

# SIGNAL AND SYSTEM

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## LINEAR TIME INVARIANT (LTI) SYSTEM

MARLINA RAMLI





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MARLINA BINTI RAMLI

POLYTECHNIC SERIES

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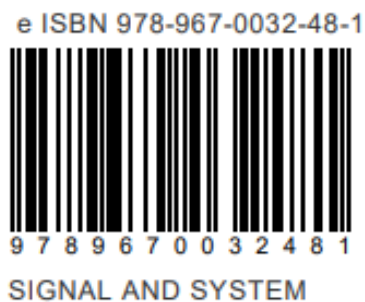
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## **SIGNAL AND SYSTEM LINEAR TIME INVARIANT (LTI) SYSTEM**

Marlina binti Ramli

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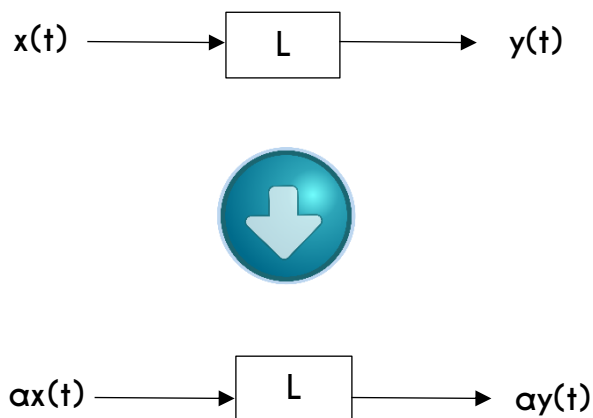
## LINEAR TIME INVARIANT SYSTEMS (LTI)

✚ Linearity and time invariance are two system properties that greatly simplify the study of systems.

✚ In signals and systems, Linear Time Invariant (LTI) systems are the primary signal-processing tool for modeling the action of a physical phenomenon on a signal such as propagation and measurement.

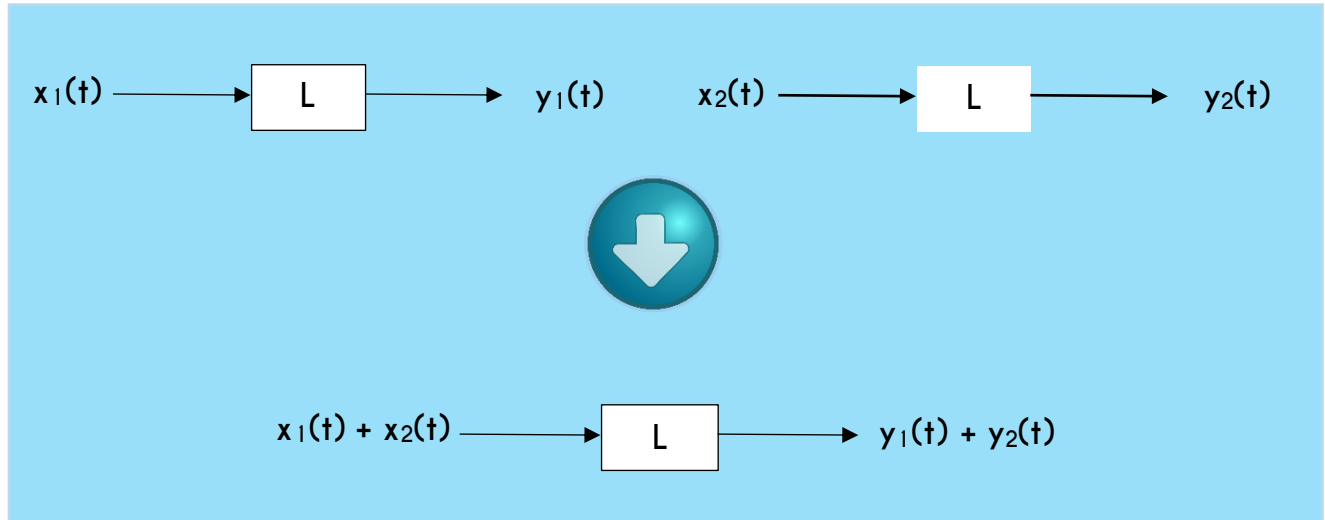
### LINEAR SYSTEMS

If a system is linear, this means that when an input to a given system is scaled by a value, the output of the system is scaled by the same amount.



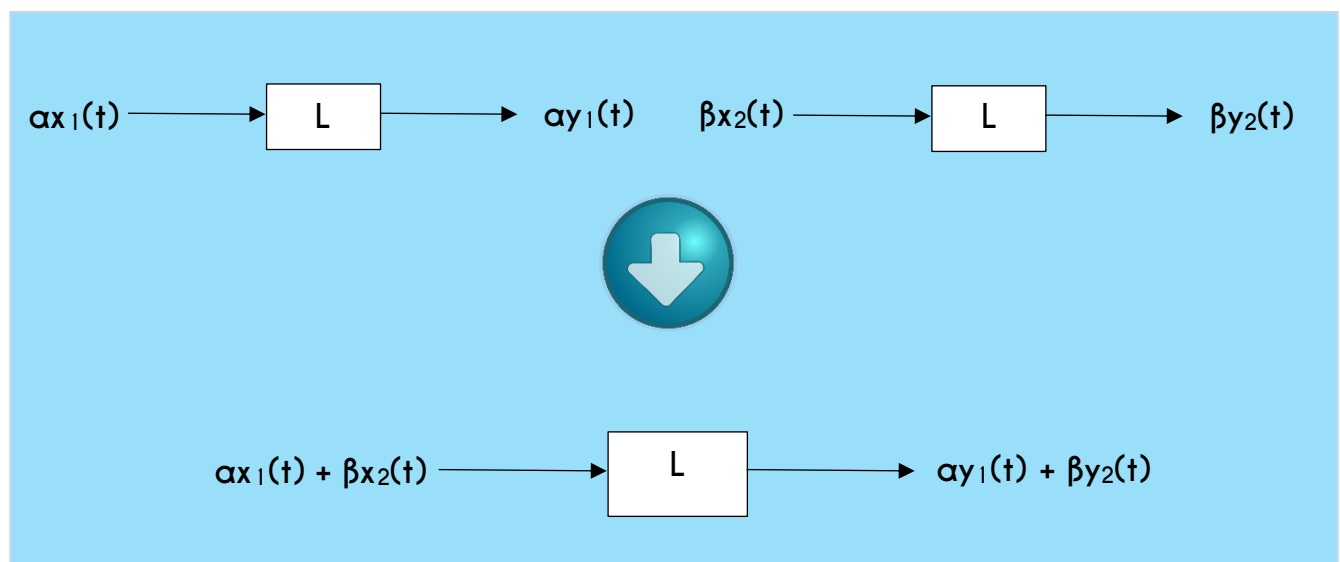
In Figure above, an input  $x(t)$  to the linear system  $L$  gives the output  $y(t)$ . If  $x(t)$  is scaled by a value  $\alpha$  and passed through this same system, as in Figure, the output will also be scaled by  $\alpha$ .

A linear system also obeys the principle of superposition. This means that if two inputs are added together and passed through a linear system, the output will be the sum of the individual inputs' outputs. It shows in Figure below.



### Superposition of Individual Input

If the inputs  $x$  and  $y$  are scaled by factors  $\alpha$  and  $\beta$ , respectively, then the sum of these scaled inputs will give the sum of the individual scaled outputs:

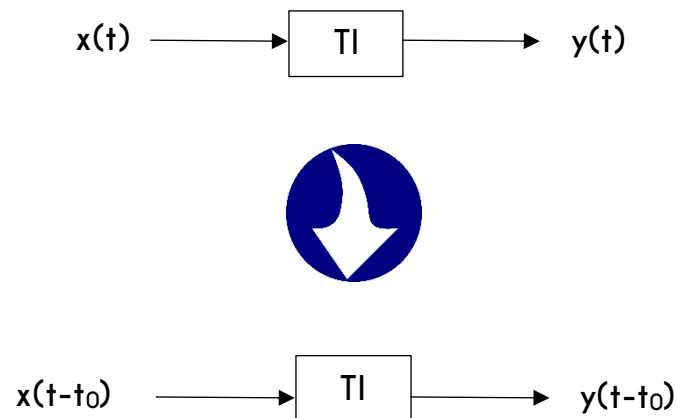


### Superposition of Scaled Input



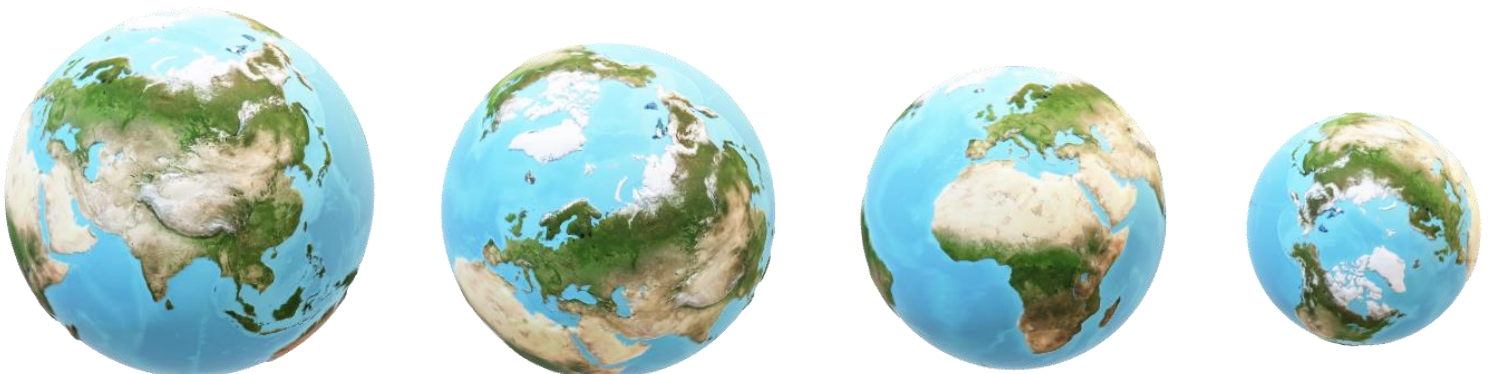
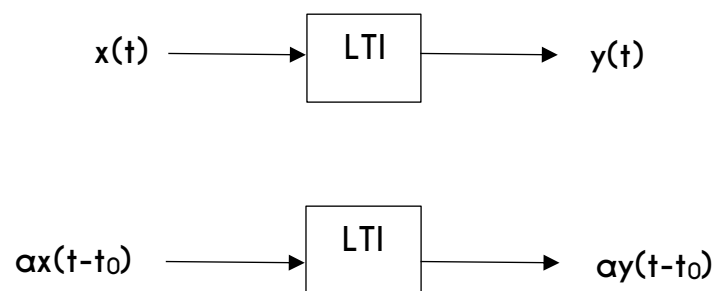
## TIME INVARIANT SYSTEMS

A time-invariant system has the property that a certain input will always give the same output (up to timing), without regard to when the input was applied to the system.



## LINEAR TIME INVARIANT SYSTEMS

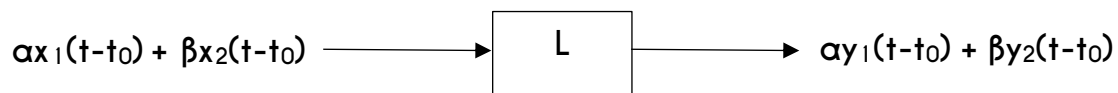
Certain systems are both linear and time-invariant, and are thus referred to as LTI systems.



As LTI systems are a subset of linear systems, they obey the principle of superposition. In the figure below, we see the effect of applying time-invariance to the superposition definition in the linear systems section above.



