \frac{3}{5}$

We equate coefficients :

s^2 : $6 = A + B$

s : $8 = 2A + C$

Gantikan $A = \frac{3}{5}$ dalam persamaan 1

$6 = A + B$

$B = 6 - \frac{3}{5} \quad = \frac{27}{5}$

Gantikan $A = \frac{3}{5}$ dalam persamaan 2

$8 = 2A + C$

$C = 8 - 2\left(\frac{3}{5}\right) \quad = \frac{34}{5}$

$$\frac{6s^2 + 8s + 3}{s(s^2 + 2s + 5)} = \frac{3}{5s} + \frac{\frac{27}{5}s + \frac{34}{5}}{(s+1)^2 + 2^2}$$

$$f(t) = \mathcal{L}^{-1}\left\{\frac{3}{5s}\right\} - \frac{27}{5}\mathcal{L}^{-1}\left\{\frac{(s+1)}{(s+1)^2 + 2^2}\right\} + \frac{7}{10}\mathcal{L}^{-1}\left\{\frac{2}{(s+1)^2 + 2^2}\right\}$$

$$f(t) = \frac{3}{5} - \frac{27}{5}e^{-t}\cos 2t + \frac{7}{10}e^{-t}\sin 2t$$

2.2.2 Completing the square

Question 1

Convert following expression function F(s) in the form of partial fractions and then determine their Inverse Laplace.

$$F(s) = \frac{s-2}{s^2 - 2s + 5}$$

Answer :

By completing the square, we get

Formula:

$$x^2 + bx + c = 0$$

$$x^2 + bx = -c$$

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = -c + \left(\frac{b}{2}\right)^2$$

$$s^2 + 2s + 5$$

$$x^2 + 2x + \left(\frac{2}{2}\right)^2 = -5 + \left(\frac{2}{2}\right)^2$$

$$(s+1)^2 + 4$$

$$s^2 + 2s + 5 = (s+1)^2 + 4$$

$$\frac{s-2}{s^2+2s+5} = \frac{(s+1)-3}{(s+1)^2+4}$$

$$\frac{(s+1)-3}{(s+1)^2+4} = \frac{(s+1)}{(s+1)^2+4} - \frac{3}{(s+1)^2+4}$$

$$= \mathcal{L}^{-1}\left\{\frac{(s+1)}{(s+1)^2+2^2}\right\} - \frac{3}{2}\mathcal{L}^{-1}\left\{\frac{2}{(s+1)^2+2^2}\right\}$$

$$= e^{-t}cos\ 2t - \frac{3}{2}e^{-t}sin\ 2t$$

Question 2

By using **completing the square**, value the **Inverse Laplace** following expression function,

$$F(s) = \frac{1}{s^2 - 4s + 7}$$

Completing the Square:

$s^2 - 4s + 7$

$x^2 - 4x + \left(\frac{4}{2}\right)^2 = -7 + \left(\frac{4}{2}\right)^2$

$(s-2)^2 + 3$

$s^2 + 2s + 5 = (s-2)^2 + 3$

$$\frac{1}{s^2 - 4s + 7} = \frac{1}{(s-2)^2 + 3}$$

$$= \frac{1}{\sqrt{3}} \mathcal{L}^{-1} \left\{ \frac{\sqrt{3}}{(s-2)^2 + 3} \right\}$$

Question 3

By using **completing the square and partial fractions**, value the **Inverse Laplace** following expression function,

$$F(s) = \frac{s^2 + 3s + 5}{(s-2)(s^2 + 8s + 22)}$$

Answer:

$$F(s) = \frac{s^2 + 3s + 5}{(s-2)(s^2 + 8s + 22)}$$

Method 1:

$$\frac{s^2 + 3s + 5}{(s-2)(s^2 + 8s + 22)} = \frac{A}{s-2} + \frac{Bs + C}{s^2 + 8s + 22}$$

$$s^2 + 3s + 5 = A(s^2 + 8s + 22) + (Bs + C)(s-2)$$

$$s^2 + 3s + 5 = A(s^2 + 8s + 22) + B(s^2 - 2s) + C(s-2)$$

We equate coefficients :

s^2	:	$1 = A + B$
s	:	$3 = 8A - 2B + C$
Constant :		$5 = 22A - 2C$

$$A = \frac{2}{5}$$

$$B = \frac{3}{5}$$

$$C = \frac{7}{5}$$

Method 2: Kaedah gantian

$$s^2 + 3s + 5 = A(s^2 + 8s + 22) + (s-2)(Bs + C)$$

Gantikan s= 2

$$s^2 + 3s + 5 = A(s^2 + 8s + 22) + (s-2)(Bs + C)$$

$$(2)^2 + 3(2) + 5 = A((2)^2 + 8(2) + 22)$$

$$9 - 12 + 5 = A(9 - 6 + 2)$$

$$2 = A(5)$$

$$A = \frac{2}{5}$$

Gantikan s= 0

$$s^2 + 4s + 5 = A(s^2 + 2s + 2) + (s+3)(Bs + C)$$

$$(0)^2 + 4(0) + 5 = \frac{2}{5}((0)^2 + 2(0) + 2) + (0+3)(B(0) + C)$$

$$5 = \frac{4}{5} + 3(C)$$

$$3C = 5 - \frac{4}{5}$$

$$3C = \frac{21}{5} \times \frac{1}{3}$$

$$C = \frac{7}{5}$$

Gantikan s= 1

$$s^2 + 4s + 5 = A(s^2 + 2s + 2) + (s+3)(Bs + C)$$

$$(1)^2 + 4(1) + 5 = \frac{2}{5}((1)^2 + 2(1) + 2) + (1+3)(B(1) + \frac{7}{5})$$

$$14+5=\frac{2}{5}(1+2+2)+\left(B+\frac{7}{5}\right)(4)$$

$$4B=10-2-\frac{28}{5}$$

$$B=\frac{12}{5}\times\frac{1}{4}$$

$$B=\frac{3}{5}$$

$$F(s)=\frac{s^2+3s+5}{(s-2)(s^2+8s+22)}$$

$$\frac{s^2+4s+5}{(s+3)(s^2+2s+2)}=\frac{\frac{2}{5}}{s+3}+\frac{\frac{3}{5}s+\frac{7}{5}}{s^2+2s+2}$$

Using completing the square:

$$\left(s+\frac{b}{2}\right)^2=-c+\left(\frac{b}{2}\right)^2$$

$$\left(s+\frac{2}{2}\right)^2=-2+\left(\frac{2}{2}\right)^2$$

$$(s+1)^2=-1$$

$$(s+1)^2+1=0$$

$$=\frac{0.4}{s+3}+\frac{0.6s+1.4}{(s+1)^2+1}$$

$$=\frac{0.4}{s+3}+\frac{0.6(s+1)+0.8}{(s+1)^2+1}$$

$$=\frac{0.4}{s+3}+\frac{0.6(s+1)}{(s+1)^2+1}+\frac{0.8}{(s+1)^2+1}$$

$$f(t) = \mathcal{L}^{-1}\left\{\frac{0.4}{s+3}\right\} + \mathcal{L}^{-1}\left\{\frac{0.6(s+1)}{(s+1)^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{0.8}{(s+1)^2+1}\right\}$$

$$f(t) = 0.4e^{-3t} + 0.6e^{-t}\cos t + 0.8e^{-t}\sin t$$

Question 4

By using **completing the square and partial fractions**, value the **Inverse Laplace** following expression function,

$$F(s) = \frac{6s^2+8s+3}{s(s^2+2s+5)}$$

Answer:

$$F(s) = \frac{6s^2+8s+3}{s(s^2+2s+5)}$$

$$\frac{6s^2+8s+3}{s(s^2+2s+5)} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+5}$$

Method 2: Kaedah gentian

$$6s^2+8s+3 = A(s^2+2s+5)+(Bs+C)(s)$$

Gantikan s= 0

$$6s^2+8s+3 = A(s^2+2s+5)+(Bs+C)(s)$$

$$6(0)^2+8(0)+3 = A((0)^2+2(0)+5)+(B(0)+C)(0)$$

$$3 = A(5)$$

$$A = \frac{3}{5}$$

Gantikan s= 1

$$6s^2+8s+3=A(s^2+2s+5)+(Bs+C)(s)$$

$$6(1)^2+8(1)+3=\frac{3}{5}((1)^2+2(1)+5)+(B(1)+C)(1)$$

$$17=\frac{24}{5}+B+C$$

$$B+C=\frac{61}{5}$$

Gantikan s= -1

$$6s^2+8s+3=A(s^2+2s+5)+(Bs+C)(s)$$

$$6(-1)^2+8(-1)+3=\frac{3}{5}((-1)^2+2(-1)+5)+(B(-1)+C)(-1)$$

$$1=\frac{12}{5}+B-C$$

$$B-C=-\frac{7}{5}$$

Persamaan serentak

$$B+C=\frac{61}{5}$$

$$B-C=-\frac{7}{5}$$

$$B=\frac{27}{5} \qquad C=\frac{34}{5}$$

$$\frac{6s^2+8s+3}{s(s^2+2s+5)}=\frac{A}{s}+\frac{Bs+C}{s^2+2s+5}$$

$$\frac{6s^2+8s+3}{s(s^2+2s+5)}=\frac{\frac{3}{5}}{s}+\frac{\frac{27}{5}s+\frac{34}{5}}{s^2+2s+5}$$

Using completing the square:

$$\left(s+\frac{b}{2}\right)^2+c-\left(\frac{b}{2}\right)^2=$$

$$\left(s+\frac{2}{2}\right)^2+5-\left(\frac{2}{2}\right)^2$$

$$(s+4)^2+2^2$$

$$=\frac{3}{5s}+\frac{\frac{27}{5}(s+1)+\frac{7}{5}}{(s+1)^2+2^2}$$

$$=\frac{3}{5s}+\frac{\frac{27}{5}(s+1)}{(s+1)^2+2^2}+\frac{\frac{7}{5}}{(s+1)^2+2^2}$$

$$=\frac{3}{5s}+\frac{\frac{27}{5}(s+1)}{(s+1)^2+2^2}+\left[\frac{\frac{7}{5}}{(s+1)^2+2^2}\times\frac{2}{2}\right]$$

$$=\frac{3}{5s}+\frac{\frac{27}{5}(s+1)}{(s+1)^2+2^2}+\left[\frac{\frac{7}{5}}{2}\times\frac{2}{(s+1)^2+2^2}\right]$$

$$f(t)=\mathcal{L}^{-1}\left\{\frac{3}{5s}\right\}+\mathcal{L}^{-1}\left\{\frac{\frac{27}{5}(s+1)}{(s+1)^2+2^2}\right\}+\mathcal{L}^{-1}\,\frac{7}{10}\left[\frac{2}{(s+1)^2+2^2}\right]$$

$$f(t)=\frac{3}{5}+\frac{27}{5}e^{-t}\cos 2t+\frac{7}{10}e^{-t}\sin 2t$$

Question 1

Determine the first derivative time domain equation below by using the Laplace Transform :

$$\frac{dv(t)}{dt} + 4v(t) + 8\int_0^t v\,dt = 4e^{-2t}$$

given at $v(0) = 1$,

Answer :

$$\mathcal{L}\left\{\frac{dv}{dt}\right\} + 4\mathcal{L}\left\{v(t)\right\} + 8\mathcal{L}\left\{\int_0^t v\,dt\right\} = \mathcal{L}\left\{4e^{-2t}\right\}$$

Dimana

$$\left[sV(s) - v(0)\right] + 4\left[V(s)\right] + 8\left[\frac{V(s)}{s}\right] = \frac{4}{s+2}$$

$$\left[sV(s) - 1\right] + 4V(s) + \frac{8}{s}V(s) = \frac{4}{s+2}$$

$$V(s)\left[s + 4 + \frac{8}{s}\right] = \frac{4}{s+2} + 1$$

$$V(s)\left[\frac{s^2 + 4s + 8}{s}\right] = \frac{4 + s + 2}{s+2}$$

$$V(s) = \frac{s+6}{s+2} \times \frac{s}{s^2 + 4s + 8}$$

$$V(s) = \frac{s^2 + 6s}{(s+2)(s^2 + 4s + 8)}$$

$$\frac{s^2 + 6s}{(s+2)(s^2 + 4s + 8)} = \frac{A}{s+2} + \frac{Bs + C}{s^2 + 4s + 8}$$

$$s^2+6s=A(s^2+4s+8)+(Bs+C)(s+2)$$

Gantikan $s=-2,$

$$s^2+6s=A(s^2+4s+8)+(Bs+C)(s+2)$$
$$(-2)^2+6(-2)=A((-2)^2+4(-2)+8)$$
$$A=-2$$

$$s^2: \quad 1=A+B$$
$$s: \quad 6=4A+2B+C$$
$$pekali: \quad 0=8A+2C$$

Gantikan $A=-2,$ dalam persamaan s^2

$$1=A+B$$
$$1=-2+B$$
$$B=1+2$$
$$B=3$$

Gantikan $A=-2,$ dalam persamaan pekali

$$0=8A+2C$$
$$0=8(-2)+2C$$
$$2C=16$$
$$C=8$$

$$\frac{s^2+6s}{(s+2)(s^2+4s+8)}=\frac{-2}{s+2}+\frac{3s+8}{s^2+4s+8}$$

Melengkapkan kuasa dua

$$\left(s+\frac{b}{2}\right)^2+c-\left(\frac{b}{2}\right)^2$$
$$\left(s+\frac{4}{2}\right)^2+8-\left(\frac{4}{2}\right)^2$$
$$(s+2)^2+8-4$$
$$(s+2)^2+4$$

$$V(s) = \frac{3s+8}{(s+2)^2+2^2} - \frac{2}{s+2}$$

$$V(s) = \frac{3(s+2)+2}{(s+2)^2+2^2} - \frac{2}{s+2}$$

$$v(t) = \zeta^{-1}\left\{\frac{3(s+2)}{(s+2)^2+2^2}\right\} + \zeta^{-1}\left\{\frac{2}{(s+2)^2+2^2}\right\} - 2\zeta^{-1}\left\{\frac{1}{s+2}\right\}$$

$$v(t) = 3e^{-2t}\zeta^{-1}\left\{\frac{s}{s^2+2^2}\right\} + e^{-2t}\zeta^{-1}\left\{\frac{2}{s^2+2^2}\right\} - 2\zeta^{-1}\left\{\frac{1}{s+2}\right\}$$

$$v(t) = 3e^{-2t}\cos 2t + e^{-2t}\sin 2t - 2e^{-2t}$$

Question 2

Transform the first derivative and second derivative time domain equation below by using the Laplace Transform:

$$\frac{dy}{dt} + 4y(t) + 3\int_0^t y(\tau)d\tau = 6e^{-2t}$$

given at t = 0 , x = -1

Answer:

i. $\frac{dy}{dt} + 4y(t) + 3\int_0^t y(\tau)d\tau = 6e^{-2t}$ given at t = 0 , x = -1

$$\mathcal{L}\left\{\frac{dy}{dt}\right\} + 4\,\mathcal{L}\left\{y(t)\right\} + 3\,\mathcal{L}\left\{\int_0^t y(\tau)d\tau\right\} = 6\mathcal{L}\left\{e^{-2t}\right\}$$

