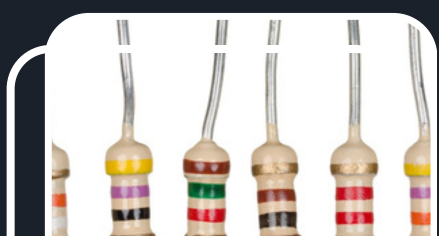
ELECTRICAL CIRCUITS

Polytechnic Chapters



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KEMENTERIAN PENDIDIKAN TINGGI
JABATAN PENDIDIKAN POLITEKNIK DAN KOLEJ KOMUNITI

POLITEKNIK MALAYSIA
SULTAN SALAHUDDIN ABDUL AZIZ SHAH

ELECTRIC CIRCUITS
Transformers
2nd Edition

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DISCLAIMER

Preface

Greeting To All.

We are very pleased to be given the opportunity to release this book as a reference for students who are enroll in Diploma in Electronic Engineering Programme.

The book contains a selected topic to ELECTRICAL CIRCUITS which includes subtopics alternating voltage and current, sinusoidal steady-state current analysis, resonance, transformers and three phase system for students' understanding. The book also provides exercises for students at the end of the topic.

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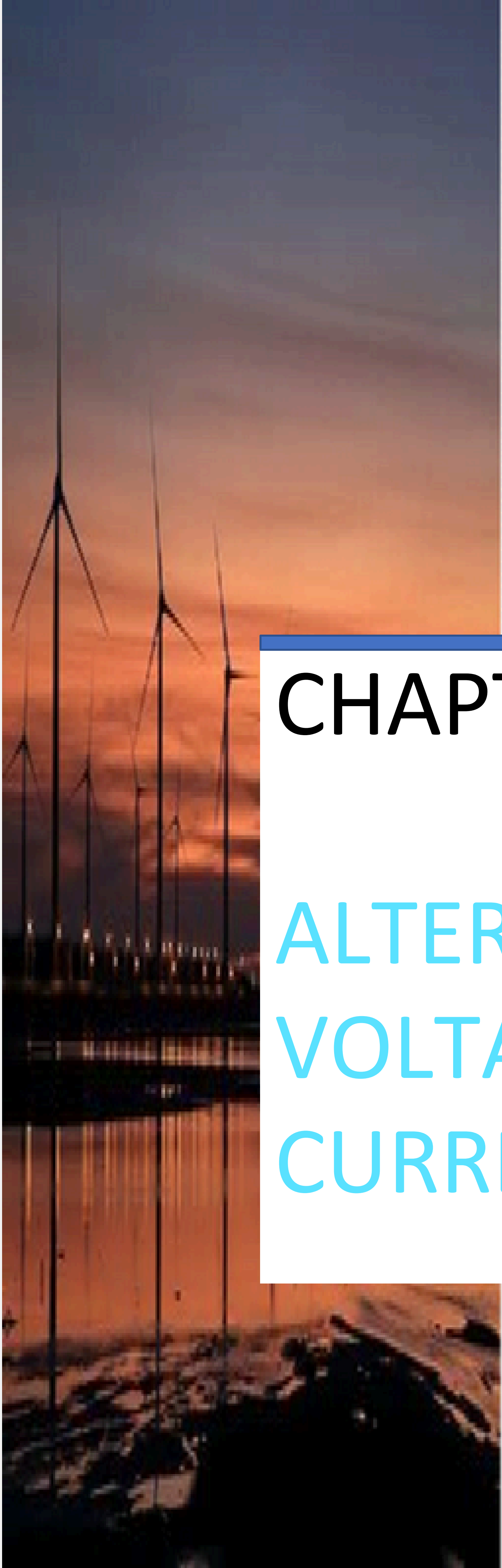