### Principle of Superposition



### NOTES:

#### IMPORTANT and USEFUL properties of systems :



#### Linearity

*The knowledge that a sum of input signals produces an output signal that is the summed original output signals and that a scaled input signal produces an output signal scaled from the original output signal.*



#### Time Invariance



*Ensures that time shifts commute with application of the system. In other words, the output signal for a time shifted input is the same as the output signal for the original input signal, except for an identical shift in time. Systems that demonstrate both linearity and time invariance, which are given the acronym LTI systems, are particularly simple to study as these properties allow to leverage some of the most powerful tools in signal processing.*



## INTRODUCTION \_ CONVOLUTION

The input-output relationship for LTI systems is described in terms of a CONVOLUTION operation. Knowledge of the response of an LTI system to the unit impulse input allows us to find its output to any input signals.

LTI system is a system satisfying both the **linearity** and the **time-invariance** property.

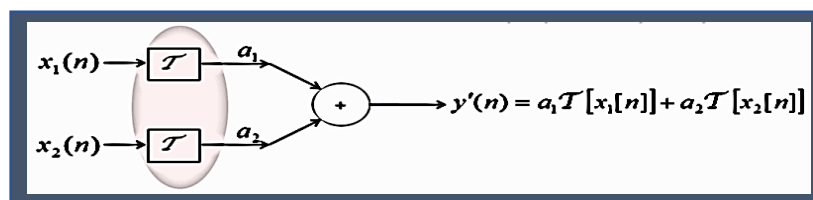
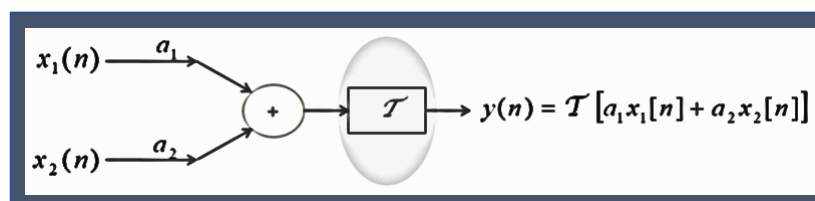
### Linearity

- response to a linear combination of inputs is the same linear combination of the individual responses.

### Time Invariance

- in effect, the system is not sensitive to the time origin

System,  $T$  is linear if and only if  $y(n)=y'(n)$   
i.e.,  $T$  satisfies the superposition principle.



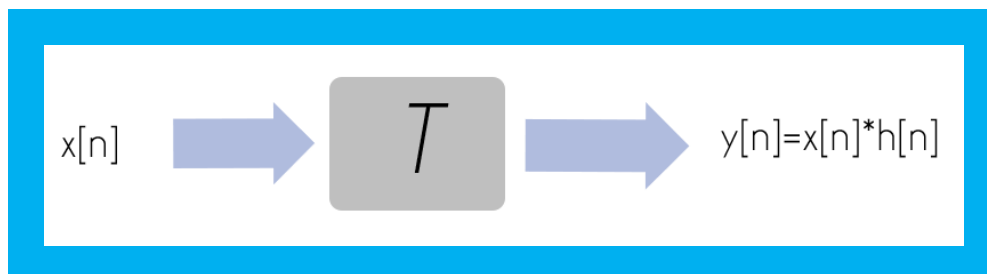


## CONVOLUTION OPERATION



The convolution of two signals is a fundamental operation in signal processing. The main use of convolution in engineering is in describing the output of a linear, time-invariant (LTI) system. By using convolution, we can construct the output of a system for any arbitrary input signal, if we know the impulse response of a system.

If the signal  $x[n]$  is applied to an LTI system with impulse response  $h[n]$ , then the output signal  $y[n]$  is:



Convolution is represented by the symbol \*

Convolution is mathematical way of combining two signals to form a third signal. Convolution is a formal mathematical operation just as multiplication, addition and integration. Addition takes two numbers and produces a third number, while convolution takes two signals and produces a third signal.



Application of convolution can be seen in digital image processing and real-time signal processing such as audio signal processing, video/image processing and large capacity data processing. Filtering is application of convolution operation to an image to achieve blurring, sharpening, edge extraction or noise removal effect.



## EFFECTS OF CONVOLUTION

1



blurring

2



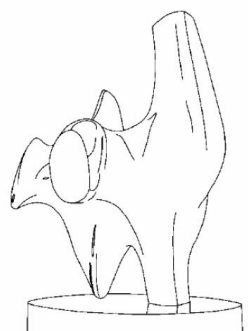
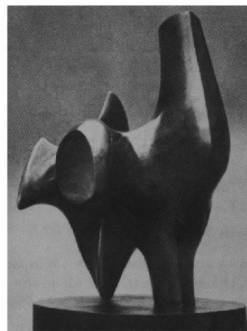
Original Image



Smart Sharpen

sharpening

3



edge extractor

4



Original Image



Modified Image

noise remover

Convolution is applicable to both continuous-time and discrete-time linear systems.

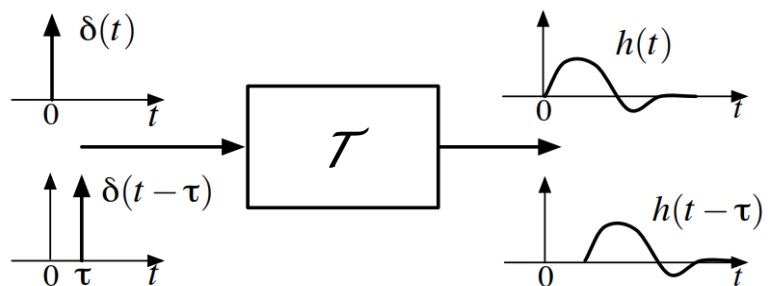
For **continuous-time** signal is called **Convolution Integral** while for **discrete-time** signal is called **Convolution Sum**.



## CONVOLUTION INTEGRAL

### Impulse Response :

- The impulse response  $h(t)$  of a continuous-time LTI system (represented by  $\mathcal{T}$ ) is defined to be the response of the system when the input is  $\delta(t)$ , that is,  $h(t) = \mathcal{T}\{\delta(t)\}$ . Transformation of  $\delta(t)$  is shown in Fig.



The convolution of two continuous-time signals  $x(t)$  and  $h(t)$  denoted by commonly called the **convolution integral**.

$$y(t) = x(t) * h(t) = \int_0^{\infty} x(\tau)h(t - \tau)d\tau$$

### CONVOLUTION INTEGRAL OPERATION



The convolution integral operation involves the following four steps :

**1**

Express each function in terms of dummy variable  $\tau$ .

**2**

Reflect one of the functions :  $h(\tau) \rightarrow h(-\tau)$

**3**

Add a time-offset,  $t$ , which allows  $h(t - \tau)$  to slide along the  $\tau$ -axis.

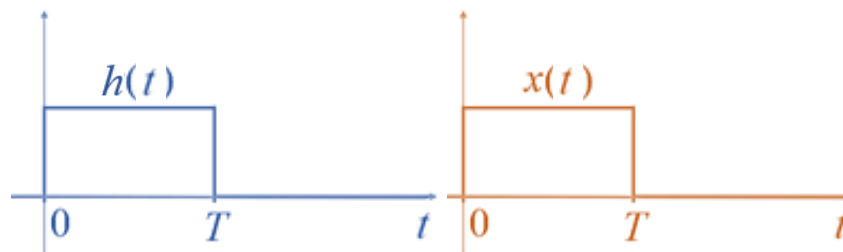
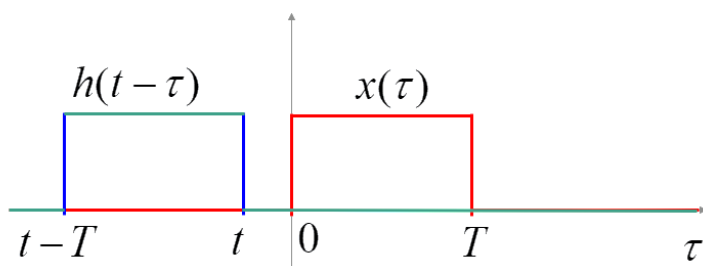
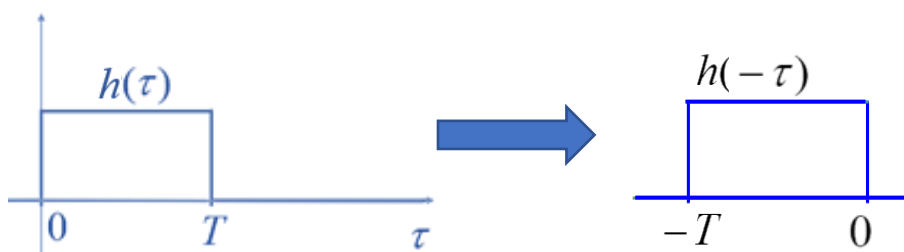
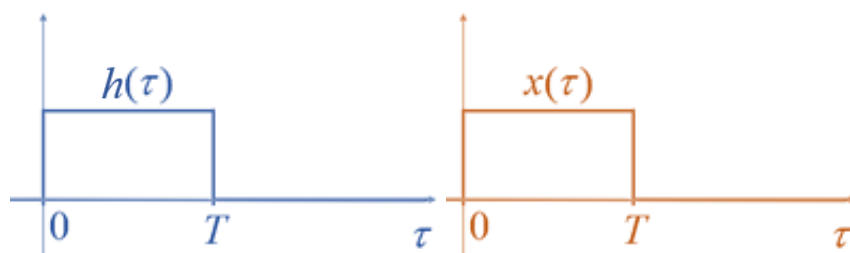
**4**

Start  $t$  at  $-\infty$ . Wherever the two functions intersect, find the integral of their product. Steps 1 to 3 are repeated as  $t$  varies over  $-\infty$  to  $\infty$  to produce the entire output  $y(t)$ .

Graphically, in order to compute the convolution integral it will decompose into a few steps.

**EXAMPLE 1**

Compute the convolution  $y(t) = x(t) * h(t)$  of the following pair of similar signals form.

**SOLUTION****Step 1 :**

- Express each function in terms of dummy variable  $\tau$ .

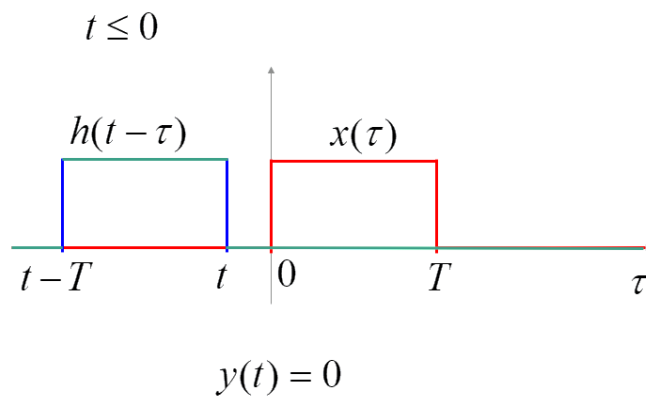
**Step 2 :**

- Reflect one of the functions ( either  $h(\tau)$  or  $x(\tau)$  ) :  $h(\tau) \rightarrow h(-\tau)$

**Step 3 :**

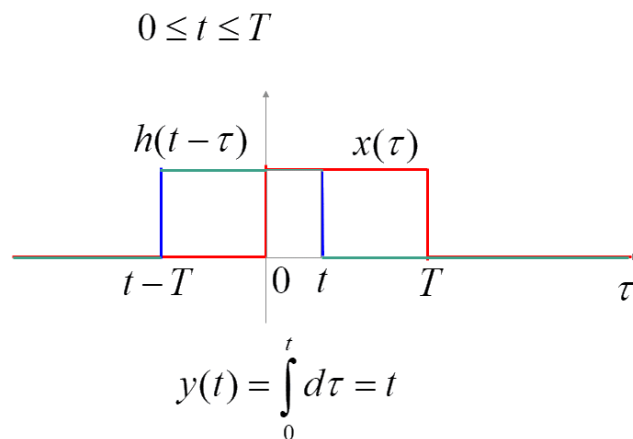
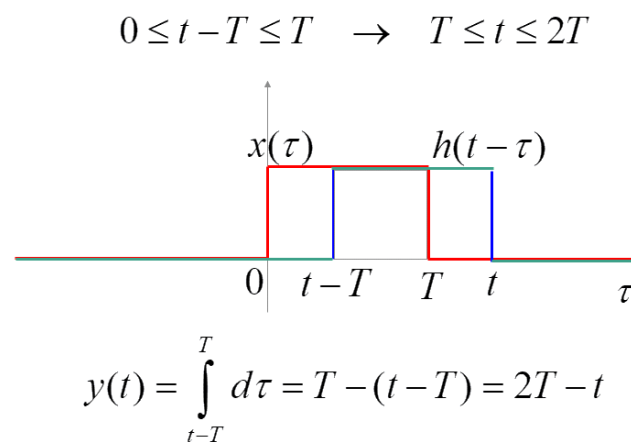
- Add a time-offset,  $t$ , which allows  $h(t-)$  to slide along the  $\tau$ -axis.