Dimana

$$\left[sY(s) - y(0)\right] + 4Y(s) + \frac{3}{s}Y(s) = \frac{6}{s+2}$$

$$sY(s) + 1 + 4Y(s) + \frac{3}{s}Y(s) = \frac{6}{s+2}$$

$$sY(s) + 4Y(s) + \frac{3}{s}Y(s) = \frac{6}{s+2} - 1$$

$$Y(s)\left[s+4+\frac{3}{s}\right]=\frac{6}{s+2}-\frac{(s+2)}{(s+2)}$$

$$Y(s)\left[\frac{s^2+4s+3}{s}\right]=\frac{4-s}{s+2}$$

$$Y(s)=\frac{s(4-s)}{(s+1)(s+2)(s+3)}$$

$$s(4-s)=\frac{A}{(s+1)}+\frac{B}{(s+2)}+\frac{C}{(s+3)}$$

$$s(4-s)=A(s+2)(s+3)+B(s+1)(s+3)+C(s+1)(s+2)$$

Gantikan $s=-1,$

$$(-1)(4-(-1))=A(-1+2)(-1+3)$$

$$A=\frac{-5}{2}$$

Gantikan $s=-2,$

$$(-2)(4-(-2))=B(-2+1)(-2+3)$$

$$B=\frac{-16}{-1}=16$$

Gantikan $s=-3,$

$$(-3)(4-(-3))=C(-3+1)(-3+2)$$

$$C=\frac{-21}{2}$$

$$\frac{s(4-s)}{(s+1)(s+2)(s+3)}=\frac{-5}{2(s+1)}+\frac{16}{(s+2)}-\frac{21}{2(s+3)}$$

$$\frac{2s+1}{(s+2)(s+3)}=+\frac{5}{(s+3)}-\frac{3}{(s+2)}$$

$$F(s)=\mathcal{L}^{-1}\left\{\frac{-5}{2(s+1)}\right\}+\mathcal{L}^{-1}\left\{\frac{16}{(s+2)}\right\}-\mathcal{L}^{-1}\left\{\frac{21}{2(s+3)}\right\}$$

$$f(t)=-2.5e^{-t}+16e^{-2t}-10.5e^{-3t}$$

Question 3

The following is an analysis of the second-order differential equations. Analysis of the second-order differential equations Transform second derivative time domain equation below by using the Laplace Transform:

$$\frac{d^2 i(t)}{dt^2} + 4\frac{di(t)}{dt} + 5i(t) = 2e^{-2t}$$

given at $i(0) = 1$, $\quad \frac{di(0)}{dt} = 2$

Answer:

$$\mathcal{L}\left\{\frac{d^2 i(t)}{dt^2}\right\} + 4\mathcal{L}\left\{\frac{di(t)}{dt}\right\} + 5\,\mathcal{L}\,\{i(t)\} = 2\,\mathcal{L}\left\{e^{-2t}\right\}$$

Dimana

$$\left[s^2 I(s) - si(0) - i'(0)\right] + 4\left[sI(s) - i(0)\right] + 5I(s) = 2\left[\frac{1}{s+2}\right]$$

$$[s^2 I(s) - s(1) - (2)] + 4[sI(s) - (1)] + 5I(s) = \frac{2}{s+2}$$

$$s^2 I(s) - s - 2 + 4sI(s) - 4 + 5I(s) = \frac{2}{s+2}$$

$$I(s)\left[s^2 + 4s + 5\right] = \frac{2}{s+2} + s + 2 + 4$$

$$I(s)\left[s^2+4s+5\right]=\frac{2}{s+2}+s+6$$

$$=\frac{2}{s+2}+\frac{s+6}{1}$$

$$=\frac{2}{s+2}+\left[\frac{(s+6)}{1}\times\frac{(s+2)}{(s+2)}\right]$$

$$=\frac{2+(s+6)(s+2)}{s+2}$$

$$=\frac{2+s^2+8s+12}{s+2}$$

$$=\frac{s^2+8s+14}{s+2}$$

$$I(s)=\frac{\frac{s^2+8s+14}{s+2}}{s^2+4s+5}$$

$$=\frac{s^2+8s+14}{s+2}\times\frac{1}{s^2+4s+5}$$

$$=\frac{s^2+8s+14}{(s+2)(s^2+4s+5)}$$

$$I(s)=\frac{s^2+8s+14}{(s+2)(s^2+4s+5)}=\frac{A}{(s+2)}+\frac{Bs+C}{(s^2+4s+5)}$$

$$s^2+8s+14=(A)(s^2+4s+5)+(Bs+C)(s+2)$$

Gantikan $s=-1,$

$$s^2+8s+14=(A)(s^2+4s+5)+(Bs+C)(s+2)$$

$$(-2)^2+8(-2)+14=(A)((-2)^2+4(-2)+5)$$

$$2=A(1)$$
$$A=2$$

Gantikan $s=0,$

$$s^2+8s+14=(A)(s^2+4s+5)+(Bs+C)(s+2)$$

$$(0)^2+8(0)+14=(A)((0)^2+4(0)+5)+(B(0)+C)(0+2)$$

$$14 = 5A + 2C$$
$$14 = 5(2) + 2C$$
$$2C = 14 - 10$$
$$C = \frac{4}{2} = 2$$

Gantikan $s = 1,$

$$s^2 + 8s + 14 = (A)(s^2 + 4s + 5) + (Bs + C)(s + 2)$$

$$(1)^2 + 8(1) + 14 = (A)((1)^2 + 4(1) + 5) + (B(1) + C)(1 + 2)$$

$$1 + 8 + 14 = A(10) + (B + C)3$$
$$3B = 23 - 20 - 6$$
$$B = \frac{-3}{3} = -1$$

$$I(s) = \frac{s^2 + 8s + 14}{(s+2)(s^2 + 4s + 5)} = \frac{2}{(s+2)} + \frac{(-s+2)}{(s^2 + 4s + 5)}$$

$$I(s) = \frac{s^2 + 8s + 14}{(s+2)(s^2 + 4s + 5)} = \frac{2}{(s+2)} - \frac{(s-2)}{(s^2 + 4s + 5)}$$

$$I(s) = \frac{s^2 + 8s + 14}{(s+2)(s^2 + 4s + 5)} = \frac{2}{(s+2)} - \frac{(s+2) - 4}{(s+2)^2 + 1^2)}$$

$$I(s) = \frac{s^2 + 8s + 14}{(s+2)(s^2 + 4s + 5)} = \frac{2}{(s+2)} - \frac{(s+2)}{(s+2)^2 + 1^2)} - \frac{4}{(s+2)^2 + 1^2)}$$

Gunakan kaedah melengkapkan kuasa dua

$$\left(s + \frac{b}{2}\right)^2 + c - \left(\frac{b}{2}\right)^2$$
$$s^2 + 4s + 5 \quad \Longrightarrow \quad a = 1, \quad b = 4, \quad c = 5$$
$$\left(s + \frac{4}{2}\right)^2 + 5 - \left(\frac{4}{2}\right)^2$$
$$(s + 2)^2 + 5 - 4$$
$$(s + 2)^2 + 1$$

Taking the inverse laplace transform:

$$i(t) = \mathcal{L}^{-1}\left\{\frac{2}{s+2}\right\} - \mathcal{L}^{-1}\left\{\frac{(s+2)}{(s+2)^2+1^2}\right\} - \mathcal{L}^{-1}\left\{\frac{4}{(s+2)^2+1^2}\right\}$$

$$i(t) = 2e^{-2t} - e^{-2t}\cos t - 4e^{-2t}\sin t$$

Question 4

The following is an analysis of the second-order differential equations. Analysis of the second-order differential equations Transform second derivative time domain equation below by using the Laplace Transform:

$$\frac{d^2 i(t)}{dt^2} + 4\frac{di(t)}{dt} + 5i(t) = 2e^{-2t}$$

given at $i(0) = 0,\quad \frac{di(0)}{dt} = 2$

Answer:

$$\mathcal{L}\left\{\frac{d^2 i(t)}{dt^2}\right\} + 4\mathcal{L}\left\{\frac{di(t)}{dt}\right\} + 5\,\mathcal{L}\,\{i(t)\} = 2\,\mathcal{L}\left\{e^{-2t}\right\}$$

Dimana

$$\left[s^2 I(s) - si(0) - i'(0)\right] + 4\left[sI(s) - i(0)\right] + 5I(s) = 2\left[\frac{1}{s+2}\right]$$

$$[s^2 I(s) - s(0) - (2)] + 4[sI(s) - (0)] + 5I(s) = \frac{2}{s+2}$$

$$s^2 I(s) - 2 + 4sI(s) + 5I(s) = \frac{2}{s+2}$$

$$I(s)\left[s^2 + 4s + 5\right] - 2 = \frac{2}{s+2}$$

$$I(s)\left[s^2 + 4s + 5\right] = \frac{2}{s+2} + 2$$

$$I(s)=\frac{\frac{2+2s+4}{s+2}}{s^2+4s+5}$$

$$I(s)=\frac{2s+6}{s+2}\times\frac{1}{s^2+4s+5}$$

$$I(s)=\frac{2s+6}{(s+2)(s^2+4s+5)}$$

$$I(s)=\frac{2s+6}{(s+2)(s^2+4s+5)}=\frac{A}{(s+2)}+\frac{Bs+C}{(s^2+4s+5)}$$

$$2s+6=(A)(s^2+4s+5)+(Bs+C)(s+2)$$

Gantikan $s=-2,$

$$2s+6=(A)(s^2+4s+5)+(Bs+C)(s+2)$$

$$2(-2)+6=(A)((-2)^2+4(-2)+5)$$

$$A=2$$

Gantikan $s=0,$

$$2s+6=(A)(s^2+4s+5)+(Bs+C)(s+2)$$

$$2(0)+6=(A)((0)^2+4(0)+5)+(B(0)+C)((0)+2)$$

$$6-10=2B$$

$$B=-2$$

Gantikan $s=1,$

$$2s+6=(A)(s^2+4s+5)+(Bs+C)(s+2)$$

$$2(1)+6=(2)((1)^2+4(1)+5)+((-2)(1)+C)((1)+2)$$

$$8 = 20 - 6 + 3C$$

$$3C = 8 - 20 + 6$$

$$C = -2$$

$$I(s) = \frac{2s+6}{(s+2)(s^2+4s+5)} = \frac{2}{(s+2)} - \frac{2s-2}{(s^2+4s+5)}$$

Gunakan kaedah melengkapkan kuasa dua

$$\left(s+\frac{b}{2}\right)^2 + c - \left(\frac{b}{2}\right)^2$$

$$s^2 + 4s + 5 \quad \Longrightarrow \; a = 1, \;\; b = 4, \quad c = 5$$

$$\left(s+\frac{4}{2}\right)^2 + 5 - \left(\frac{4}{2}\right)^2$$

$$(s+2)^2 + 5 - 4$$

$$(s+2)^2 + 1$$

$$I(s) = \frac{2}{(s+2)} - \frac{2(s+2)}{(s+2)^2+1} + \frac{2}{(s+2)^2+1}$$

Taking the inverse laplace transform

$$I(s) = \frac{2}{(s+2)} - \frac{2(s+2)}{(s+2)^2+1} + \frac{2}{(s+2)^2+1}$$

$$i(s) = \mathcal{L}^{-1}\left\{\frac{2}{s+2}\right\} - \mathcal{L}^{-1}\left\{\frac{2(s+2)}{(s+2)^2+1}\right\} - \mathcal{L}^{-1}\left\{\frac{2}{(s+2)^2+1}\right\}$$

$$i(t) = 2e^{-2t} - 2e^{-2t}\cos t + 2e^{-2t}\sin t$$

Question 5

The following is an analysis of the second-order differential equations. Analysis of the second-order differential equations Transform second derivative time domain equation below by using the Laplace Transform:

$$\frac{d^2v(t)}{dt^2} + 5\frac{dv(t)}{dt} + 6v(t) = 10e^{-t}$$

given at $v(0) = 2, \quad \frac{dv(0)}{dt} = 4$

Answer:

$$\mathcal{L}\left\{\frac{d^2v}{dt^2}\right\} + 5\mathcal{L}\left\{\frac{dv}{dt}\right\} + 6\,\mathcal{L}\,\{v(t)\} = \mathcal{L}\{10e^{-t}\}$$