CHAPTER 1

ALTERNATING VOLTAGES AND CURRENT

COURSE LEARNING OUTCOME

1. Explain AC circuit concept and their analysis using AC circuit law.

2. Apply the knowledge of AC circuit in solving problem related to AC electrical circuit.

OBJECTIVE

1. Remember an alternating current
2. Understand the generation of an alternating current
3. Apply sinusoidal voltage and current values of a sine wave
4. Apply a sinusoidal wave for an angular measurement.
5. Apply the understand of phasor to represent a sin.e wave
6. Apply the basic circuit laws of resistive AC circuits

Differentiate between direct current and alternating current

DC & AC

Overview

Direct Current (DC)

The flow of electrical charge is only in one direction. The output voltage will remain essentially constant over time.

Alternating Current(AC)

The movement of electrical charge periodically reverses directions.
AC source of electrical power charges constantly in amplitude & regularly changes polarity.

Direct Current (DC)

Direct current (DC), always flows in the same direction (always positive or always negative), but it may increase and decrease.

Electronic circuits normally require a steady DC which is constant at one value or a smooth DC supply which has a small variation called ripple.

Cells, batteries and regulated power supplies provide steady DC which is ideal for electronic circuits.

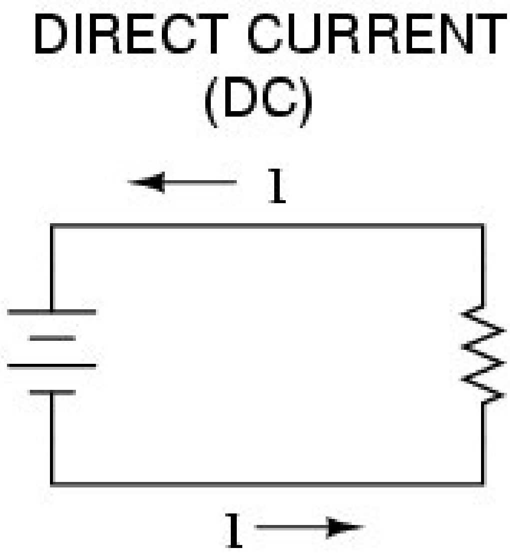
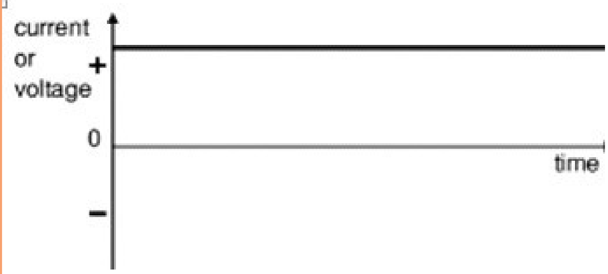


Diagram 1: DC circuit

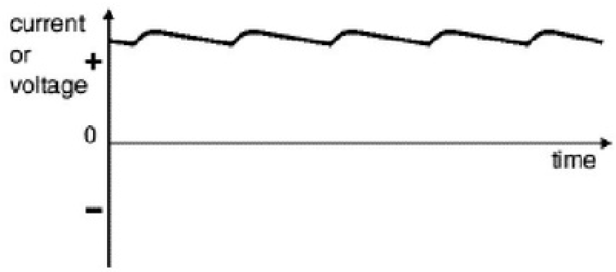
D C Waveforms

01



Steady DC
from a battery or regulated power supply,
this is ideal for electronic circuits.

02



Smooth DC
from a smoothed power supply,
this is suitable for some electronics.

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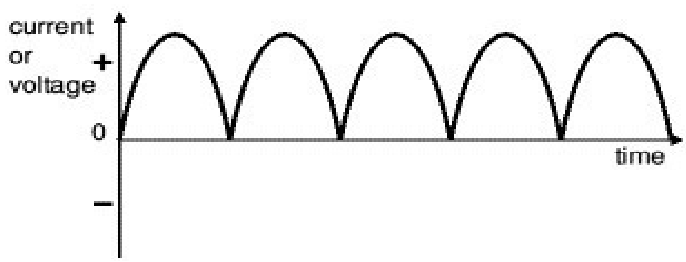


Varying DC
from a power supply without smoothing,
this is not suitable for electronics.