Case 1

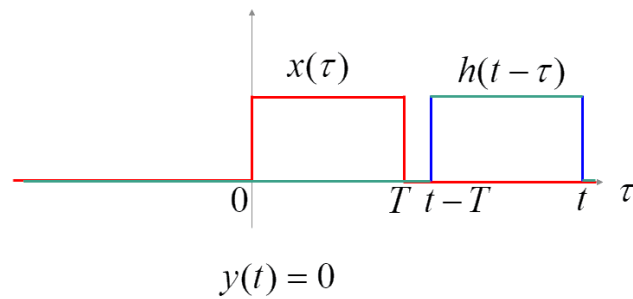
**Step 4 :**

- Start  $t$  at  $-\infty$ . Wherever the two functions intersect, find the integral of their product. Steps 1 to 3 are repeated as  $t$  varies over  $-\infty$  to  $\infty$  to produce the entire output  $y(t)$ .

Case 2Case 3

Case 4

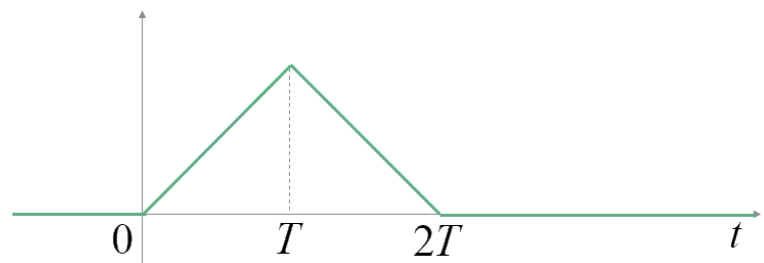
$$T \leq t - T \rightarrow 2T \leq t$$

**OUTPUT**

Convolution  $y(t) = x(t) * h(t)$

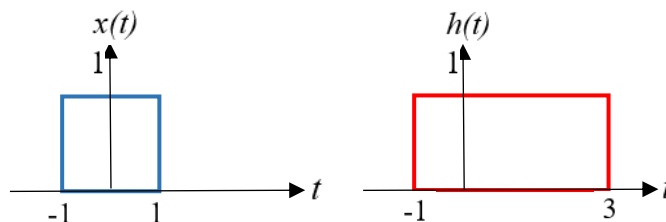
$$y(t) = \begin{cases} 0 & t < 0 \\ t & 0 \leq t < T \\ 2T - t & T \leq t < 2T \\ 0 & t \geq 2T \end{cases}$$

$$y(t) = x(t) * h(t)$$

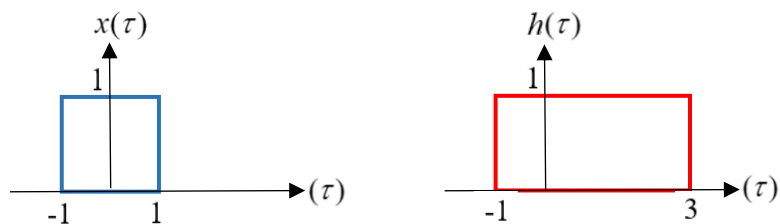


**EXAMPLE 2**

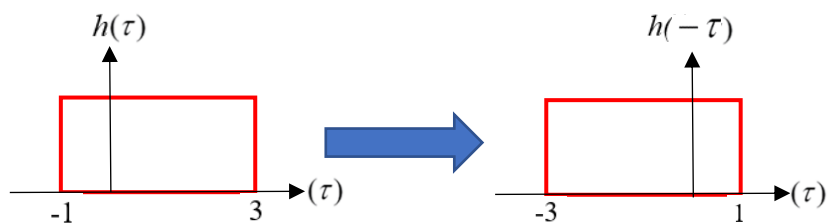
Compute the output  $y(t)$  for a continuous-time LTI system whose impulse response  $h(t)$  and the input  $x(t)$  are shown in Figure.

**SOLUTION****Step 1 :**

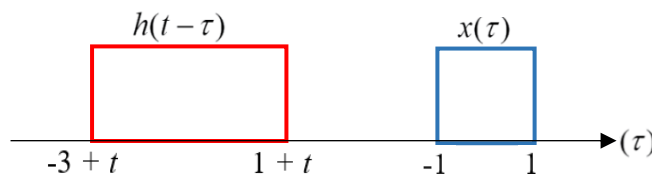
Express each function in terms of dummy variable  $\tau$ .

**Step 2 :**

Reflect one of the functions ( either  $h(\tau)$  or  $x(\tau)$  ) :  $h(\tau) \rightarrow h(-\tau)$

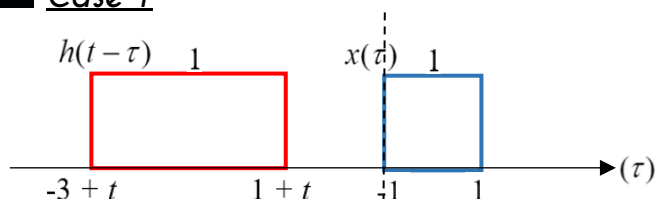
**Step 3 :**

Add a time-offset,  $t$ , which allows  $h(t-)$  to slide along the  $-$ axis.



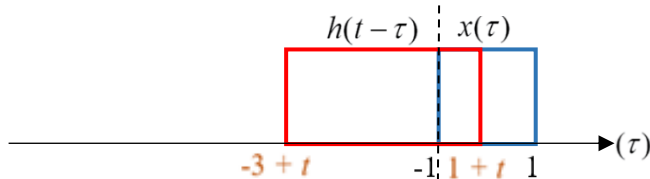
**Step 4:**

Start  $t$  at  $-\infty$ .  
Wherever the two functions intersect, find the integral of their product.  
Steps 1 to 3 are repeated as  $t$  varies over  $-\infty$  to  $\infty$  to produce the entire output  $y(t)$ .

**Case 1**

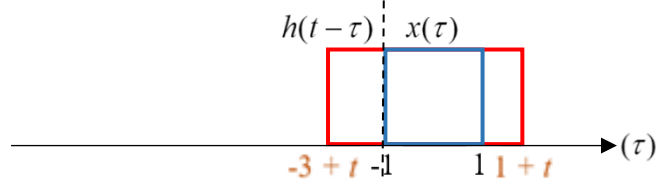
$$1+t < -1; t < -2$$

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\ &= \int_{1+t}^{-1} 0.0 d\tau = 0 \end{aligned}$$

**Case 2**

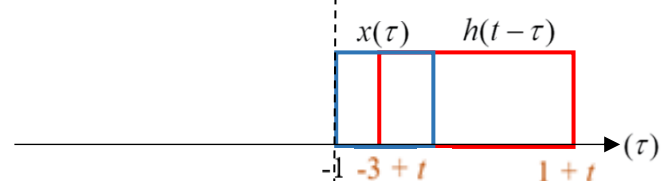
$$\begin{aligned} 1+t \geq -1; t \geq -2 \\ 1+t < 1; t < 0 \end{aligned} \quad -2 \leq t < 0$$

$$\begin{aligned} y(t) &= \int_{-1}^{1+t} x(\tau)h(t-\tau)d\tau \\ &= \int_{-1}^{1+t} 1.1 d\tau = [\tau]_{-1}^{1+t} \\ &= [(1+t) - (-1)] = 2+t \end{aligned}$$

**Case 3**

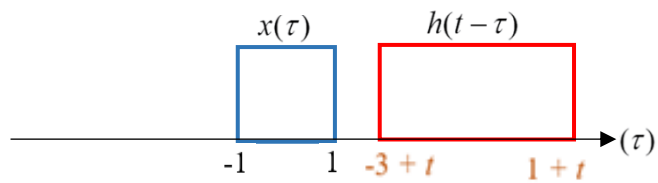
$$\begin{aligned} 1+t \geq 1; t \geq 0 \\ -3+t < -1; t < 2 \end{aligned} \quad 0 \leq t < 2$$

$$\begin{aligned} y(t) &= \int_{-1}^1 x(\tau)h(t-\tau)d\tau \\ &= \int_{-1}^1 1.1 d\tau = [\tau]_{-1}^1 \\ &= [1 - (-1)] = 2 \end{aligned}$$

**Case 4**

$$\begin{aligned} -3+t \geq -1; t \geq 2 \\ -3+t < 1; t < 4 \end{aligned} \quad 2 \leq t < 4$$

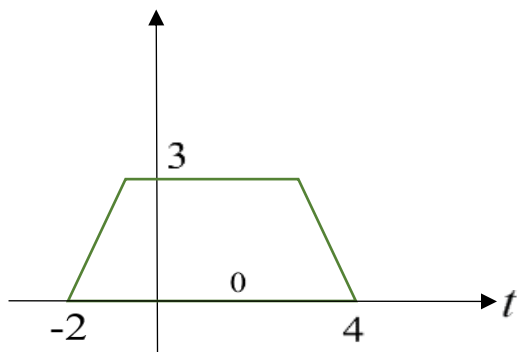
$$\begin{aligned} y(t) &= \int_{-3+t}^1 x(\tau)h(t-\tau)d\tau \\ &= \int_{-3+t}^1 1.1 d\tau = [\tau]_{-3+t}^1 \\ &= [1 - (-3+t)] = 4-t \end{aligned}$$

Case 5**OUTPUT**

Output  $y(t)$  for a continuous-time LTI system whose impulse response  $h(t)$

$$y(t) = \begin{cases} 0 & t < -2 \\ 2+t & -2 \leq t < 0 \\ 2 & 0 \leq t < 2 \\ 4-t & 2 \leq t < 4 \\ 0 & t \geq 4 \end{cases}$$

$$y(t) = x(t) * h(t)$$



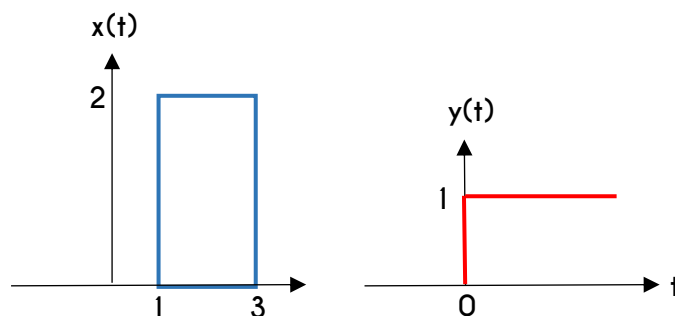
$$-3+t \geq 1; t \geq 4$$

$$y(t) = \int_1^{-3+t} x(\tau)h(t-\tau)d\tau$$

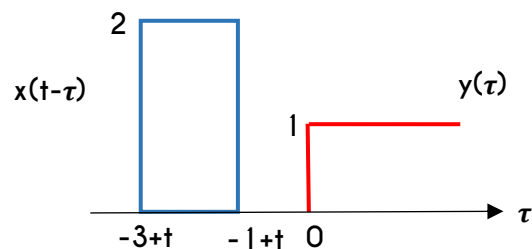
$$= \int_1^{-3+t} 0.0 d\tau = 0$$

## EXAMPLE 3

Consider the continuous-time signals,  $x(t)=u(t)$  and  $y(t)$  are shown in Fig. Compute these two signals and sketch the output,  $z(t)$