Dimana

$$\left[s^2V(s) - sv(0) - v'(0)\right] + 5\left[sV(s) - v(0)\right] + 6V(s) = \frac{10}{s+1}$$

$$[s^2V(s) - s(2) - (4)] + 5[sV(s) - (2)] + 6V(s) = \frac{10}{s+1}$$

$$s^2V(s) - 2s - 4 + 5sV(s) - 10 + 6V(s) = \frac{10}{s+1}$$

$$V(s)\left[s^2 + 5s + 6\right] = \frac{10}{s+1} + 2s + 14$$

$$V(s)\left[s^2 + 5s + 6\right] = \frac{2s^2 + 16s + 24}{(s+1)}$$

$$V(s) = \frac{\frac{2s^2 + 16s + 24}{(s+1)}}{s^2 + 5s + 6}$$

$$V(s) = \frac{2s^2 + 16s + 24}{(s+1)} \times \frac{1}{s^2 + 5s + 6}$$

$$V(s) = \frac{2(s+6)(s+2)}{(s+1)(s+2)(s+3)}$$

$$V(s) = \frac{2(s+6)}{(s+1)(s+3)}$$

$$\frac{2(s+6)}{(s+1)(s+3)} = \frac{A}{s+1} + \frac{B}{s+3}$$

$$2(s+6) = (A)(s+3) + (B)(s+1)$$

Gantikan $s = -1,$

$$2(-1+6) = A(-1+3)$$
$$A = 5$$

Gantikan $s = -3,$

$$2(-3+6) = B(-3+1)$$
$$C = \frac{6}{-2} = -3$$

Gantikan $s = 0,$

$$(0)^2 + 6(0) + 6 = 1(0+2) + B(0+1)(0+2) + 2(0+1)$$
$$B = \frac{6-6}{2} = 0$$

$$\frac{2(s+6)}{(s+1)(s+3)} = \frac{5}{s+1} - \frac{3}{s+3}$$

$$F(s) = \mathcal{L}^{-1}\left\{\frac{5}{(s+1)}\right\} - \mathcal{L}^{-1}\left\{\frac{3}{s+3}\right\}$$

$$f(t) = 5e^{-t} - 3te^{-3t}$$

2.3 Laplace Transform in RLC circuit

Question 1

Diagram 1, shows the connections of several electronic components. By using Mesh Analysis, evaluate the circuit to fine the current i(t), using Laplace Transform. Assuming zero initial condition.

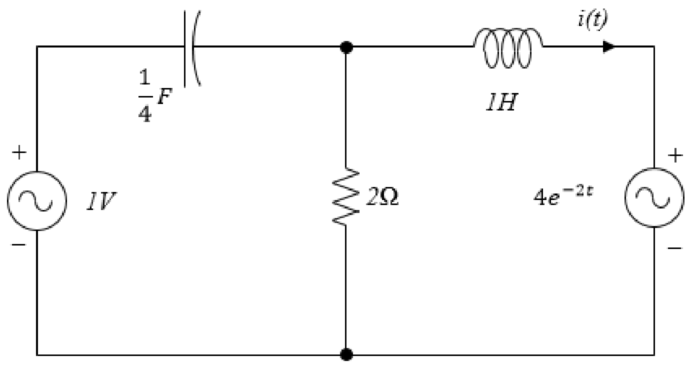


Diagram 1

Answer:

$$\zeta\{1\} = \frac{1}{s}$$

$$\zeta\{1H\} = sL = s$$

$$\zeta\left\{\frac{1}{4}F\right\} = \frac{1}{sC} = \frac{1}{s\frac{1}{4}} = \frac{1}{\frac{s}{4}} = 1\times\frac{4}{s} = \frac{4}{s}$$

$$\zeta\left\{4e^{-2t}\right\} = \frac{4}{s+2}$$

Loop 1

$$\left(\frac{4}{s}+2\right)I_1 - (2)I_2 = \frac{1}{s}$$

Loop 2

$$-(2)I_1 + (s+2)I_2 = \frac{-4}{s+2}$$

Persamaan serentak

$$\left(\frac{4}{s}+2\right)I_1-(2)I_2=\frac{1}{s}$$

$$-(2)I_1+(s+2)I_2=\frac{-4}{s+2}$$

we use Maxwell's equation to get current I_1 and I_2

$$\begin{bmatrix}\frac{2s+4}{2} & -2\\ -2 & s+2\end{bmatrix}\begin{bmatrix}I_1\\ I_2\end{bmatrix}=\begin{bmatrix}\frac{1}{s}\\ \frac{-4}{s+2}\end{bmatrix}$$

$$\Delta=\begin{bmatrix}\frac{2s+4}{2} & -2\\ -2 & s+2\end{bmatrix}$$

$$=\left(\frac{2s+4}{2}\right)(s+2)-(-2)(-2)$$

$$\Delta=\frac{(2s+4)(s+2)}{s}-4$$

$$\Delta=\frac{2s^2+8s+4}{s}-\left[\frac{4}{1}\times\frac{s}{s}\right]$$

$$=\frac{2s^2+8s+4-4s}{s}$$

$$\Delta=\frac{2(s^2+2s+4)}{s}$$

Nilai ΔI_2

$$\Delta I_2=\begin{bmatrix}\frac{2s+4}{s} & \frac{1}{s}\\ -2 & \frac{-4}{s+2}\end{bmatrix}$$

$$=\left(\frac{2s+4}{s}\right)\left(\frac{-4}{s+2}\right)-(-2)\left(\frac{1}{s}\right)$$

$$\Delta I_2=\left(\frac{-8}{s}\right)+\left(\frac{2}{s}\right)$$

$$\Delta I_2 = -\frac{6}{s}$$

$$I_2 = \frac{-\frac{6}{s}}{\frac{2(s^2+2s+4)}{s}}$$

$$= -\frac{6}{s} \times \frac{s}{2(s^2+2s+4)}$$

$$= \frac{-3}{(s^2+2s+4)}$$

Gunakan kaedah melengkapkan kuasa dua

$$\left(s+\frac{b}{2}\right)^2 + c - \left(\frac{b}{2}\right)^2$$

$$s^2+2s+4 \quad ==\triangleright \; a=1, \; b=2, \quad c=4$$

$$\left(s+\frac{2}{2}\right)^2 + 4 - \left(\frac{2}{2}\right)^2$$

$$(s+1)^2 + 4 - 1$$

$$(s+1)^2 + 3$$

$$I_2 = \frac{-3}{(s+1)^2+3}$$

$$I_2 = \frac{\Delta I_2}{\Delta}$$

$$i_2(t) = \zeta^{-1}\left\{\frac{-\left(\sqrt{3}\right)^2}{\left((s+1)^2+\left(\sqrt{3}\right)^2\right)}\right\}$$

$$i_2(t) = -\sqrt{3}e^{-t}\sin\sqrt{3}t \;\; Amp$$

Question 2

Diagram 2, shows the connections of several electronic components. By using Mesh Analysis, evaluate the circuit to fine the current i(t), using Laplace Transform. Assuming zero initial condition.

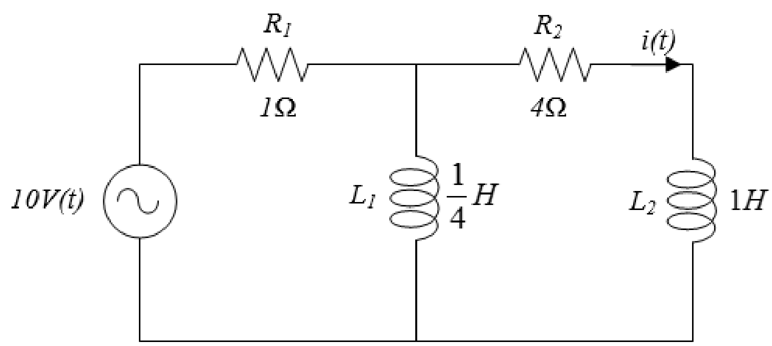


Diagram 2

Answer:

$$\zeta\{10\} = \frac{10}{s}$$
$$\zeta\{1H\} = sL = s$$
$$\zeta\left\{\frac{1}{4}H\right\} = sL = \frac{s}{4}$$

Loop 1

$$\left(1+\frac{s}{4}\right)I_1 - \left(\frac{s}{4}\right)I_2 = \frac{10}{s}$$

Loop 2

$$-\left(\frac{s}{4}\right)I_1 + \left(4+s+\frac{s}{4}\right)I_2 = 0$$

Persamaan serentak

$$\left(\frac{4+s}{4}\right)I_1 - \left(\frac{s}{4}\right)I_2 = \frac{10}{s}$$

$$-\left(\frac{s}{4}\right)I_1 + \left(4+\frac{5s}{4}\right)I_2 = 0$$

we use Maxwell's equation to get current I_1, I_2 and I_3.

$$\begin{bmatrix} \frac{4+s}{4} & -\frac{s}{4} \\ -\frac{s}{4} & 4+\frac{5s}{4} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{10}{s} \\ 0 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} \frac{4+s}{4} & -\frac{s}{4} \\ -\frac{s}{4} & 4+\frac{5s}{4} \end{bmatrix}$$

$$\Delta = \left(\frac{4+s}{4}\right)\left(4+\frac{5s}{4}\right) - \left(-\frac{s}{4}\right)\left(-\frac{s}{4}\right)$$

$$\Delta = \frac{64+20s+16s+5s^2}{16} - \frac{s^2}{16}$$

$$\Delta = \frac{4s^2+36s+64}{16} = \frac{4(s^2+9s+16)}{16}$$

$$\Delta = \frac{s^2+9s+16}{4}$$

Nilai ΔI_2

$$\Delta I_2 = \begin{bmatrix} \frac{4+s}{4} & \frac{10}{s} \\ -\frac{s}{4} & 0 \end{bmatrix}$$

$$\Delta I_2 = \left(\frac{4+s}{4}\right)(0) - \left(-\frac{s}{4}\right)\left(\frac{10}{s}\right)$$

$$\Delta I_2 = \frac{5}{2}$$

$$I_2 = \frac{\Delta I_2}{\Delta}$$

$$I_2 = \frac{\frac{5}{2}}{\frac{s^2 + 9s + 16}{4}}$$

$$I_2 = \frac{5}{2} \times \frac{4}{s^2 + 9s + 16}$$

$$I_2 = \frac{10}{s^2 + 9s + 16}$$

Gunakan kaedah melengkapkan kuasa dua

$$\left(s + \frac{b}{2}\right)^2 + c - \left(\frac{b}{2}\right)^2$$

$$s^2 + 9s + 16 \quad \Longrightarrow \; a = 1, \; b = 9, \; c = 16$$

$$\left(s + \frac{9}{2}\right)^2 + 16 - \left(\frac{9}{2}\right)^2$$

$$\left(s + \frac{9}{2}\right)^2 + 16 - \left(\frac{9}{2}\right)^2$$

$$\left(s + \frac{9}{2}\right)^2 + \frac{17}{4}$$

$$I_2 = \frac{10}{\left(s + \frac{9}{2}\right)^2 + \frac{17}{4}}$$

$$i_2(t) = 10\zeta^{-1}\left\{\frac{1}{\left(\left(s + \frac{9}{2}\right)^2 + \left(\sqrt{\frac{17}{4}}\right)^2\right)}\right\}$$

$$= 10\zeta^{-1}\left\{\frac{1}{\left(\left(s+\frac{9}{2}\right)^2+\left(\frac{17}{4}\right)^2\right)}\right\}\times\frac{\frac{17}{4}}{\frac{17}{4}}$$

$$=10\times\frac{2}{\sqrt{17}}\zeta^{-1}\left\{\frac{\frac{17}{4}}{\left(\left(s+\frac{9}{2}\right)^2+\frac{17}{4}\right)}\right\}$$

$$i_2(t)=\frac{20}{\sqrt{17}}e^{-\frac{9}{2}t}\sinh\frac{9}{2}tA$$

Question 3

Diagram 3, shows the connections of several electronic components. By using Mesh Analysis, evaluate the circuit to fine the voltage $V_O(t)$, using Laplace Transform. Assuming zero initial condition.