Alternating Current (AC)

Alternating Current (AC) flows one way, then the other way, continually reversing direction
An AC voltage is continually changing between positive (+) and negative (-).

The rate of changing direction is called frequency of the AC and it is measured in Hertz (Hz) which is the number of forwards-backwards cycles per second.

An AC supply is suitable for powering some devices such as lamps and heaters.

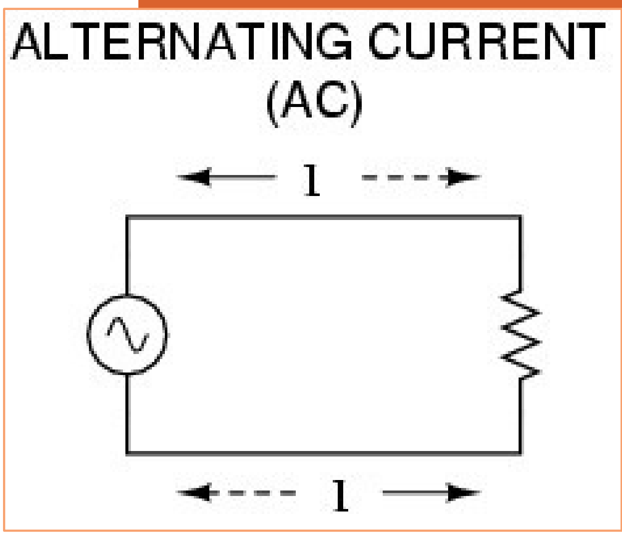
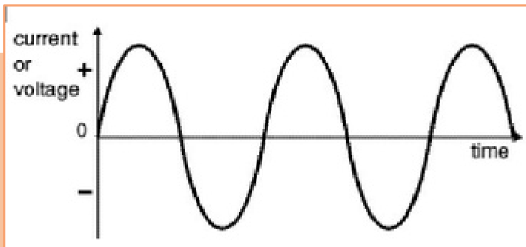


Diagram 2: AC circuit

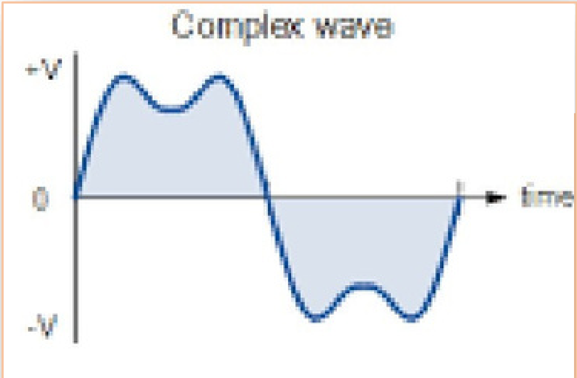
A C Waveforms

01



AC from a power supply
This shape is called a **sine wave**.

03



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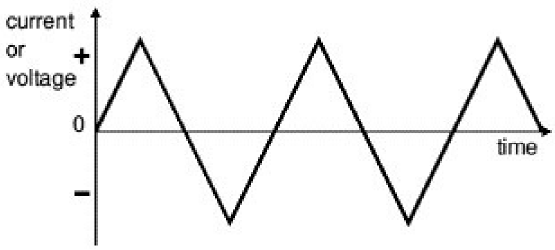
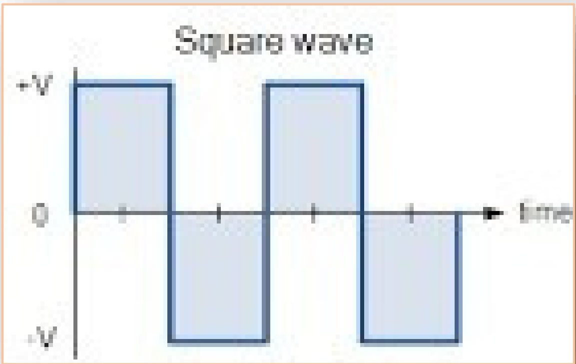


This triangular signal is AC because it changes between positive (+) and negative (-).