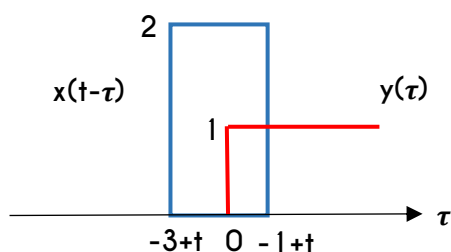


## SOLUTION



$$-1+t \leq 0; t \leq 1$$

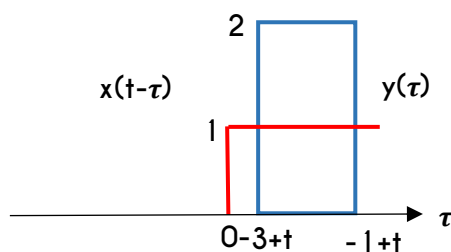
$$\begin{aligned} y(t) &= \int_0^{\infty} x(\tau)h(t-\tau)d\tau \\ &= \int_{-1+t}^0 0.0 d\tau = 0 \end{aligned}$$



$$-1+t \geq 0; t \geq 1$$

$$-3+t < 0; t \leq 3 \quad 1 \leq t \leq 3$$

$$\begin{aligned} y(t) &= \int_0^{-1+t} x(\tau)h(t-\tau)d\tau \\ &= \int_0^{-1+t} 2.1 d\tau = [2\tau]_0^{-1+t} \\ &= [2(-1+t) - 2(0)] = -2 + 2t \end{aligned}$$



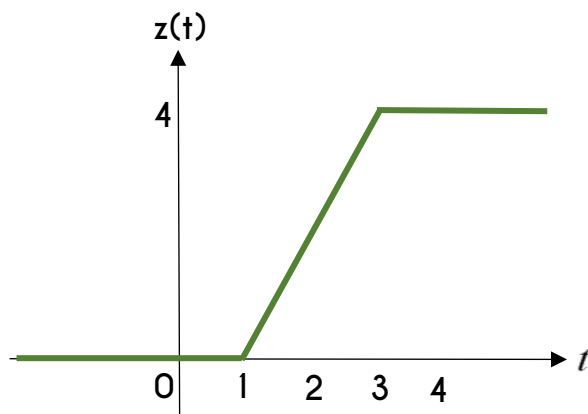
$$3+t \geq 0 \quad t \geq -3$$

$$\begin{aligned} y(t) &= \int_{-3+t}^{-1+t} x(\tau)h(t-\tau)d\tau \\ &= \int_{-3+t}^{-1+t} 2.1 d\tau = [2\tau]_{-3+t}^{-1+t} \\ &= [2(-1+t) - 2(-3+t)] \\ &= -2 + 2t + 6 - 2t = 4 \end{aligned}$$



## OUTPUT

$$z(t) = \begin{cases} 0 & t \leq 1 \\ -2 & 1 \leq t \leq 3 \\ 4 & t \geq 3 \end{cases}$$

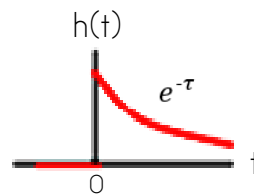
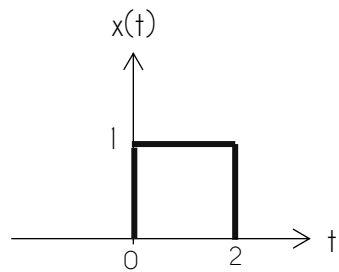


**KEEP  
TRYING  
TRYING  
TRYING  
TRYING**

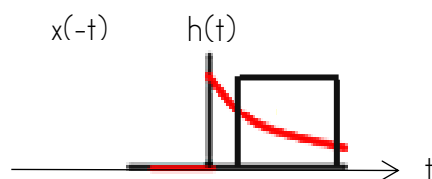
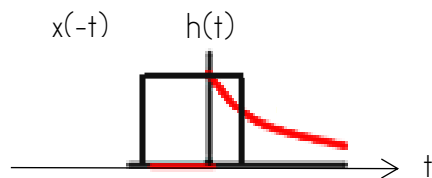
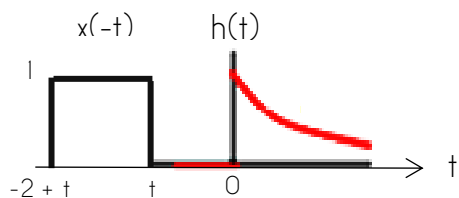


## EXAMPLE 4

Compute and sketch the output  $y(t)$  using convolution integral for input signal,  $x(t)$  and impulse response  $h(t)$ .



## SOLUTION



$$t < 0$$

$$y(t) = 0$$

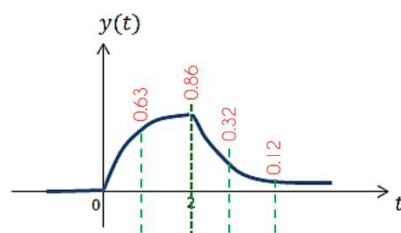
$$0 \leq t \leq 2$$

$$y(t) = \int_0^t 1 \cdot e^{-\tau} d\tau = 1 - e^{-t}$$

$$t \geq 2$$

$$y(t) = \int_{t-2}^t 1 \cdot e^{-\tau} d\tau = 6.39e^{-t}$$

## OUTPUT

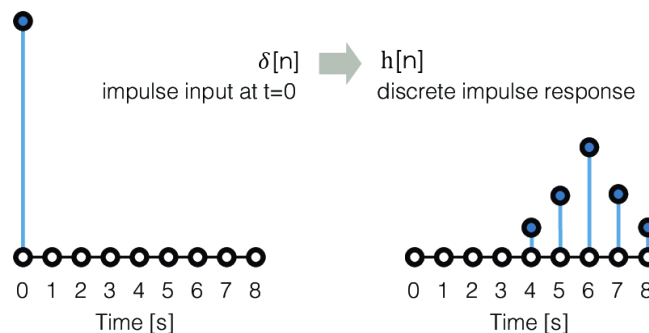




## CONVOLUTION SUM

### Impulse Response :

- The impulse response  $h[n]$  of a continuous-time LTI system (represented by  $T$ ) is defined to be the response of the system when the input is  $\delta[n]$ , that is,  $h[n] = T\{\delta[n]\}$ . Transformation of  $\delta[n]$  is shown in Fig.



The convolution of two discrete-time signals  $x[n]$  and  $h[n]$  denoted by commonly called the **convolution sum**.

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$



The convolution integral operation involves the following four steps :

1

The impulse response  $h[k]$  is time-reversed (that is, reflected about the origin) to obtain  $h[-k]$  and then shifted by  $n$  to form  $h[n-k] = h[-(k-n)]$ , which is a function of  $k$  with parameter  $n$ .

2

Two sequences  $x[k]$  and  $h[n-k]$  are multiplied together for all values of  $k$  with  $n$  fixed at some value.

3

The product  $x[k]h[n-k]$  is summed over all  $k$  to produce a single output sample  $y[n]$ .

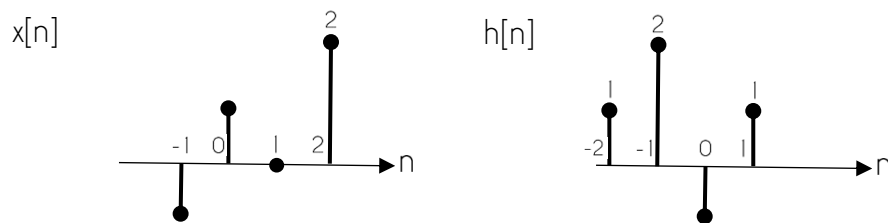
4

Steps 1 to 3 are repeated as  $n$  varies over  $-$  to  $+$  to produce the entire output  $y[n]$ .

CONVOLUTION  
SUM  
OPERATION

## EXAMPLE 5

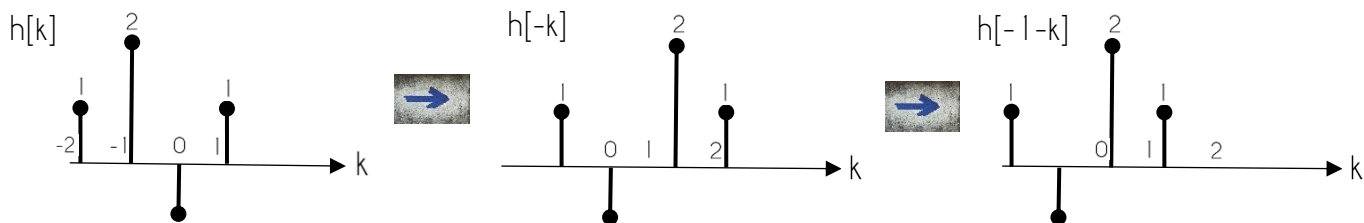
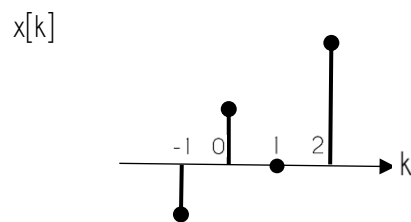
The input  $x[n]$  and the impulse response  $h[n]$  of a discrete-time LTI system are given by figure below. Find the  $y[n]$  graphically.



## SOLUTION

## Step 1 :

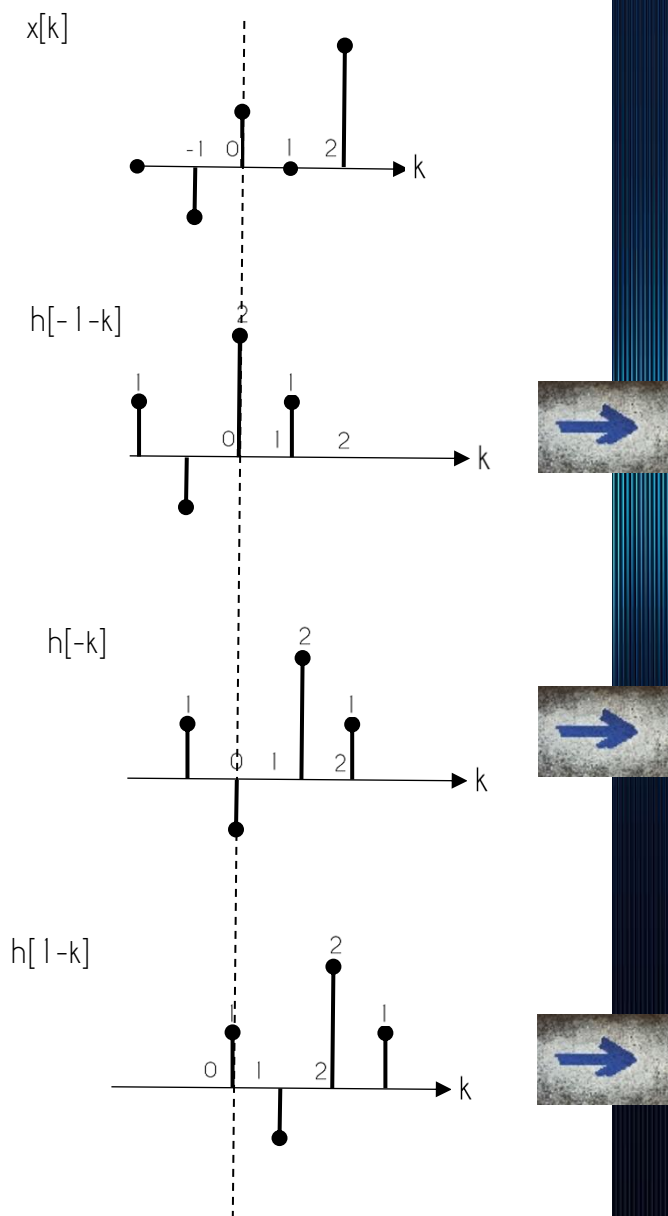
The impulse response  $h[k]$  is time-reversed (that is, reflected about the origin) to obtain  $h[-k]$  and then shifted by  $n$  to form  $h[n - k] = h[-(k - n)]$ , which is a function of  $k$  with parameter  $n$ .