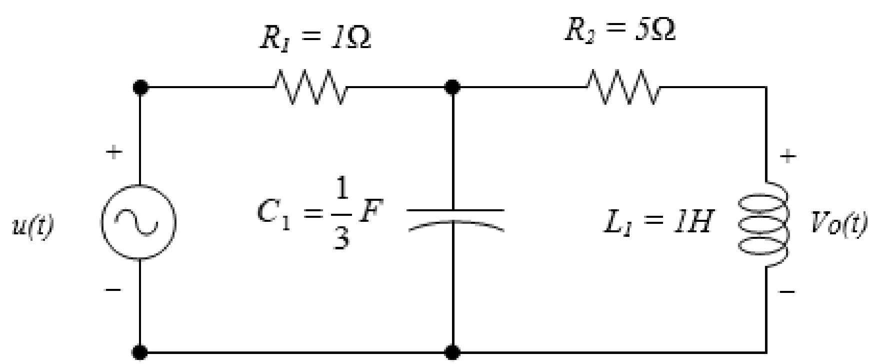
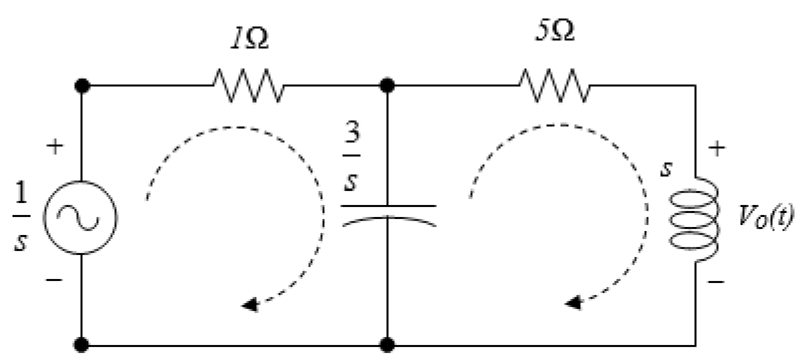


Diagram 3

Answer:



$$\zeta\{u(t)\} = \frac{1}{s}$$
$$\zeta\{1H\} = sL = s$$
$$\zeta\left\{\frac{1}{3}F\right\} = \frac{1}{sC} = \frac{3}{s}$$

Loop 1

$$\frac{1}{s} = I_1\left(1+\frac{3}{s}\right) - I_2\left(\frac{3}{s}\right)$$

Loop 2

$$0 = -I_1\left(\frac{3}{s}\right) + I_2\left(s+5+\frac{3}{s}\right)$$

Jadikan I_1 tajuk rumus pada loop 2

$$0 = -I_1\left(\frac{3}{s}\right) + I_2\left(s+5+\frac{3}{s}\right)$$
$$I_1\left(\frac{3}{s}\right) = I_2\left(s+5+\frac{3}{s}\right)$$
$$I_1 = \left(\frac{s^2+5s+3}{3}\right)I_2$$
$$\frac{1}{s} = I_2\left[\left(\frac{s^2+5s+3}{3}\right)\left(1+\frac{3}{s}\right)\right] - I_2\left(\frac{3}{s}\right)$$
$$= I_2\left[\left(\frac{s^2+5s+3}{3}\right)\left(1+\frac{3}{s}\right) - \left(\frac{3}{s}\right)\right]$$
$$\frac{1}{s} = I_2\left[\frac{s^3+8s^2+18s}{3s}\right]$$
$$I_2 = \frac{1}{s} \times \frac{3s}{s^3+8s^2+18s}$$
$$I_2 = \frac{3}{s(s^2+8s+18)}$$

Dapatkan nilai Vo(s)

$$Vo(s) = I_2 \times s$$

$$I_1 = \frac{3}{s(s^2 + 8s + 18)} \times s$$

$$= \frac{3}{s^2 + 8s + 18}$$

Gunakan kaedah melengkapkan kuasa dua

Gantikan I_1 dalam persamaan I_2

$$\left(s + \frac{b}{2}\right)^2 + c - \left(\frac{b}{2}\right)^2$$

$$s^2 + 8s + 18 \quad \Longrightarrow \quad a = 1, \quad b = 8, \quad c = 18$$

$$\left(s + \frac{8}{2}\right)^2 + 18 - \left(\frac{8}{2}\right)^2$$

$$(s+4)^2 + 18 - 16$$

$$(s+4)^2 + 2$$

$$V_0(s) = \frac{3}{(s+4)^2 + 2}$$

$$V_0(s) = \zeta^{-1}\left\{\frac{3}{(s+4)^2 + 2}\right\}$$

$$= e^{-4t}\zeta^{-1}\left\{\frac{3}{s^2 + 2}\right\} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= e^{-4t} \times \frac{3}{\sqrt{2}}\zeta^{-1}\left\{\frac{\sqrt{2}}{s^2 + (\sqrt{2})^2}\right\}$$

$$V_0(t) = \frac{3}{\sqrt{2}} e^{-4t} \sin\sqrt{2}tV$$

Chapter 3

3.0 Fourier Series

3.1 Trigonometric Fourier Series
3.2 Fourier series odd and even functions
3.5 The Fourier sine and cosine series for odd and even functions
3.6 Fourier half range of sine and cosine series

3.1 Trigonometric Fourier Series

Question 1

Diagram 1, shows the square wave. Evaluate the Trigonometric Fourier Series expansion of the waveform to find coefficients of a_o, a_n *and* b_n

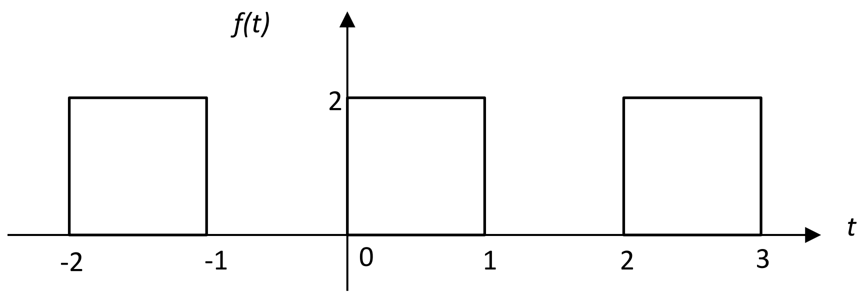


Diagram 1

Answer:

The function is described:

$$f(t) = \begin{cases} 2 & 0 < t < 1 \\ 0 & 1 < t < 2 \end{cases}$$

$$Since \quad T = 2, \;\; \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$$

Coefficients of a_0 , a_n *and* b_n.

$$a_0 = \frac{1}{T}\int_0^T f(t) \; dt$$

$$= \frac{1}{2}\left[\int_0^1 2 \; dt + \int_1^2 0 \; dt\right]$$

$$= \frac{1}{2} 2t \Big|_0^1$$

$$= \frac{1}{2}[2(1) - 2(0)]$$

$$= \frac{1}{2}[2 - 0]$$

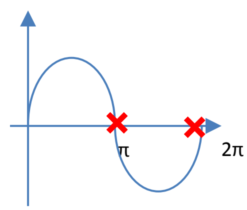
$$= 1$$

Nilai a_n

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_0 t\, dt$$

$$= \frac{2}{2}\left[\int_0^1 2\cos n\pi t + \int_1^2 0\cos n\pi t \quad dt\right]$$

$$= \int_0^1 2\cos n\pi t \;\, dt$$



$$= \frac{2}{n\pi}\sin n\pi t \Big|_0^1$$

$$= \frac{2}{n\pi}\sin n\pi(1) - \frac{2}{n\pi}\sin n\pi(0)$$

$$= \frac{2}{n\pi}\sin n\pi - \frac{2}{n\pi}\sin n\pi$$

$$= \frac{1}{n\pi}(0) - \frac{1}{n\pi}(0)$$

$$= 0$$

$$\int \sin ax\, dx = -\frac{1}{a}\cos ax + C$$

$$\int \cos ax\, dx = \frac{1}{a}\sin ax + C$$

Nilai b_n

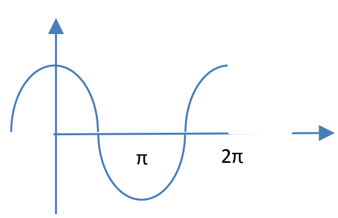
$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_0 t\, dt$$

$$= \frac{2}{2}[2\sin n\pi t \;\, dt + 0\sin n\pi t \quad dt]$$

$$= \frac{2}{2}\left[\int_0^1 2\sin n\pi t \;\, dt\right]$$

$$= 2\int_0^1 \sin n\pi t \;\, dt$$

$$= -\frac{2}{n\pi}\cos n\pi t \Big|_0^1$$



$$= -\frac{2}{n\pi}[\cos n\pi(1) - \cos n\pi(0)]$$

$$= -\frac{2}{n\pi}[(-1) - 1]$$

$$= \frac{4}{n\pi}$$

$$= \frac{4}{n\pi}\left[(-1)^n - 1\right] = \begin{cases} \frac{4}{n\pi}, & n = odd \\ 0, & n = even \end{cases}$$

Question 2

Diagram 1, shows the square wave. Evaluate the Trigonometric Fourier Series expansion of the waveform to find coefficients of a_o, a_n *and* b_n

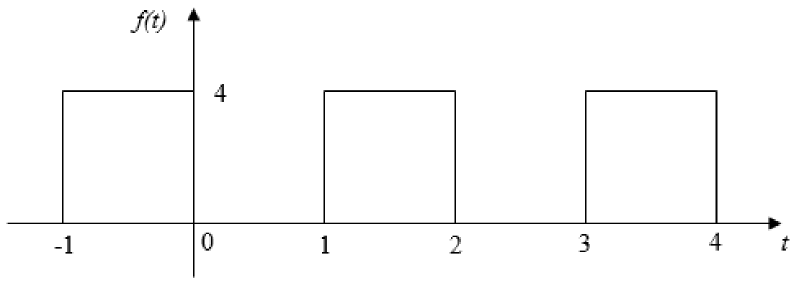


Diagram 1

Answer:

The function is described :

$$f(t) = \begin{cases} 0 & 0 < t < 1 \\ 4 & 1 < t < 2 \end{cases}$$

$$Since \quad T = 2, \ \ \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$$

Coefficients of a_0 , a_n *and* b_n.

$$a_0 = \frac{1}{T}\int_0^T f(t) \ \ dt$$

$$= \frac{1}{2}\left[\int_0^1 0 \ \ dt + \int_1^2 4 \ \ dt\right]$$

$$= \frac{1}{2} 4t\Big|_1^2$$

$$= \frac{1}{2}[4(2) - 4(1)]$$

$$= \frac{1}{2}[8 - 4]$$

$$= 2$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax + C$$

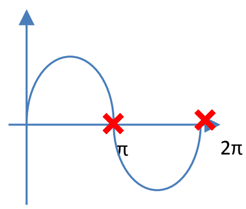
$$\int \cos ax\,dx = \frac{1}{a}\sin ax + C$$

Nilai a_n

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_0 t\,dt$$

$$= \frac{2}{2}\left[0\cos n\pi t\ dt + \int_1^2 4\cos n\pi t\ \ dt\right]$$

$$= \int_1^2 4\cos n\pi t\ dt$$



$$= \frac{4}{n\pi}\sin n\pi t\Big|_1^2$$

$$= \frac{4}{n\pi}\sin n\pi(2) - \frac{4}{n\pi}\sin n\pi(1)$$

$$= \frac{1}{n\pi}\sin n\pi - \frac{1}{n\pi}\sin n0$$

$$= \frac{1}{n\pi}(0) - \frac{1}{n\pi}(0)$$

$$= 0$$

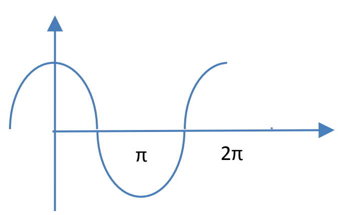
Nilai b_n

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_0 t\,dt$$

$$= \frac{2}{2}[0\sin n\pi t\ dt + 4\sin n\pi t\ \ dt]$$

$$= \frac{2}{2}\left[\int_1^2 4\sin n\pi t\ dt\right]$$

$$= 4\int_1^2 \sin n\pi t\ dt$$



$$= -\frac{4}{n\pi}\cos n\pi t\Big|_1^2$$

$$= -\left[\frac{4}{n\pi}\cos n\pi(2) - \frac{4}{n\pi}\cos n\pi(1)\right]$$

$$= \frac{4}{n\pi}\left[1-(-1)^n\right] = \begin{cases} \dfrac{8}{n\pi}, & n = odd \\ 0, & n = even \end{cases}$$

Question 3

Diagram A2 (c) shows the waves in an even symmetry. Evaluate the Fourier coefficients series of the wave.

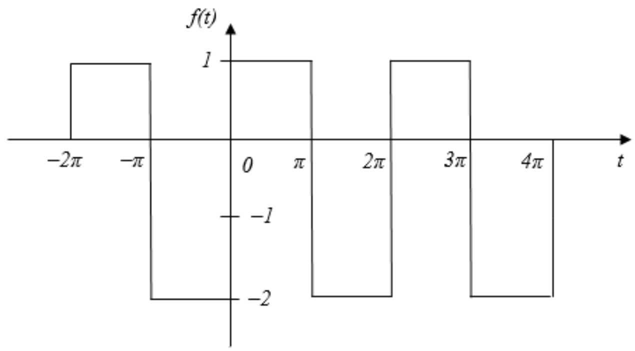


Diagram A2 (c)

Answer :

The function is described as :

$$f(t) = \begin{cases} 1 & 0 < t \leq \pi \\ -2 & \pi < t \leq 2\pi \end{cases}$$

Siner *T=2π* ,
$$w_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi}$$
$$= 1$$

Value a_o :

$$a_O = \frac{1}{T}\int_0^t f(t)dt$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} 1dt + \int_{\pi}^{2\pi}(-2)dt\right]$$

$$= \frac{1}{2\pi}\left\{[t]_0^{\pi} - [2t]_{\pi}^{2\pi}\right\}$$

$$= \frac{1}{2\pi}\{[\pi - 0] - [2(2\pi) - 2\pi]\}$$

$$= \frac{1}{2\pi}[\pi - 4\pi + 2\pi]$$

$$= -\frac{1}{2} \quad @ \quad -0.5$$

Value a_n :

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(\omega_O nt)dt$$

$$= \frac{2}{2\pi}\left[\int_0^{\pi}\cos(nt)dt + \int_{\pi}^{2\pi}\cos(nt)dt\right]$$

$$= \frac{1}{\pi}\left\{\left[\frac{\sin\ nt}{n}\right]_0^{\pi} - 2\left[\frac{\sin\ nt}{n}\right]_{\pi}^{2\pi}\right\}$$

$$= \frac{1}{\pi n}[\sin\ \pi - \sin(0)] - 2[\sin 2\pi - \sin\pi]$$

$$= 0$$

Value b_n :

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(\omega_O nt)dt$$

$$= \frac{2}{2\pi}\int_0^{\pi}(1)\sin(nt)dt + \int_{\pi}^{2\pi}(-2)\sin(nt)dt$$

$$= \frac{1}{\pi}\left[\left[-\frac{1}{n}\cos(nt)\right]_0^{\pi} - \left[\frac{2}{n}\cos(nt)\right]_{\pi}^{2\pi}\right]$$

$$= \frac{1}{n\pi}\{[-\cos(n\pi) + \cos(0)] - 2[-\cos(\pi) - \cos(2\pi)]\}$$

$$= \frac{1}{n\pi}\{[1+1] - 2[-1-1]\}$$

$$= \frac{6}{n\pi}$$

$$b_n = \begin{cases} \dfrac{6}{n\pi}\ , & n = odd\,(ganjil) \\ 0\ , & n = even(genap) \end{cases}$$

$$f(t) = -\frac{1}{2} + \sum_0^{\infty}\frac{6}{n\pi}\sin\ nt \quad , n = odd$$

Question 4
By referring to Figure 10:

i) Write an analytical equation for the waveform $f(t)$.

ii) Calculate the Fourier Series coefficients of a_0, a_n *and* b_n.

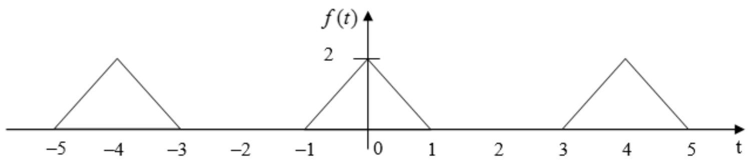


Figure 10

<u>Answer</u>

The function is described :

$$f(t) = \begin{cases} 2(1-t), & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$$

$\text{Since} \quad T = 4,$

$$\omega_0 = \frac{2\pi}{T} = \frac{\pi}{2}$$

$\text{Since} \quad T = 4,$

$$a_0 = \frac{2}{T}\int_0^{T/2} f(t)\,dt$$

$$= \frac{2}{4}\int_0^1 2(1-t)\,dt$$

$$= \left[t - \frac{t^2}{2} \right]_0^1$$

$$= \left[(1) - \frac{(1)^2}{2} \right] - \left[(0) - \frac{(0)^2}{2} \right]$$

$$= 0.5$$

$\text{Since} \quad T = 4,$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(\omega_o nt)dt$$

$$= \frac{4}{4}\int_0^1 2(1-t)\cos\left(\frac{n\pi t}{2} \right)dt$$

$$= \frac{8}{n^2\pi^2}\left(1 - \cos\left(\frac{n\pi}{2} \right) \right)$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_0 t\,dt$$

$$= 0$$

Fourier Series equation $f(t)$

$$f(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{8}{n^2\pi^2}\left[1 - \cos\left(\frac{n\pi}{2} \right)\cos\left(\frac{n\pi t}{2} \right) \right]$$

Question 5

Referring to Figure 2:

i) Write an analytical equation for the waveform $f(t)$.

ii) Calculate the Fourier Series coefficients of a_0, a_n *and* b_n.

iii) Produce the Fourier Series equation $f(t)$ for $n = 1 \, to \, 3$.

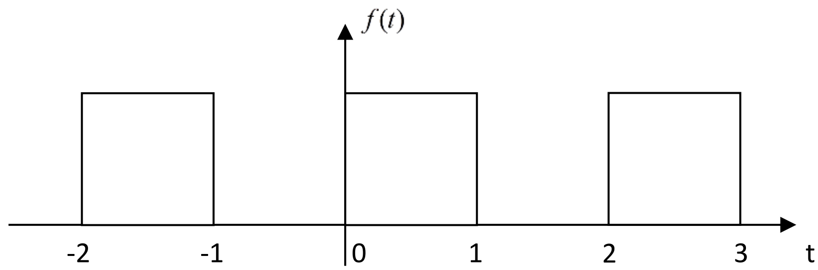


Figure 10

<u>Answer</u>

i) The function is described :

$$f(t) = \begin{cases} 1, & 0 < t < 1 \\ 0 & 1 < t < 2 \end{cases}$$

$$Since \quad T = 2, \quad \omega_0 = \frac{2\pi}{T} = \pi$$

ii) Coefficients of a_0, a_n *and* b_n.

$$a_0 = \frac{1}{T}\int_0^T f(t) \; dt$$

$$= \frac{1}{2}\left[\int_0^1 1 \; dt + \int_1^2 0 \; dt\right]$$

$$= \frac{1}{2} t \Big|_0^1$$

$$= \frac{1}{2}$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_0 t\,dt$$

$$= \frac{2}{2}\left[\int_0^1 1\cos n\pi t\ \ dt + \int_1^2 0\cos n\pi t\ \ \ dt\right]$$

$$= \frac{1}{n\pi}\sin n\pi t\Big|_0^1$$

$$= \frac{1}{n\pi}\sin n\pi\ = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_0 t\,dt$$

$$= \frac{2}{2}\left[\int_0^1 1\sin n\pi t\ \ dt + \int_1^2 0\sin n\pi t\ \ \ dt\right]$$

$$= \frac{1}{n\pi}\cos n\pi t\Big|_0^1$$

$$= -\frac{1}{n\pi}(\cos n\pi - 1), \quad \cos n\pi = (-1)^n$$

$$= \frac{1}{n\pi}\left[1-(-1)^n\right] = \begin{cases} \dfrac{2}{n\pi}, & n = odd \\ 0, & n = even \end{cases}$$

iii) Fourier Series equation $f(t)$

$n = 1$

$$= \frac{1}{\pi}\left[1-(-1)\right] \qquad = \frac{2}{\pi}$$

$n = 2$

$$= \frac{1}{2\pi}\left[1-(-1)^2\right] \qquad = 0$$

$n = 3$

$$= \frac{1}{3\pi}\left[1-(-1)^3\right] \qquad = \frac{2}{3\pi}$$

$n = 5$

$$= \frac{1}{5\pi}\left[1 - (-1)^5\right] \qquad = \frac{2}{5\pi}$$

$$f(t) = \frac{1}{2} + \frac{2}{\pi}\sin \pi t + \frac{2}{3\pi}\sin 3\pi t + \frac{2}{5\pi}\sin 5\pi t + ...$$

3.2 Fourier series odd and even functions

Question 1

Diagram 2 shows the waves in an odd symmetry. Evaluate the Fourier coefficients series of the wave.

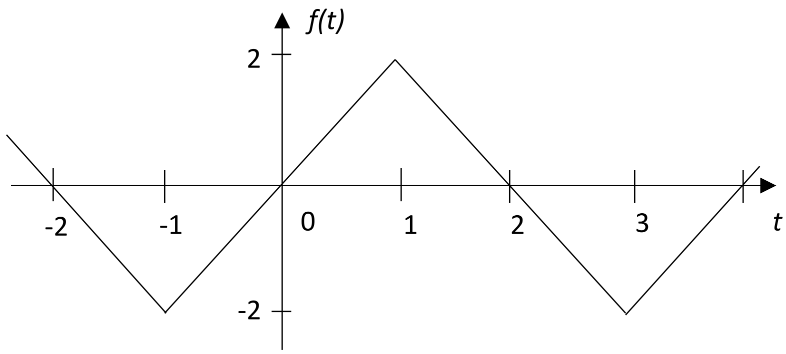


Diagram 2

Answer :

The function is described as :

$$f(t) = 2t \qquad -1 < t < 1$$

$$w_0 = \frac{2\pi}{T} = \frac{2\pi}{4}$$

$$= \frac{\pi}{2}$$

Since *T=4*

Value a_o :

$$a_O = \frac{1}{T}\int_{-T/2}^{T/2} f(t)dt \qquad = \frac{1}{T}\left[\int_{-T/2}^{0} f(t)dt + \int_{0}^{T/2} f(t)dt\right]$$

$$= \frac{1}{4}\left[\int_{-2}^{0} 2tdt + \int_{0}^{2} 2tdt\right]$$

$$= \frac{1}{4}\left\{\left[\frac{2t^2}{2}\right]_{-2}^{0} - \left[\frac{2t^2}{2}\right]_{0}^{2}\right\}$$

$$= \frac{1}{4}\{[2(0) - 2(-2)] - [2(2) - 2(0)]\}$$

$$=\frac{1}{4}[0]$$

$$=0$$

Value a_n :

$$a_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)dt \qquad = \frac{1}{T}\left[\int_{-T/2}^{0} f(t)dt + \int_{0}^{T/2} f(t)dt\right]$$

$$= \frac{2}{2\pi}\left[\int_0^{\pi} \cos(nt)dt + \int_{\pi}^{2\pi} (-2)\cos(nt)dt\right]$$

$$= \frac{1}{\pi}\left\{\left[\frac{\sin\ nt}{n}\right]_0^{\pi} - 2\left[\frac{\sin\ nt}{n}\right]_{\pi}^{2\pi}\right\}$$

$$= \frac{1}{\pi n}[\sin\ \pi - \sin(0)] - 2[\sin 2\pi - \sin\pi]$$

$$=0$$

Value b_n :

$$b_n = \frac{2}{T}\int_0^{T} f(t)\sin(\omega_O nt)dt$$

$$= \frac{2}{2\pi}\int_0^{\pi} (1)\sin(nt)dt + \int_{\pi}^{2\pi} (-2)\sin(nt)dt$$

$$= \frac{1}{\pi}\left[\left[-\frac{1}{n}\cos(nt)\right]_0^{\pi} - \left[\frac{2}{n}\cos(nt)\right]_{\pi}^{2\pi}\right]$$

$$= \frac{1}{n\pi}\{[-\cos(n\pi) + \cos(0)] - 2[-\cos(\pi) - \cos(2\pi)]\}$$

$$= \frac{1}{n\pi}\{[1+1] - 2[-1-1]\}$$

$$= \frac{6}{n\pi}$$

$$b_n = \begin{cases} \dfrac{6}{n\pi} , & n = odd(ganjil) \\ 0 , & n = even(genap) \end{cases}$$

$$f(t) = -\frac{1}{2} + \sum_{0}^{\infty} \frac{6}{n\pi} \sin \ nt \quad , n = odd$$

Question 2

The signal displayed by a medical device can be approximated by the waveform shown in Diagram A1 (a). Determine the Fourier series representation of the signal.

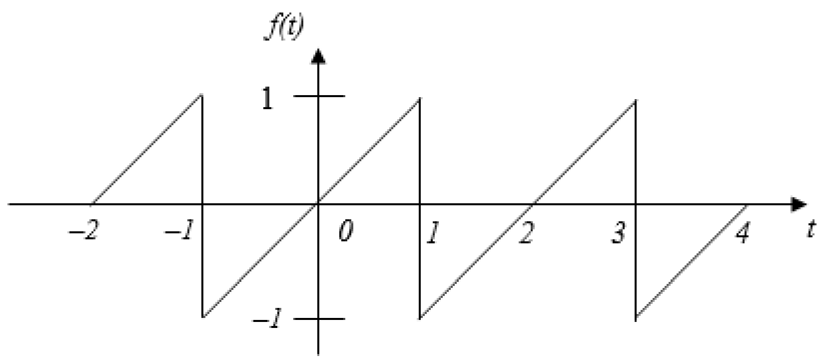


Diagram A1 (a)

<u>Answer</u>

The function is described as :

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-0}{1-0} = 1$$

Equation of the line :

$$y = mx + c$$
$$y = t + 0$$
$$f(t) = t, \qquad -1 < t < 1$$

$$T=2 \quad , \quad w_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$$

$a_n = a_o = 0$ =➔ This an odd function

Value b_n :

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin(\omega_O nt)dt$$

$$= \frac{4}{2}\int_0^{2/2} t\sin(n\omega_o t)dt$$

$$= 2\int_0^{1} t\sin(n\pi t)dt$$

$$= \frac{2}{n^2\pi^2}\left[\sin(n\pi t) - n\pi t\cos(n\pi t)\right]_0^1$$

$$= \frac{2}{n^2\pi^2}\left\{\begin{matrix}\left[\sin \pi t(1) - n\pi(1)\ \cos n\pi(1)\right] - \\ \left[\sin \pi t(0) - n\pi(0)\ \cos n\pi(0)\right]\end{matrix}\right\}$$

$$= \frac{-2}{n\pi}\left[\cos(n\pi)\right]$$

Question 3

Diagram 2 shows the waves in an odd symmetry. Evaluate the Fourier coefficients series of the wave.