**Step 2-4 :** Two sequences  $x[k]$  and  $h[n - k]$  are multiplied together for all values of  $k$  with  $n$  fixed at some value.

The product  $x[k] h[n - k]$  is summed over all  $k$  to produce a single output sample  $y[n]$ .

Steps 1 to 3 are repeated as  $n$  varies over  $-$  to  $+$  to produce the entire output  $y[n]$ .



$$\begin{aligned}
 y[-1] &= \sum_{k=-\infty}^{\infty} x[k]h[n-k] \\
 &= [0][1] + [-1][-1] + [1][2] + [0][1] + [2][0] \\
 &= 3
 \end{aligned}$$

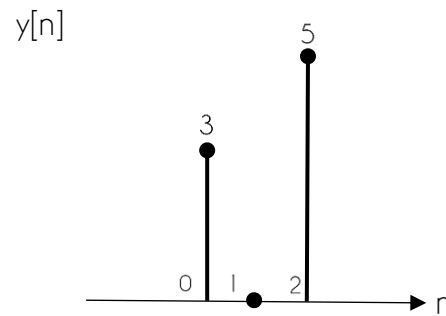
$$\begin{aligned}
 y[0] &= \sum_{k=-\infty}^{\infty} x[k]h[n-k] \\
 &= [-1][1] + [1][-1] + [0][2] + [2][1] \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 y[1] &= \sum_{k=-\infty}^{\infty} x[k]h[n-k] \\
 &= [-1][0] + [1][1] + [0][-1] + [2][2] \\
 &= 5
 \end{aligned}$$



## OUTPUT

$y[n]$  graphically



$$y[n] = \{3, 0, 5\}$$

## EXAMPLE 4

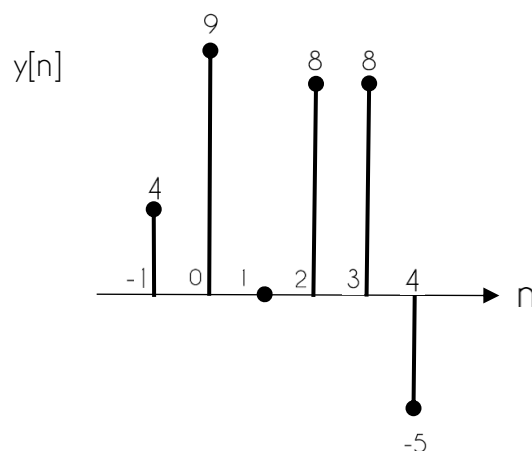
The input  $x[n]$  and the impulse response  $h[n]$  of a discrete-time LTI system are given by figure below. Find  $y[n]$  by suitable method.

$$h[n] = \{1, \underline{2}, -1\}$$

$$x[n] = \{\underline{4}, 1, 2, 5\}$$

## SOLUTION

$h[n]$	1	<u>2</u>	-1			
$x[n]$	X	<u>4</u>	1	2	5	
	4	8	-4			
		1	2	-1		
			2	4	-2	
+				5	10	-5
$y[n]$	4	<u>9</u>	0	8	8	-5



$$y[n] = \{4, \underline{9}, 0, 8, 8, -5\}$$

## OUTPUT







## PROPERTIES OF CONVOLUTION INTEGRAL

### THE COMMUTATIVE PROPERTY

$$\begin{aligned}
 x(t) * h(t) &= h(t) * x(t) \\
 x(t) * h(t) &= \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau \\
 x(t) * h(t) &= \int_{-\infty}^{\infty} x(t - \gamma) h(\gamma) d\gamma \\
 &= h(t) * x(t)
 \end{aligned}$$

### THE DISTRIBUTIVE PROPERTY

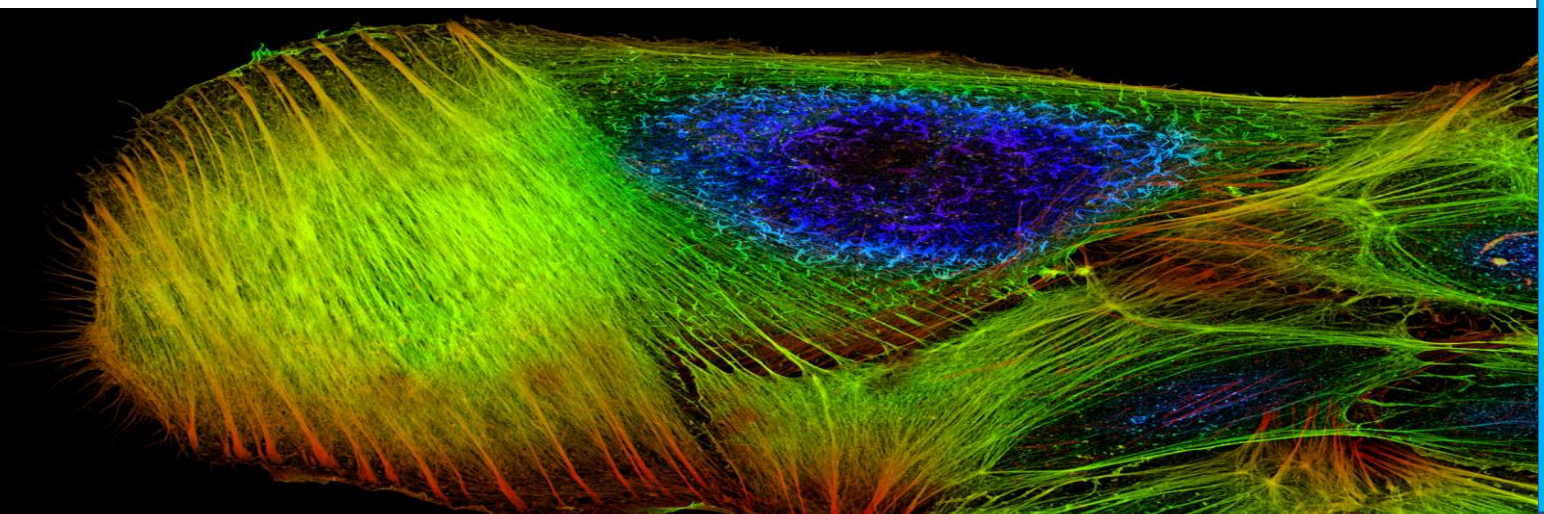
$$[x(t) * h_1(t)] * h_2(t) = x(t) * [h_1(t) * h_2(t)]$$

### THE ASSOCIATIVE PROPERTY

$$x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t)$$

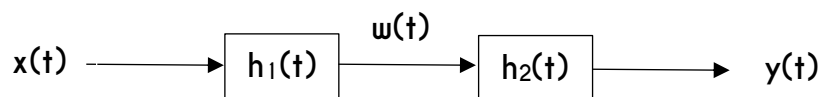
### THE TIME-SHIFT PROPERTY

$$\text{If } y(t) = x(t) * h(t) \text{ then } x(t - t_0) * h(t) = y(t - t_0)$$



**EXAMPLE 6**

The system shown in figure is formed by connecting two systems in **cascade**. The impulse responses of the systems are given by  $h_1(t)$  and  $h_2(t)$  respectively. Find the impulse response  $h(t)$  of the overall system.

**SOLUTION**

Let  $w(t)$  be the output of the first system.

$$w(t) = x(t) * h_1(t)$$

Then we have

$$y(t) = w(t) * h_2(t) = [x(t) * h_1(t)] * h_2(t)$$

But by the associativity property of convolution, it can be rewritten as

$$y(t) = x(t) * [h_1(t) * h_2(t)]$$







## PROPERTIES OF CONVOLUTION SUM

### THE COMMUTATIVE PROPERTY

$$x[n] * h[n] = h[n] * x[n]$$

### THE DISTRIBUTIVE PROPERTY

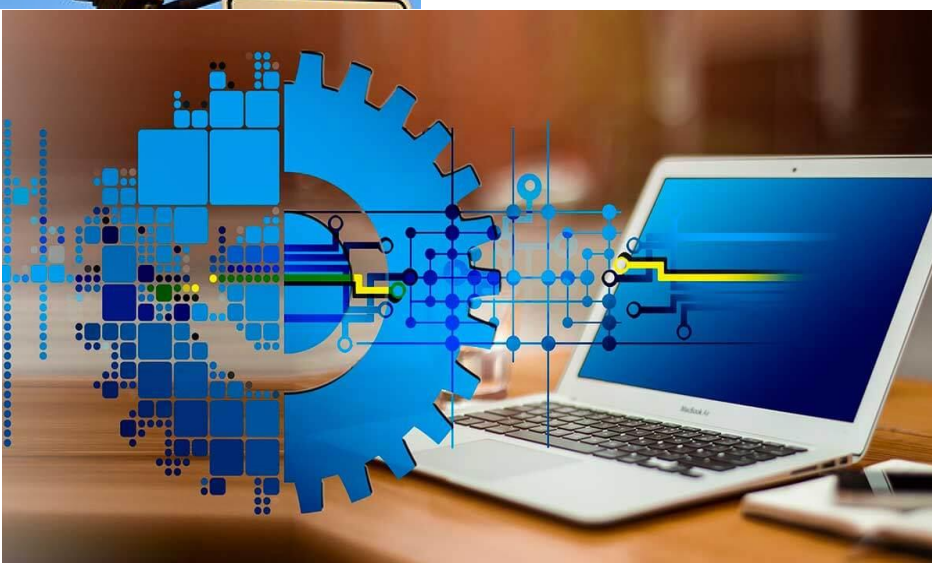
$$x[n] * \{h_1[n] + h_2[n]\} = x[n] * h_1[n] + x[n] * h_2[n]$$

### THE ASSOCIATIVE PROPERTY

$$\{x[n] * h_1[n]\} * h_2[n] = x[n] * \{h_1[n] * h_2[n]\}$$

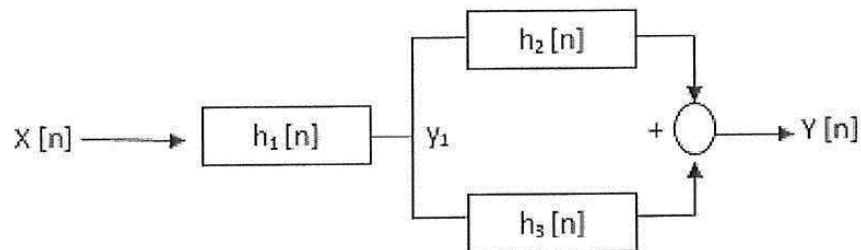
### THE TIME-SHIFT PROPERTY

$$\text{If } y[n] = x[n] * h[n] \text{ then } x[n - k] * h[n] = y[n - k]$$



**EXAMPLE 7**

Express the input-output relationship for a block diagram of LTI system shown in figure below

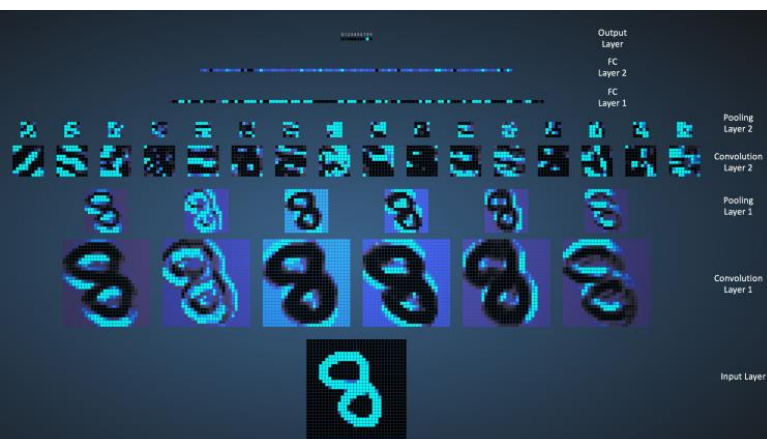
**SOLUTION**

$$y_1[n] = x[n] * h_1[n]$$

$$y[n] = y_1[n] h_2[n] + y_1[n] h_3[n]$$

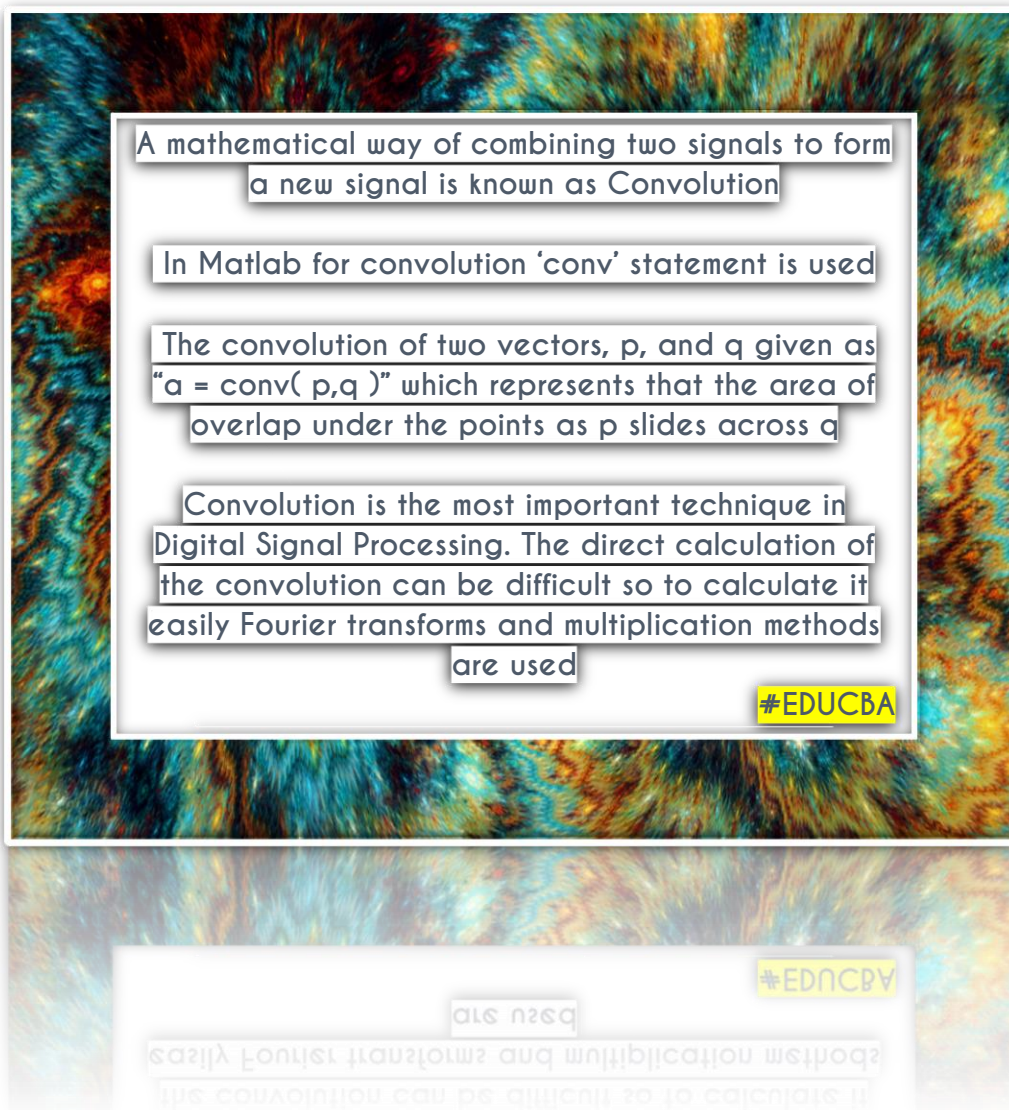
$$= x[n] h_1[n] h_2[n] + x[n] h_1[n] h_3[n]$$

$$= x[n] h_1[n] (h_2[n] + h_3[n])$$





## CONVOLUTION MATLAB



## HOW TO DO CONVOLUTION MATLAB

- 
- **Step 1:** Take an input signal and also define its length
- **Step 2:** Take an impulse response signal and defined its length
- **Step 3:** Perform a convolution using a conv function on Matlab
- **Step 4:** If we want to plot three signals we use a subplot and stem functions.





## LINEAR CONVOLUTION

*If the input and impulse response of a system is  $x[n]$  and  $h[n]$  respectively, the convolution is given by the expression,*

$$x[n] * h[n] = \sum x[k] h[n-k]$$

*Where  $k$  ranges between  $-\infty$  and  $\infty$*

*In this equation,  $x(k)$ ,  $h(n-k)$  and  $y(n)$  represent the input to and output from the system at time  $n$ . One of the inputs is shifted in time by a value every time. It is multiplied by the other input signal.*

### LINEAR CONVOLUTION

*is quite often used as a method of implementing filters of various types.*

*In mathematics and in particular functional analysis:*

### CONVOLUTION

*is similar to cross-correlation that has applications that include probability, statistics, computer vision, natural language processing, image and signal processing, engineering, and differential equations.*

# EXAMPLE

## Linear convolution integrals

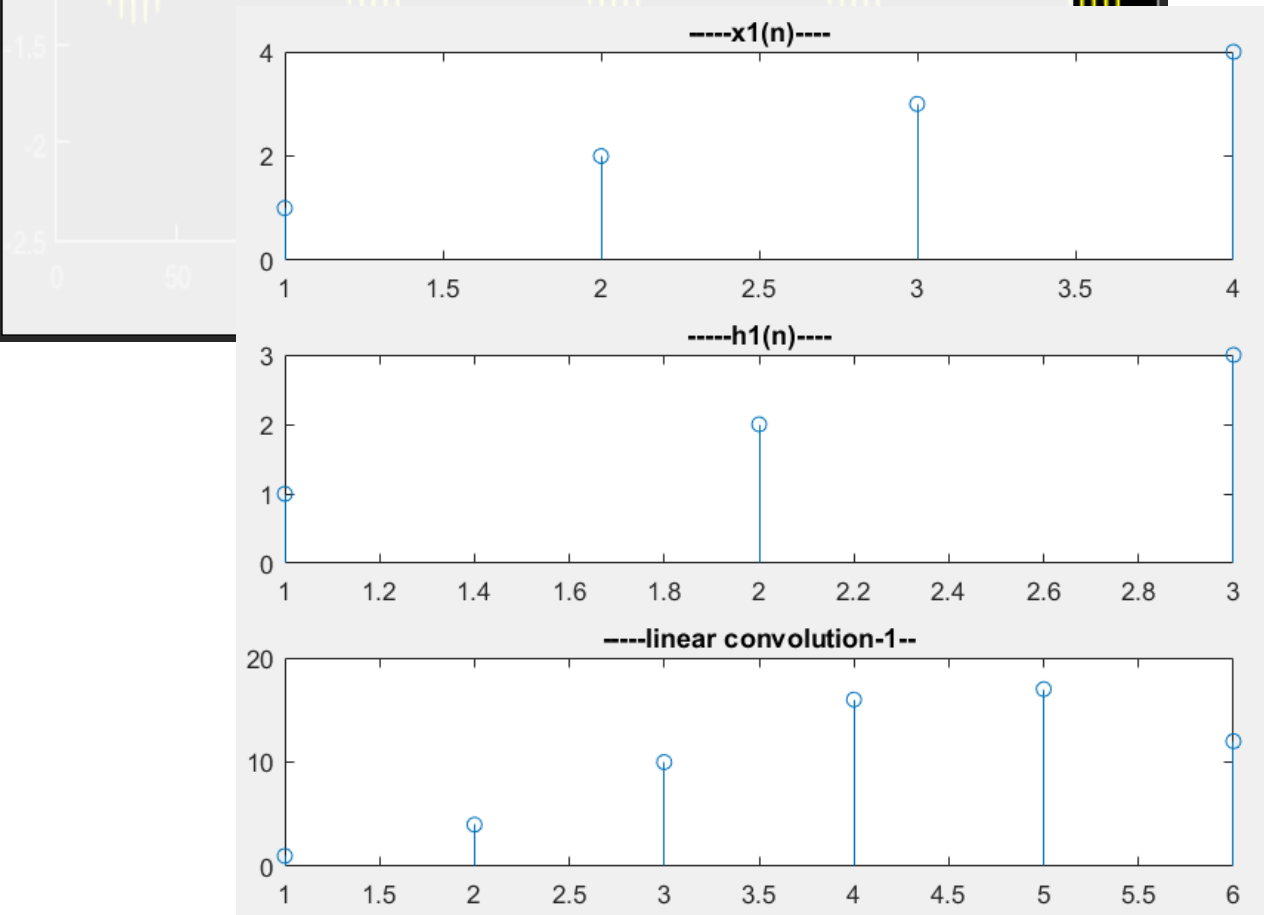
```
n = -10:10;
x1 = [1 2 3 4]; % the data value
subplot(4,2,1); % This the first subplot
stem(x1); % stem plot of the x1 data without performing
convolution operation to it.
title('-----x1(n)-----');
h1 = [1,2,3]; %This is the second data value.

subplot(4,2,3);
stem(h1); %stem plot of the second data value without
performing convolution operation to it.
title('-----h1(n)-----');

%%%%linear convolution%%%%%%%%
l1 = conv(x1,h1) % %conv gives the convolution of x1 and
h1 vectors. l1 stores these values.
subplot(4,2,5);
stem(l1); %gives the stem plot of the l1 data values.

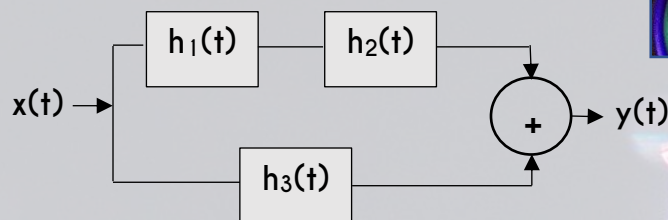
title('-----linear convolution-1--');
```

Amplitude



**Question 1 :**

Express the input-output relationship for a block diagram of LTI system shown in Figure.



$$y(t) = x(t)*h_3(t) + [x(t)*h_1(t)*h_2(t)]$$

$$= x(t)[h_1(t)*h_2(t)] + h_3(t)$$

$$y(t) = x(t) * h(t) = \int_0^{\infty} x(\tau)h(t - \tau)d\tau$$

**Question 2 :**

Define mathematically the convolution of two continuous signals  $x(t)$  and  $h(t)$ .