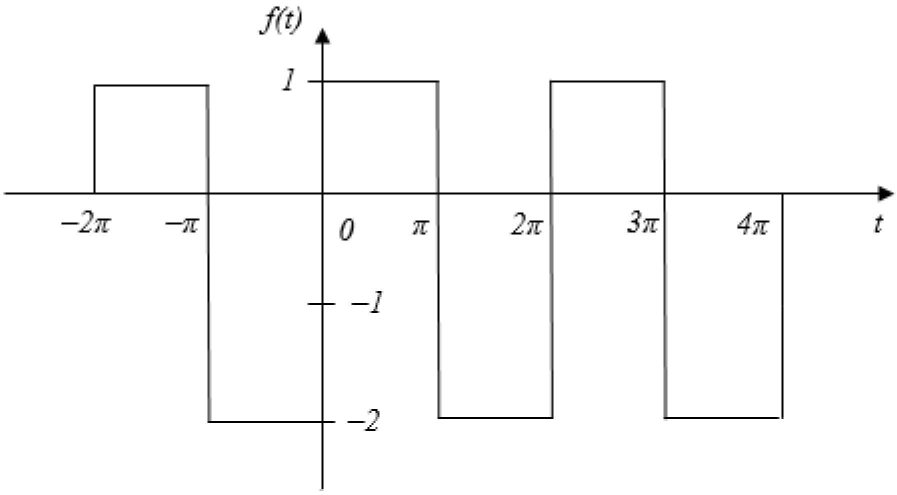


Diagram 2

<u>Answer</u>

The function is described as :

$$f(t)=\begin{cases}1 & 0<t\le\pi\\ -2 & \pi<t\le 2\pi\end{cases}$$

$$w_0=\frac{2\pi}{T}=\frac{2\pi}{2\pi}$$

Siner *T=2π* ,
$$=1$$

Value a_o :

$$a_O=\frac{1}{T}\int_0^t f(t)dt$$

$$=\frac{1}{2\pi}\left[\int_0^{\pi}1dt+\int_{\pi}^{2\pi}(-2)dt\right]$$

$$=\frac{1}{2\pi}\left\{[t]_0^{\pi}-[2t]_{\pi}^{2\pi}\right\}$$

$$=\frac{1}{2\pi}\{[\pi-0]-[2(2\pi)-2\pi]\}$$

$$= \frac{1}{2\pi}\left[\pi - 4\pi + 2\pi\right]$$

$$= -\frac{1}{2} \quad @ \quad -0.5$$

Value a_n :

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(\omega_O nt)dt$$

$$= \frac{2}{2\pi}\left[\int_0^{\pi}\cos(nt)dt + \int_{\pi}^{2\pi}(-2)\cos(nt)dt\right]$$

$$= \frac{1}{\pi}\left\{\left[\frac{\sin\ nt}{n}\right]_0^{\pi} - 2\left[\frac{\sin\ nt}{n}\right]_{\pi}^{2\pi}\right\}$$

$$= \frac{1}{\pi n}\left[\sin\ \pi - \sin(0)\right] - 2\left[\sin 2\pi - \sin\pi\right]$$

$$= 0$$

Value b_n :

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(\omega_O nt)dt$$

$$= \frac{2}{2\pi}\int_0^{\pi}(1)\sin(nt)dt + \int_{\pi}^{2\pi}(-2)\sin(nt)dt$$

$$= \frac{1}{\pi}\left[\left[-\frac{1}{n}\cos(nt)\right]_0^{\pi} - \left[\frac{2}{n}\cos(nt)\right]_{\pi}^{2\pi}\right]$$

$$= \frac{1}{n\pi}\{[-\cos(n\pi) + \cos(0)] - 2[-\cos(\pi) - \cos(2\pi)]\}$$

$$= \frac{1}{n\pi}\{[1+1] - 2[-1-1]\}$$

$$= \frac{6}{n\pi}$$

$$b_n = \begin{cases} \dfrac{6}{n\pi}\ , & n = odd(ganjil) \\ 0\ , & n = even(genap) \end{cases}$$

$$f(t) = -\frac{1}{2} + \sum_{0}^{\infty} \frac{6}{n\pi} \sin\ nt \quad , n = odd$$

Question 4

Diagram A1 (b) shows the waves in an odd symmetry. Evaluate the Fourier coefficients series of the wave.

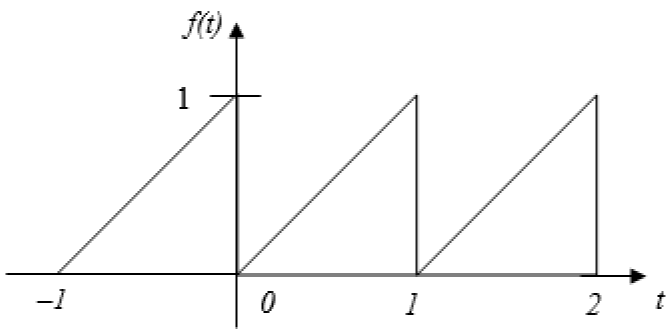


Diagram A1 (b)

<u>Answer</u>

The function is described as :

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-0}{1-0} = 1$$

Equation of the line :

$$y = mx + c$$
$$y = t + 0$$

The function is described as :

$$f(t) = t, \qquad 0 < t < 1$$

T=1 , $w_0 = \frac{2\pi}{T} = \frac{2\pi}{1}$

$$= 2\pi$$

Value a_o :

$$a_O = \frac{1}{T}\int_0^t f(t)dt$$

$$= \frac{1}{1}\left[\int_0^1 tdt\right]$$

$$= \left\{\left[\frac{t^2}{2}\right]\begin{matrix}1\\0\end{matrix}\right\}$$

$$= \left[\frac{1}{2} - 0\right]$$

$$= \frac{1}{2}$$

Value a_n :

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(\omega_O nt)dt$$

$$= \frac{2}{1}\left[\int_0^1 t\cos(nt)dt\right]$$

===> $\int_0^t t\cos n\omega_o t\ dt$ *dari rumus*

===> $\int_0^t x\cos axdx \quad = \frac{1}{a^2}(\cos ax + ax\sin ax) + c)$

$$= \frac{1}{2^2 n^2 \pi^2}\left[\cos 2n\pi t - 2n\pi t\sin 2n\pi t\right]_0^\pi$$

$$= \frac{1}{4n^2\pi^2}\left\{\begin{matrix}\left[\cos 2n\pi(1) - 2n\pi(1)\sin 2n\pi(1)\right] - \\ \left[\cos 2n\pi(0) - 2n\pi(0)\sin 2n\pi(0)\right]\end{matrix}\right\}$$

$$= \frac{1}{4n^2\pi^2}\{[1+0]-[1+0]\}$$

$$= 0$$

Value b_n :

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(\omega_O nt)dt$$

$$= \frac{2}{1}\int_0^1 t \ \sin(n2\pi t)dt$$

===> $\int_0^t t\sin n\omega_o t\ dt$ *dari rumus*

===> $\int_0^t x\sin axdx \quad = \frac{1}{a^2}\left(\sin ax - ax\cos ax) + c\right)$

$$= \frac{1}{2^2 n^2 \pi^2}\left[\sin(2n\pi t) + -2n\pi t\cos(2n\pi t)\right]_0^1$$

$$= \frac{1}{4n^2\pi^2}\begin{Bmatrix}\left[\sin 2n\pi(1) - 2n\pi(1)\cos 2n\pi(1)\right] - \\ \left[\sin 2n\pi(0) - 2n\pi(0)\cos 2n\pi(0)\right]\end{Bmatrix}$$

$$= \frac{1}{4n^2\pi^2}\left[0 + 2n\pi\right] - \left[0 + 0\right]$$

$$= \frac{1}{4n^2\pi^2}\left[-2n\pi\right]$$

$$= \frac{-1}{n\pi}$$

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n\cos n\omega_0 t + b_n \sin n\omega_0 t\right)$$

$$f(t) = \frac{1}{2} - \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin \ 2\pi nt$$

Question 5

Diagram A1(c) shows the waves in an even symmetry. Construct the Fourier coefficients series of the wave.

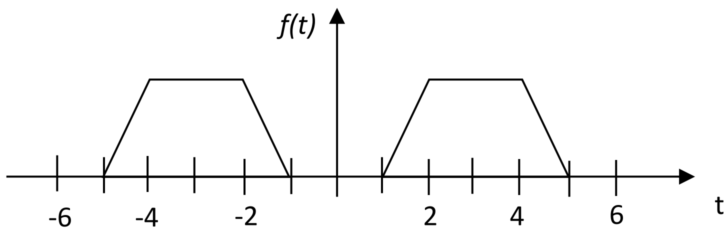


Diagram A1(c)

Answer

Gradient :

$$m = \frac{4-0}{2-1} = 4$$

$y = mx + c$

$f(t) = 4t + c$ melalui titik (1,0)

$f(t) = 4$

The function is described :

$$f(t) = \begin{cases} 4t-4, & 1 < t < 2 \\ 4 & 2 < t < 3 \end{cases}$$

$$Since \quad T = 6, \;\; \omega_0 = \frac{2\pi}{6} = \frac{\pi}{3}$$

Coefficients of a_0 , a_n *and* b_n.

$$a_0 = \frac{1}{T}\int_0^T f(t)\ \ dt$$

$$= \frac{2}{6}\left[\int_1^2 (4t-4)\ \ dt + \int_2^3 4\ \ dt\right]$$

$$= \frac{1}{3}\left[2t^2 - 4t\Big|_1^2\right] + 4(3-2)$$

$$= 2$$

$$a_n = \frac{4}{T}\int_0^{T/4} f(t)\cos(n\omega_0 t/3)\,dt = \frac{4}{6}\left[\int_1^2 (4t-4)\cos(n\pi t/3)\;dt + \int_2^3 4\cos(n\pi t/3)\quad dt\right]$$

$$= \frac{16}{6}\left[\frac{9}{n^2\pi^2}\cos\frac{n\pi t}{3} + \frac{3t}{n\pi}\sin\frac{n\pi t}{3} - \frac{3}{n\pi}\sin\frac{n\pi t}{3}\right]\Bigg|_1^2 + \frac{16}{6}\left[\frac{3}{n\pi}\sin\frac{n\pi t}{3}\right]\Bigg|_2^3$$

$$= \frac{24}{n^2\pi^2}(\cos\frac{2n\pi}{3} - \cos\frac{n\pi}{3})$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_0 t\,dt$$
$$= 0$$

At t=2,

Fourier Series equation $f(t)$

$$f(2) = 2 + \frac{24}{\pi^2}\begin{bmatrix} \left(\cos\frac{2\pi}{3}\right) - \left(\cos\frac{\pi}{3}\right)\left(\cos\frac{2\pi}{3}\right) \\ + \frac{1}{4}\left(\cos\frac{4\pi}{3}\right) - \left(\cos\frac{2\pi}{3}\right)\left(\cos\frac{4\pi}{3}\right) \\ + \frac{1}{9}\left(\cos 2\pi\right) - \left(\cos\pi\right)\left(\cos 2\pi\right) + ... \end{bmatrix}$$

$$f(2) = 2 + 2.432(0.5 + 0 + 0.222 + ...)$$

$$f(2) = 3.756$$

Question 6

Figure 9 shows the waves in an even symmetry. Construct the Fourier coefficients series of the wave.

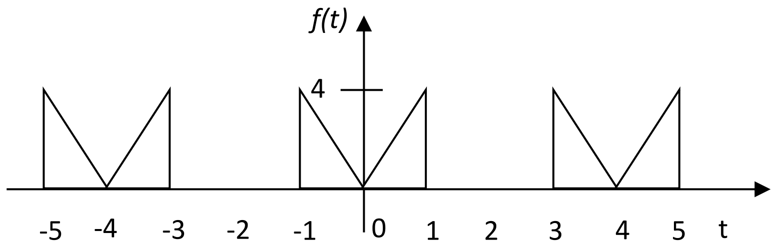


Figure 9

<u>Answer</u>

$$\text{Since} \quad T = 4,$$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{4}$$

$$= \frac{\pi}{2}$$

F(t) is an odd function

$$\text{Since} \quad T = 4,$$

$$a_0 = \frac{2}{T}\int_0^{T/2} f(t)\,dt$$

$$= \frac{2}{4}\int_0^{1} 4t\,dt$$

$$= \frac{2}{4} \times \frac{4t^{1+1}}{2}\Bigg|_0^1$$

$$= t^2\Bigg|_0^1$$

$$= [1 - 0]$$

$$= 1$$

$\text{Since} \quad T = 4,$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(\omega_o nt)dt$$

$$= \frac{4}{4}\int_0^1 4t\cos\left(\frac{n\pi t}{2}\right)dt$$

$$= 4\left[\frac{4}{n^2\pi^2}\cos\left(\frac{n\pi t}{2}\right)+\frac{2t}{n\pi}\sin\left(\frac{n\pi t}{2}\right)\right]\Bigg|_0^1$$

$$= 4\left[\begin{matrix}\left[\frac{4}{n^2\pi^2}\cos\left(\frac{n\pi(1)}{2}\right)+\frac{2t}{n\pi}\sin\left(\frac{n\pi(1)}{2}\right)\right]- \\ \left[\frac{4}{n^2\pi^2}\cos\left(\frac{n\pi(0)}{2}\right)+\frac{2t}{n\pi}\sin\left(\frac{n\pi(0)}{2}\right)\right]\end{matrix}\right]$$

$$= \frac{16}{n^2\pi^2}\left(\cos\left(\frac{n\pi}{2}\right)-1\right)+\frac{8}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

Chapter 4

4.0 Two Port Network Parameters

4.1 z-parameters
4.2 y-parameters
4.3 h-parameters

4.1 z-Parameters

Question 1

Construct the z parameters for the two-port network shown in Diagram A3(a).

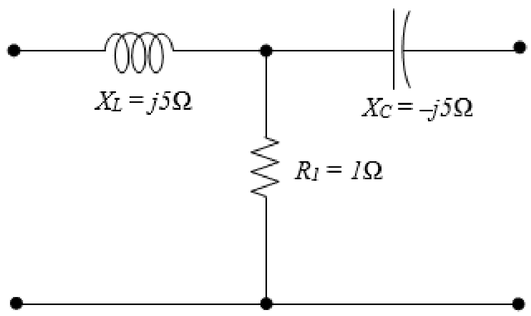


Diagram A3(a)

Answer:

$$Z_{11} = \frac{V_1}{I_1} \quad = \frac{(j5+1)}{I_1} I_1 \quad = 1 + j5$$

$$Z_{22} = \frac{V_2}{I_2} \quad = \frac{(-j5+1)}{I_2} I_2 \quad = 1 - j5$$

$$Z_{21} = \frac{V_2}{I_1} \quad = \frac{(1)}{I_1} I_1 \quad = 1$$

$$Z_{12} = \frac{V_1}{I_2} \quad = \frac{(1)}{I_2} I_2 \quad = 1$$

Question 2

Construct the *z* parameters for the two-port network shown in Diagram 2

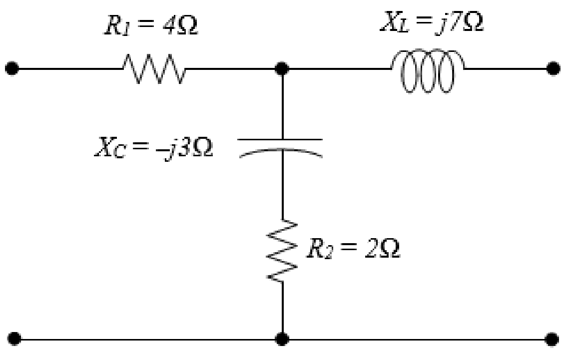


Diagram 2

Answer

$$Z_{11} = \frac{V_1}{I_1} \quad = \frac{(4+2-j3)}{I_1} I_1 \quad = 6 - j3$$

$$Z_{21} = \frac{V_2}{I_1} \quad = \frac{(2-j3)}{I_1} I_1 \quad = 2 - j3$$

$$Z_{12} = \frac{V_1}{I_2} \quad = \frac{(2-j3)}{I_2} I_2 \quad = 2 - j3$$

$$Z_{22} = \frac{V_2}{I_2} \quad = \frac{(j7-j3+2)}{I_2} I_2 \quad = 2 + j4$$

Question 3

Diagram A3(c) shows a two-port network circuit, evaluate parameters z_{21} the circuit using *z* parameters of the two-port network as functions of *s domain*.

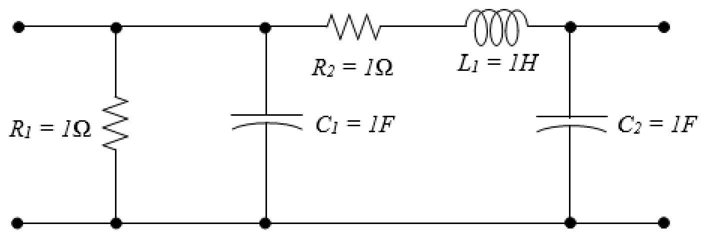
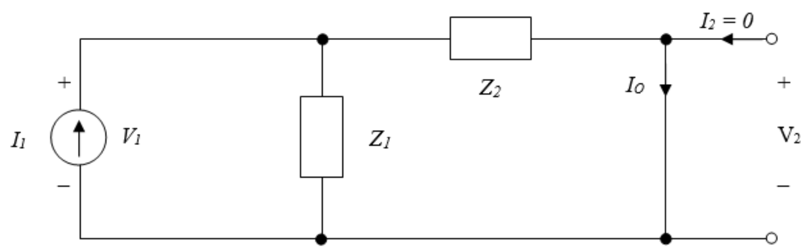


Diagram A3(c)

Answer:

Value Z_{11} :



$$Z_1 = 1 // \frac{1}{s}$$

$$= \frac{1 \times \frac{1}{s}}{1 + \frac{1}{s}}$$

$$= \frac{1}{s} \times \frac{s}{1+s}$$

$$= \frac{1}{1+s}$$

$$Z_2 = 1 + s + \frac{1}{s}$$

$$= \frac{1}{1} + \frac{s}{1} + \frac{1}{s}$$

$$= \frac{s^2 + s + 1}{s}$$

$$Z_{11} = Z_1 // Z_2$$

$$= \frac{\frac{1}{s+1} \times \frac{s^2+s+1}{s}}{\frac{1}{s+1} + \frac{s^2+s+1}{s}}$$

$$= \frac{\frac{s^2+s+1}{s(s+1)}}{\frac{s+(s^2+s+1)(s+1)}{s(s+1)}}$$

$$= \frac{s^2+s+1}{s(s+1)} \times \frac{s(s+1)}{s+(s^2+s+1)(s+1)}$$

$$= \frac{s^2+s+1}{s^3+2s^2+3s+1}$$

Value I_O :

Apply KCL

$$I_O = \frac{I_1 \times Z_1}{Z_1 + Z_2}$$

$$= \frac{I_1 \times \frac{1}{s+1}}{\frac{1}{s+1} + 1 + s + \frac{1}{s}}$$

$$= \frac{s}{s^3+2s^2+3s+1} \times I_1$$

Value V_2 :

$$V_2 = I_0 \times \frac{1}{s}$$

$$= \frac{sI_1}{s^3+2s^2+3s+1} \times \frac{1}{s}$$

$$= \frac{I_1}{s^3+2s^2+3s+1}$$

Value Z_{21} :

$$Z_{21} = \frac{V_2}{I_1}$$

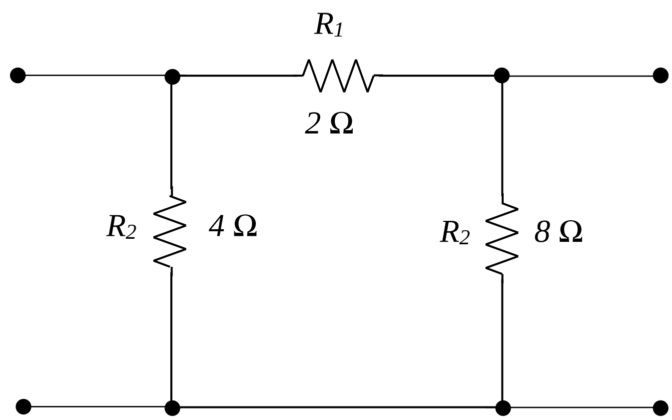
$$= \frac{I_1}{s^3 + 2s^2 + 3s + 1} \times \frac{1}{I_1}$$

$$= \frac{1}{s^3 + 2s^2 + 3s + 1}$$

4.2 y-Parameters

Question 1

Obtain the y parameter for the π network shown in Diagram 1



$$V_2 = I_2\left(8//2\right) = \frac{8\times 2}{8+2}I_2$$
$$= \frac{16}{10}$$
$$= \frac{8}{5}I_2$$

$$y_{22} = \frac{I_2}{V_2}$$
$$= \frac{I_2}{\frac{8}{5}I_2}$$
$$= \frac{1}{\frac{8}{5}}$$
$$y_{22} = \frac{5}{8} = 0.625S$$

$$V_1 = I_1\left(4//2\right) = \frac{4\times 2}{4+2}I_1$$
$$= \frac{8}{6} = \frac{4}{3}I_1$$

$$y_{11} = \frac{I_1}{V_1} \quad = \frac{I_1}{\frac{4}{3}I_1}$$

$$0.75S = \frac{1}{\frac{4}{3}} \quad = \frac{3}{4}$$
$$= 0.75S$$

$$= \frac{I_1}{\frac{4I_1}{3}}$$
$$= I_1 \times \frac{3}{4I_1}$$

$$-I_2 = \frac{I_1 \times 4}{4+2}$$
$$= \frac{4}{6}I_1 = \frac{2}{3}I_1$$
$$I_2 = -\frac{2}{3}I_1$$

$$y_{21} = \frac{I_2}{V_1} = \frac{-\frac{2}{3}I_1}{\frac{4}{3}I_1}$$
$$= -\frac{2}{3}\times\frac{3}{4} = -\frac{2}{4} = -\frac{1}{2}$$
$$= -0.5S$$

$$-I_1 = \frac{I_2 \times 8}{8+2}$$

$$= \frac{8}{10} I_2 = \frac{4}{5} I_2$$

$$I_1 = -\frac{4}{5} I_2$$

$$y_{12} = \frac{I_1}{V_2} = \frac{-\frac{4}{5} I_2}{\frac{8}{5} I_2}$$

$$= -\frac{4}{5} \times \frac{5}{8} = -\frac{4}{8} = -\frac{1}{2}$$

$$= -0.5S$$

$$y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

$$y = \begin{bmatrix} 0.75 & -0.5 \\ -0.5 & 0.625 \end{bmatrix}$$

Question 2

Determine the parameters y as function of s parameter, for the two-port network shown in Diagram A2(c).

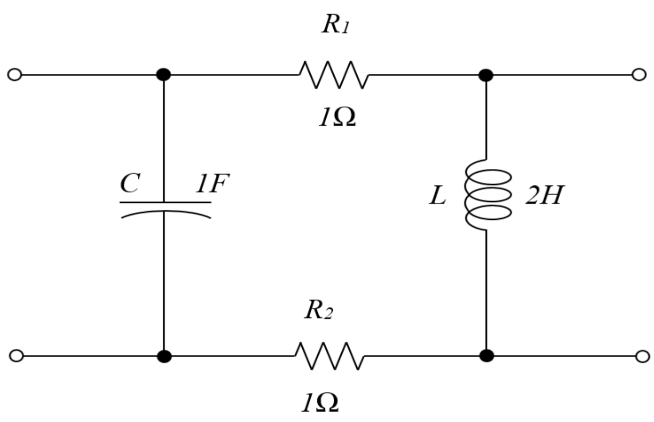
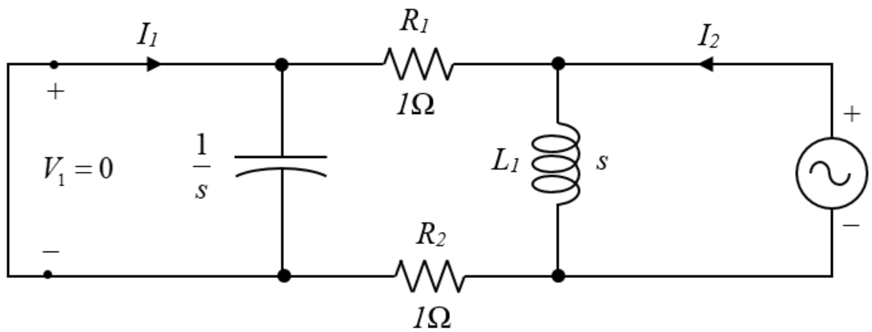


Diagram A2(c).

Answer



$\zeta\{1H\} = sL = s$

$\zeta\{1F\} = \frac{1}{sC} = \frac{1}{s}$

Obtain h_{11} dan h_{21} :

$$V_1 = \left(\left(\frac{1}{s}//2\right)\right) I_1$$

$$= \left[\frac{\left[\frac{1}{s}\right]\times 2}{\left[\frac{1}{s}\right]+2}\right] I_1$$

$$= \left(\frac{\frac{2}{s}}{\frac{1+2s}{s}}\right) I_1$$

$$= \left(\frac{2}{s}\times\frac{s}{1+2s}\right) I_1$$

$$= \left(\frac{2}{2s+1}\right) I_1$$

$$y_{11} = \frac{I_1}{V_1}$$

$$= \frac{I_1}{\left(\frac{2}{2s+1}\right) I_1}$$

$$= \frac{2s+1}{2}$$

$$= s + \frac{1}{2}$$

By using current division :

$$I_2 = \left[\frac{-\frac{1}{s}}{2 + \frac{1}{s}} \right] I_1$$

$$= \left(\frac{-\frac{1}{s}}{\frac{2s+1}{s}} \right) I_1$$

$$= \left(-\frac{1}{s} \times \frac{s}{2s+1} \right) I_1$$

$$= \frac{-I_1}{2s+1}$$

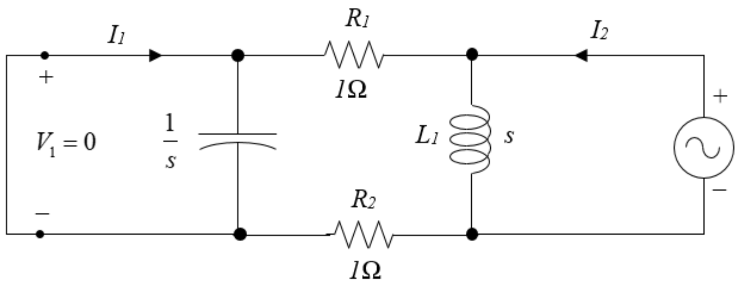
$$y_{21} = \frac{I_2}{V_1}$$

$$= \frac{\frac{-I_1}{2s+1}}{\left(\frac{2}{2s+1}\right) I_1}$$

$$= \frac{-I_1}{2s+1} \times \frac{2s+1}{(2) I_1}$$

$$= -\frac{1}{2}$$

Obtain y_{22} dan y_{12} :



$$V_2 = (s // 2) I_2$$
$$= \frac{s \times 2}{s+2} I_2$$
$$= \frac{2s}{s+2} I_2$$

Obtain y_{22}

$$y_{22} = \frac{I_2}{V_2} \quad = \frac{I_2}{\frac{2s}{s+2} I_2}$$
$$= \frac{s+2}{2s}$$
$$= \frac{1}{2} + \frac{1}{s}$$

$$V_2 = \frac{2s}{s+2} I_2$$
$$I_2 = \frac{s+2}{2s} V_2$$

$$I_1 = \frac{-s}{s+2} I_2$$

$$= \frac{-s}{s+2} \times \frac{s+2}{2s} V_2$$

$$= \frac{-V_2}{2}$$

$$y_{12} = \frac{I_1}{V_2}$$

$$= \frac{\frac{-V_2}{2}}{V_2}$$

$$= \frac{-V_2}{2} \times \frac{1}{V_2}$$

$$= \frac{-1}{2}$$

4.3 h-Parameters

Question 1

Determine the parameters y as function of s parameter, for the two-port network shown in Diagram A2(c).