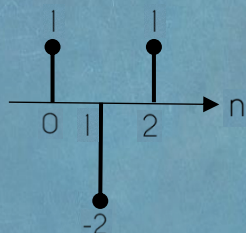
**Question 3 :**

Describe the steps involved in graphical convolution of  $x(t)$  and  $h(t)$ .

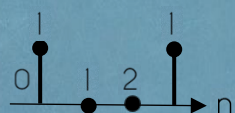
1. Express each function in terms of dummy variable  $\tau$ .
2. Reflect one of the functions :  $h(\tau) \rightarrow h(-\tau)$
3. Add a time-offset,  $t$ , which allows  $h(t-\tau)$  to slide along the  $\tau$ -axis.
4. Start  $t$  at  $-\infty$ . Wherever the two functions intersect, find the integral of their product. Steps 1 to 3 are repeated as  $t$  varies over  $-\infty$  to  $\infty$  to produce the entire output  $y(t)$ .

i.

$x[n]$



$h[n]$



**Question 4 :**

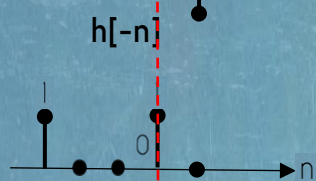
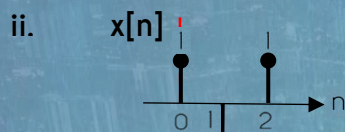
Consider an LTI system with an impulse,  $h[n]$  and the input signal,  $x[n]$  as follows;

$$x[n] = \delta[n] - 2\delta[n-1] + \delta[n-2]$$

$$h[n] = \delta[n] + \delta[n-3]$$

- i. Interpret the input,  $x[n]$  and impulse,  $h[n]$  in graphical term.
- ii. Calculate the output of the system,  $y[n]$  using convolution sum.
- iii. Draw the output of the system,  $y[n]$ .





$$y[0] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$= [1][1] = 1$$



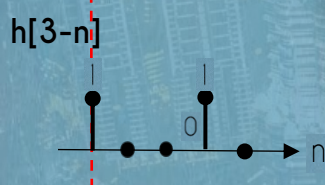
$$y[0] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$= [-2][1] = -2$$



$$y[0] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$= [1][1] = 1$$



$$y[0] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$= [1][1] = 1$$

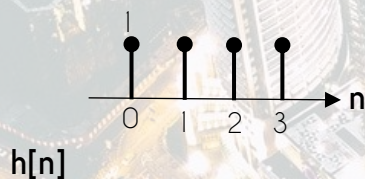
iii  $y[n]$



$$y[n] = \{1, -2, 1, 1\}$$

Question 5 :

Determine  $y[n] = x[n]*h[n]$  for  $0 \leq n \leq 3$  when  $x[n]$  and  $h[n]$  are shown in Fig.

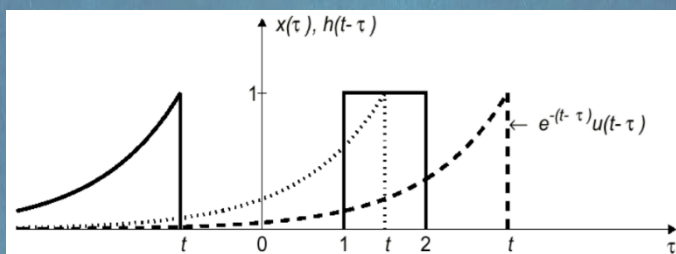


$x[n]$	1	1	1	1		
$h[n]$	1	1	1			
	1	1	1	1		
		1	1	1	1	
+			1	1	1	1
	1	2	3	3	2	1

$$y[n] = \{1, 2, 3, 3, 2, 1\}$$







$$t \leq 1$$

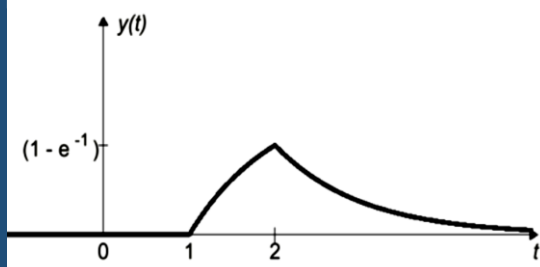
$$y(t) = 0$$

$$1 \leq t \leq 2$$

$$y(t) = \int_1^t 1 \cdot e^{-\tau} d\tau = 1 - e^{-(t-1)}$$

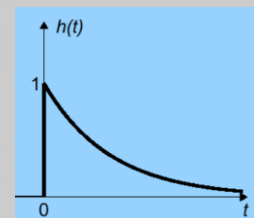
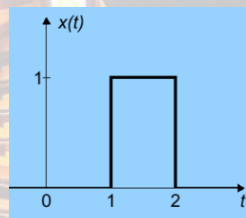
$$t \geq 2$$

$$y(t) = \int_1^2 1 \cdot e^{-\tau} d\tau = e^{-(t-2)} - e^{-(t-1)}$$



### Question 6:

An input signal,  $f(t)$  and impulse response,  $g(t)$  are shown in Fig. Convolve these two signals and sketch the output,  $q(t)$



### Question 7 :

Express the convolution in the time range from 0 to 1 if  $h(t) = e^{-\alpha t} u(t)$  and  $x(t) = u(t)$ .

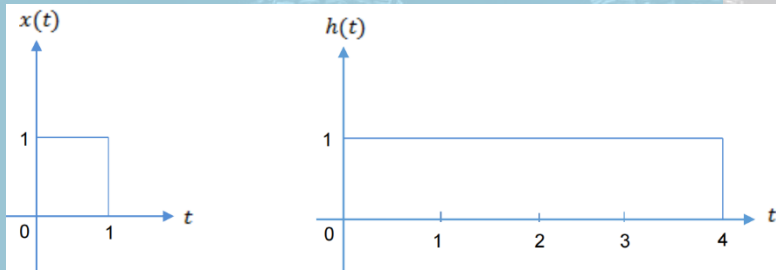
$$\begin{aligned} y(t) &= x(t) * h(t) \\ &= \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau \\ &= \int_0^t e^{-\alpha \tau} d\tau \\ &= \left[ -\frac{1}{\alpha} e^{-\alpha \tau} \right]_0^t \\ &= \frac{1}{\alpha} [1 - e^{-\alpha t}] \end{aligned}$$

**Question 8:**

Display the output of

$$y(t) = x(t) * h(t)$$

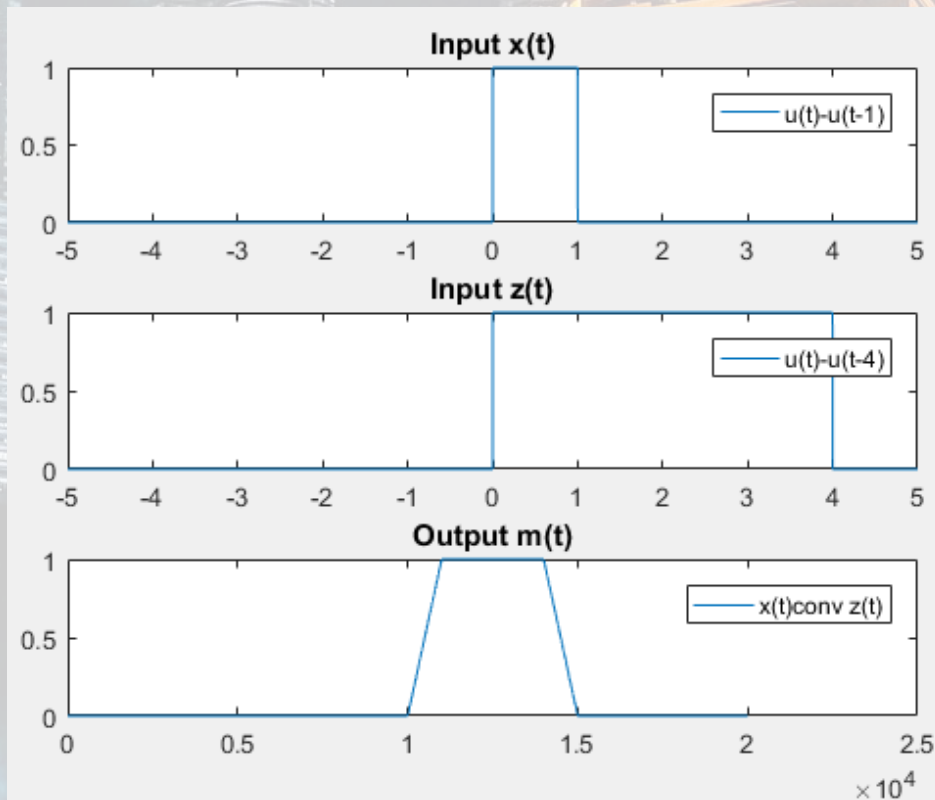
using convolution integral where  $x(t)$  and  $h(t)$  are shown below. Execute the correct command arrays.



```
t=-5:0.001:5;
x=heaviside(t)-heaviside(t-1);
subplot(3,1,1);
plot (t,x);
title('Input x(t)')
legend('u(t)-u(t-1)');
```

```
h=heaviside(t)- heaviside(t-4);
subplot(3,1,2);
plot (t,h);
title('Input h(t)')
legend('u(t)-u(t-4)');
```

```
m=conv(x,h) .*0.001;
subplot(3,1,3);
plot (m);
title('Output m(t)')
legend('x(t)conv h(t)');
```





Question 9:

Sketch graph of

$$x(t) = 3u(t) - 3u(t-1)$$

and

$$h(t) = 2u(t) - 2u(t-4)$$

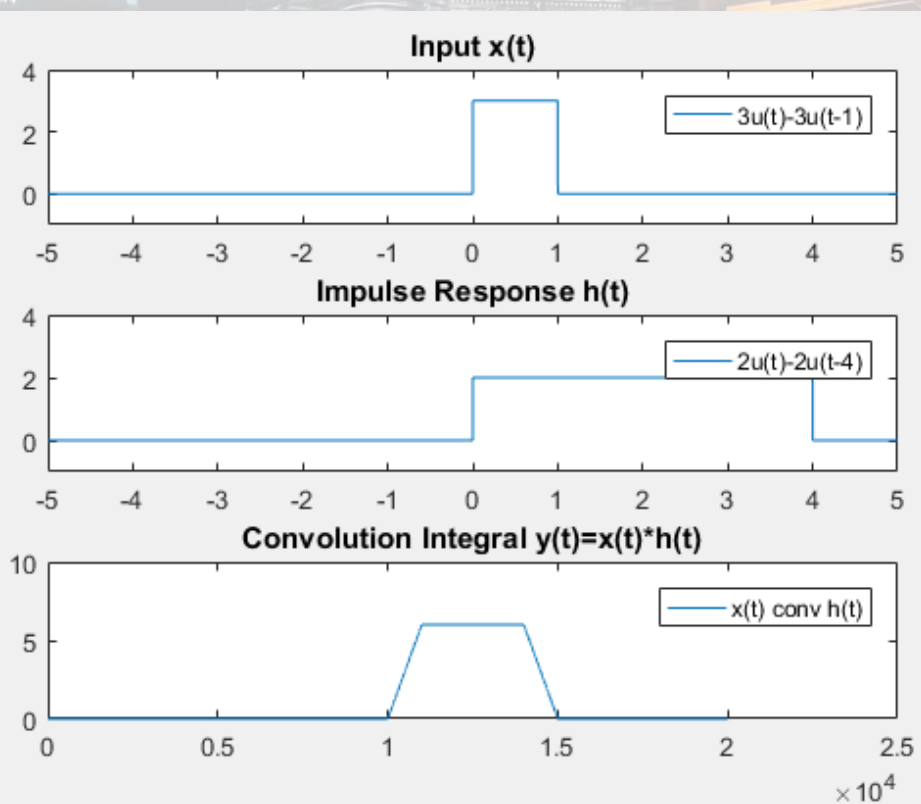
in subplot(3,1,row). Execute the command using Matlab.

```
t=-5:0.001:5;
x=3*heaviside(t)-3*heaviside(t-1);
subplot(3,1,1);
plot (t,x);
title('Input x(t)')
legend('3u(t)-3u(t-1)');
axis([-5 5 -1 4]);
```

```
h=2*heaviside(t)-2*heaviside(t-4);
subplot(3,1,2);
plot (t,h);
title('Impulse Response h(t)')
legend('2u(t)-2u(t-4)');
axis([-5 5 -1 4]);
```

```
m=conv(x,h).*0.001;
subplot(3,1,3);
plot (m);
title('Impulse Response h(t)')
legend('x(t) conv h(t)');
```

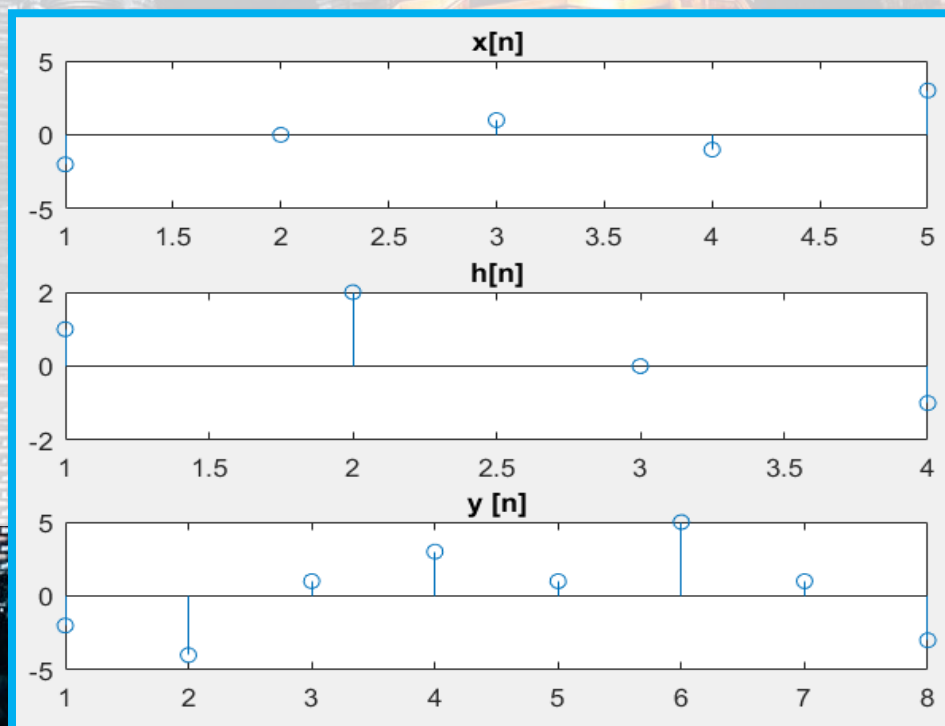
```
title('Convolution Integral
y(t)=x(t)*h(t)')
```



**Question 10 :**

Determine and sketch the convolution sum of  $x(n) = [-2 \ 0 \ 1 \ -1 \ 3]$  and  $h(n) = [1 \ 2 \ 0 \ -1]$  in subplot (3,1, row). Put an appropriate title for every axis.

```
x = [-2 0 1 -1 3]; % x( n)
h = [1 2 0 -1]; % h( n)
y = conv (x,h); % convolution
subplot (311)
stem(x), title ('x[n]')
subplot (312)
stem(h), title ('h[n]')
subplot (313)
stem(y), title ('y[n]')
```





**Question 11:**

Determine and sketch the convolution sum of  $x(n) = \{ 1 \ 2 \ 3 \}$  and  $h(n) = \{ 2 \ 4 \ 3 \ 5 \}$  using the MATLAB script with following line style and colour for :

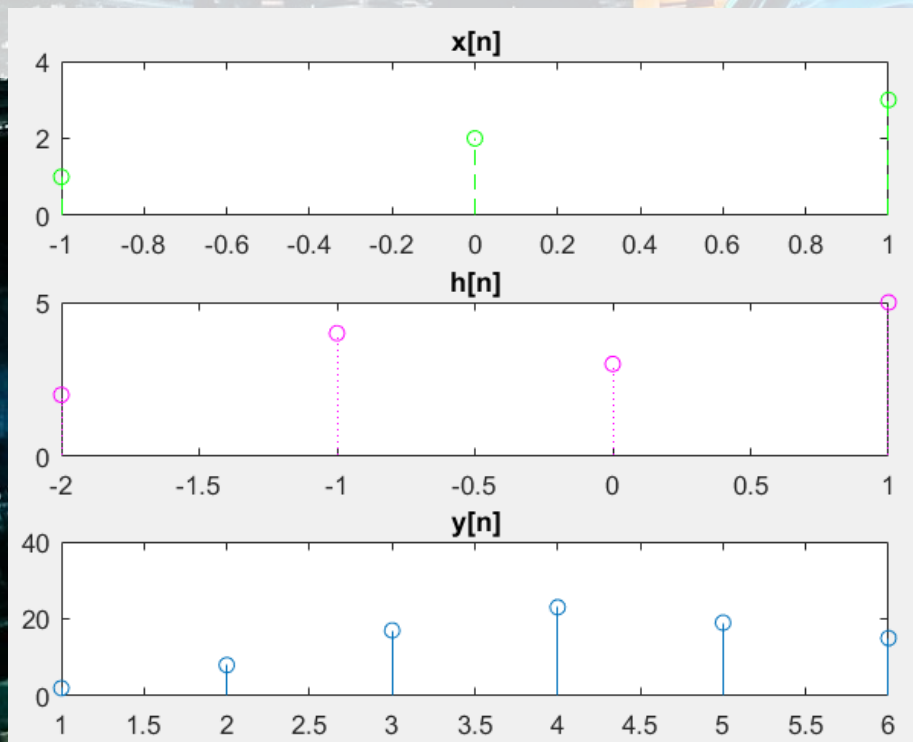
$x(n)$  : **green**, dashed line style

$h(n)$  : **magenta**, dotted line style

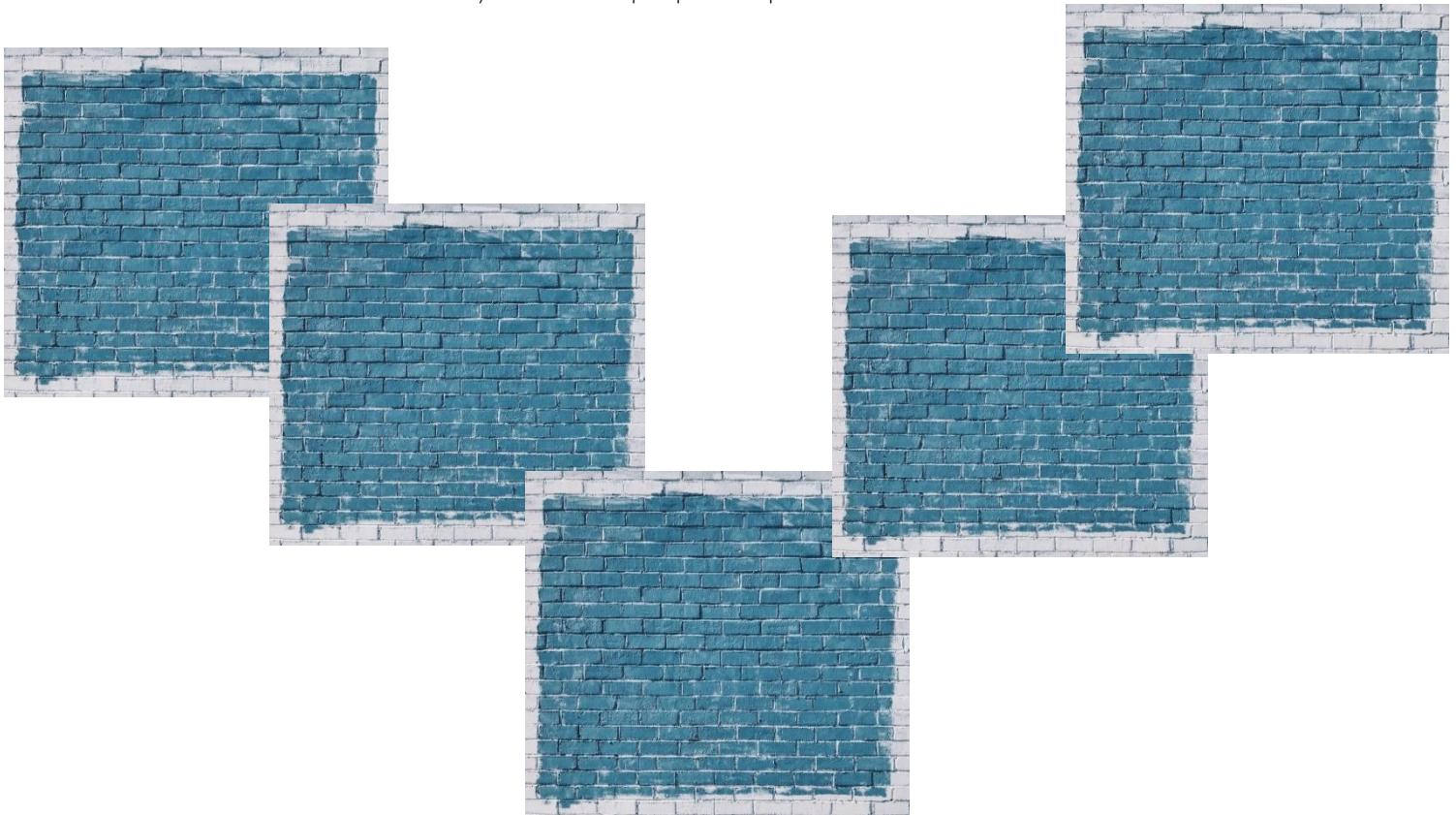
```
x = [1 ,2 ,3]; % x( n)
nx = -1:1;
subplot (311)
stem(nx,x,'g','--');
title ('x[n]')

h = [2 ,4 ,3 ,5]; % h( n )
nh = -2:1;
subplot (312)
stem(nh,h,'m',':');
title ('h[n]')

y = conv (x,h); % convolution
subplot (313)
stem(y)
title ('y[n]')
```



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- [9] Richard Baraniuk. (2015). *Signals and Systems*. Creative Commons Attribution License 4.0. <https://cnx.org/contents/cal10064/1.15/>
- [10] Simon Haykin. (1999). *Signals and Systems*. John Wiley & Sons. <https://cnx.org/contents/pgrh5dEe@15.5:D7g4NQMa@2/Properties-of-Convolution-Integrals> Convolution (Sec 1.4 from Hayes DSP). [https://elec3004.uqcloud.net/2013/lectures/Convolution%20\(Sec%201.4%20from%20Hayes%20DSP\).pdf](https://elec3004.uqcloud.net/2013/lectures/Convolution%20(Sec%201.4%20from%20Hayes%20DSP).pdf) (uqcloud.net)





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