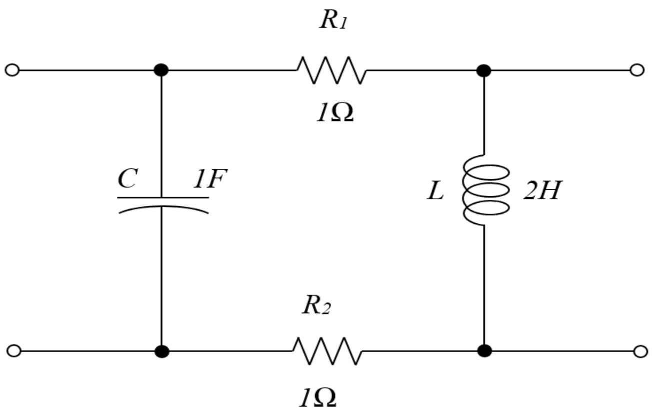
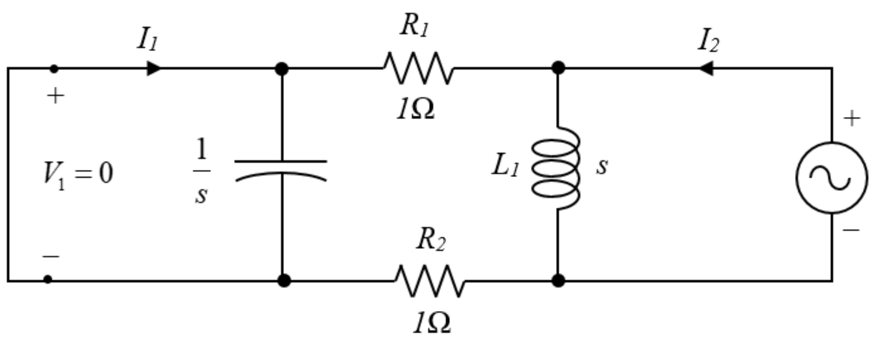


Diagram A2(c).

<u>Answer</u>



$$\zeta\{1H\} = sL = s$$

$$\zeta\{1F\} = \frac{1}{sC} = \frac{1}{s}$$

Obtain h_{11} dan h_{21} :

$$V_1 = \left(\left(\frac{1}{s} // 2\right)\right) I_1$$

$$= \left[\frac{\left[\frac{1}{s}\right] \times 2}{\left[\frac{1}{s}\right] + 2}\right] I_1$$

$$= \left(\frac{\frac{2}{s}}{\frac{1+2s}{s}}\right) I_1$$

$$= \left(\frac{2}{s} \times \frac{s}{1+2s}\right) I_1$$

$$= \left(\frac{2}{2s+1}\right) I_1$$

$$y_{11} = \frac{I_1}{V_1}$$

$$= \frac{I_1}{\left(\frac{2}{2s+1}\right) I_1}$$

$$= \frac{2s+1}{2}$$

$$= s + \frac{1}{2}$$

By using current division :

$$I_2 = \left[\frac{-\frac{1}{s}}{2+\frac{1}{s}}\right] I_1$$

$$= \left(\frac{-\frac{1}{s}}{\frac{2s+1}{s}}\right) I_1$$

$$= \left(-\frac{1}{s}\times\frac{s}{2s+1}\right) I_1$$

$$= \frac{-I_1}{2s+1}$$

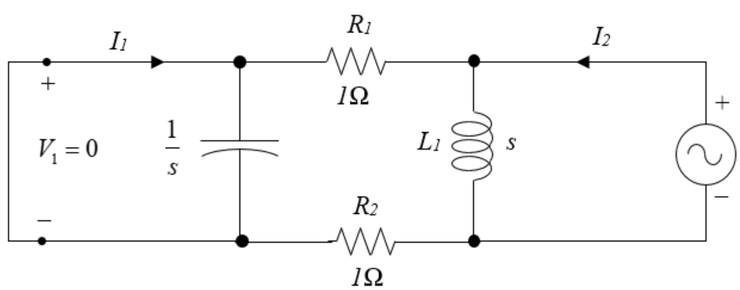
$$y_{21} = \frac{I_2}{V_1}$$

$$= \frac{\frac{-I_1}{2s+1}}{\left(\frac{2}{2s+1}\right) I_1}$$

$$= \frac{-I_1}{2s+1}\times\frac{2s+1}{(2) I_1}$$

$$= -\frac{1}{2}$$

Obtain y_{22} dan y_{12} :



$$V_2 = (s // 2) I_2$$

$$= \frac{s \times 2}{s+2} I_2$$

$$= \frac{2s}{s+2} I_2$$

Obtain y_{22}

$$y_{22} = \frac{I_2}{V_2} \quad = \frac{I_2}{\frac{2s}{s+2} I_2}$$

$$= \frac{s+2}{2s}$$

$$= \frac{1}{2} + \frac{1}{s}$$

$$V_2 = \frac{2s}{s+2} I_2$$

$$I_2 = \frac{s+2}{2s} V_2$$

$$I_1 = \frac{-s}{s+2} I_2$$

$$= \frac{-s}{s+2} \times \frac{s+2}{2s} V_2$$

$$= \frac{-V_2}{2}$$

$$y_{12} = \frac{I_1}{V_2}$$

$$= \frac{\frac{-V_2}{2}}{V_2}$$

$$= \frac{-V_2}{2} \times \frac{1}{V_2}$$

$$= \frac{-1}{2}$$

4.3 h-Parameters

Determine the parameters h_{11} and h_{22} as function of s parameter, for the two-port network shown in Diagram A3(b).

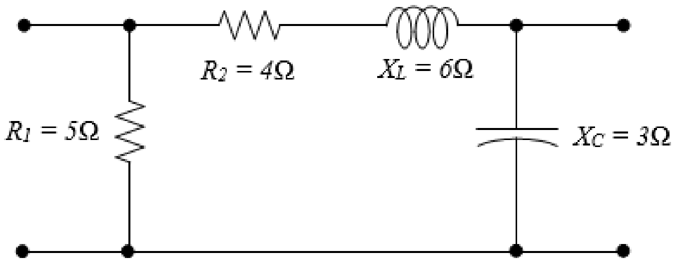
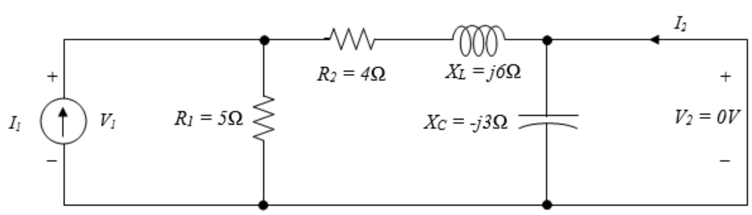


Diagram A3(b)

Answer



Value V_1 :

$$V_1 = 5//(4 + j6)$$

$$= \frac{5 \times (4 + j6)}{5 + (4 + j6)}$$

$$= \frac{20 + j30}{9 + j6}$$

$$E_{TH}' = 9.9083 + j33.0275$$

or

$$E_{TH}' = 34.4817\angle 73.3008^{\circ}$$

Value h_{11} :

$$h_{11} = \frac{V_1}{I_1}$$

$$= \frac{(3.0769 + j1.2821)}{I_1} I_1$$

$$= 3.0769 + j1.2821$$

Value V_1 :

$$V_2 = -j3 // (5 + 4 + j6) I_2$$

$$= \left[\frac{-j3 \times (9 + j6)}{-j3 \times (9 + j6)} \right] \times I_2$$

$$= \left[\frac{-18 + j27}{9 - j6} \right] \times I_2$$

Value h_{11} :

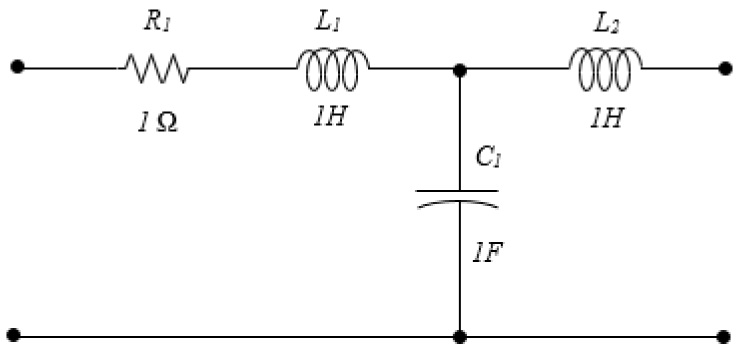
$$h_{11} = \frac{I_2}{V_2}$$

$$= I_2 \times \left[\frac{9 + j3}{-18 + j27} \right] \times \frac{1}{I_2}$$

$$= 0.0769 + j0.2821$$

Question 2

Determine the parameters h as function of s parameter, for the two-port network shown in Diagram A2(b).



Answer

$$\zeta\{1H\} = sL = s$$

$$\zeta\{1F\} = \frac{1}{sC} = \frac{1}{s}$$

Obtain h_{11} dan h_{21} :

$$V_1 = \left(1 + s + \left(s // \frac{1}{s}\right)\right) I_1$$

$$= \left[1 + s + \left(\frac{s \times \frac{1}{s}}{s + \frac{1}{s}}\right) I_1\right]$$

$$= \left[1 + s + \left(\frac{s}{s^2 + 1}\right) I_1\right]$$

$$h_{11} = \frac{V_1}{I_1} = s + 1 + \frac{s}{s^2 + 1}$$

By using current division :

$$I_2 = \left[\frac{-\frac{1}{s}}{s + \frac{1}{s}}\right] I_2$$

$$= \frac{-I_1}{s + 1}$$

$$h_{21} = \frac{I_2}{I_1} \quad = \frac{-1}{s^2+1}$$

Obtain h_{22} dan h_{12} :

$$V_1 = \frac{\frac{1}{s}}{s+\frac{1}{s}} V_2$$

$$= \frac{V_2}{s^2+1}$$

$$h_{12} = \frac{V_1}{V_2} \quad = \frac{1}{s^2+1}$$

$$V_2 = s + \frac{1}{s} I_2$$

$$h_{22} = \frac{I_2}{V_2} \quad = \frac{1}{s+\frac{1}{s}}$$

$$= \frac{s}{s^2+1}$$

References

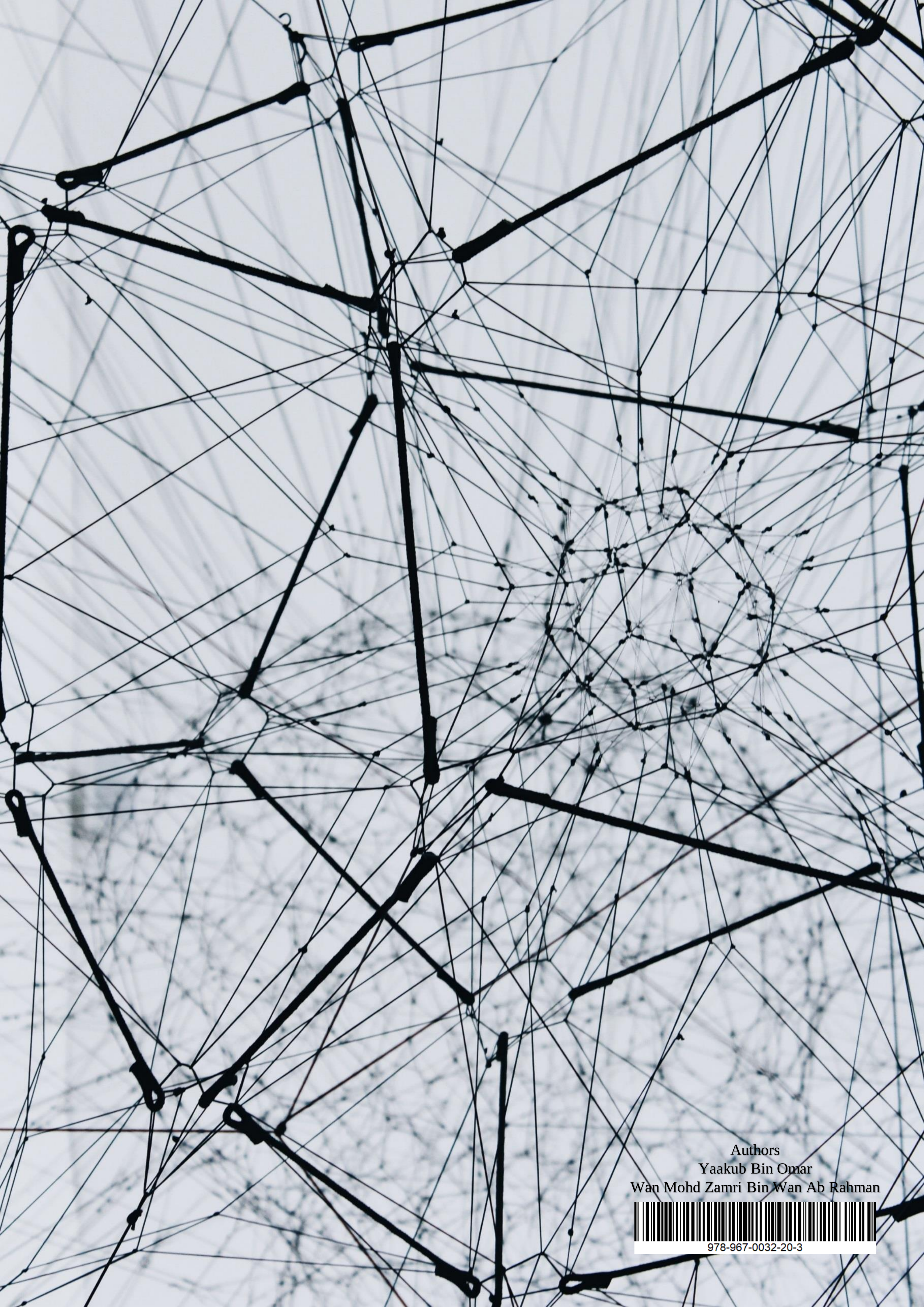
Boylestad, Robert L. (2015), Introductory Circuit Analysis. Pearson Education India.

Alexander Sadiku (2021), Fundamental of Electric Circuits. 7th. Edition. Mc Graw Hill.

Nilsson, James W & Riedel, Susan A (2008), Electric Circuits (8^{th}), Pearson Prentice Hall. (ISBN-13: 978-0-13-198925-2)

William H. Hayt Jr, Jack E. Kennedy and Steven M. Durbin (2007), Engineering Circuit Analysis (7^{th}), Mc Graw Hill. (ISBN-13: 978-0-07-286611-7)

UA Bakshi & AV Bakshi (2009), Circuit Analysis II, Technical Publication Pune. (ISBN: 978-8-18-431597-4)



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