04



Describe Why AC Is Used In Preference To DC

Advantages of ac electricity

Overview

- The ability to readily transform voltages is the main reason to use AC instead of DC
- Since high voltages are more efficient for sending electricity great distances, AC electricity has an advantage over DC.
- This is because the high voltages from the power station can be easily reduced to a safer voltage for use in the house by using the transformer

Alternating Current (AC)

AC is commonly use to power our television, lights and computers. In AC electricity, the current alternates in direction.

The motors that using AC are smaller, more durable and difficult to damage because AC motor doesn’t have commutator.

AC supply more easier to converted to DC by using rectifier

It is easier and cheaper to generate alternating current (ac) than direct current (dc) and ac is more conveniently distributed than dc since its voltage can be readily altered using transformers.

Sinusoidal AC voltages are available from variety of sources

The most common source is the typical home outlet, which provides an ac voltage that originates at a power plant; such a power plant is most commonly fueled by water power, oil, gas, or nuclear fusion.

Explain faraday's and lenz's law involved in generating AC current

ALTERNATOR

- AC can be produced using a device called an alternator
- This device is a special type of electrical generator designed to produce alternating current
- The rotation of the wire can come from any number of means : a wind turbine, a steam turbine, flowing water etc

Generation of an alternating current

Alternating voltage may be generated by rotating a coil in the magnetic field or by rotating a magnetic field within a stationary coil.

The value of the voltage generated depends on :
- The number of turns in the coil.
- Strength of the field.
- The speed at which the coil or magnetic field rotates.

GENERATION OF AN ALTERNATING CURRENT

FARADAY'S LAW

Overview

Faraday's observations can be stated as follows:

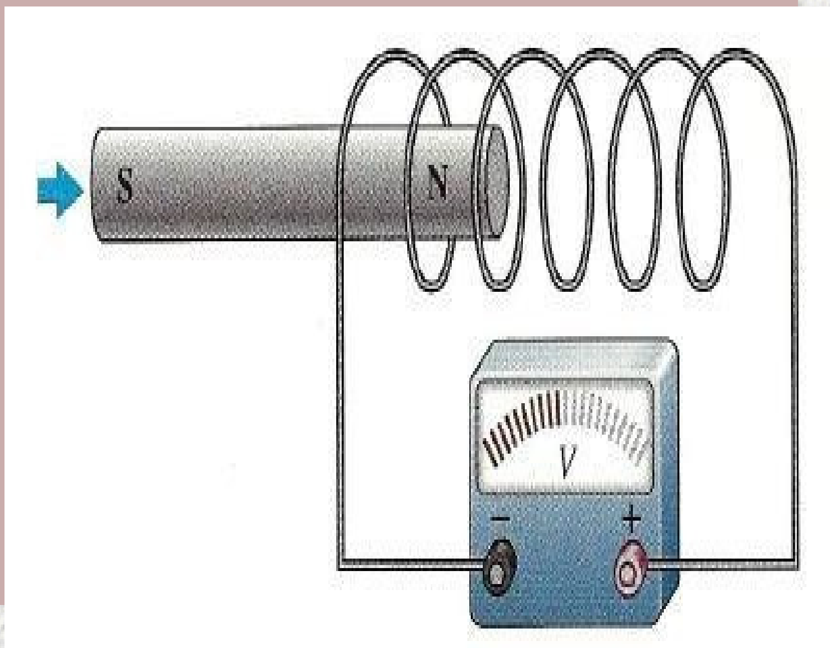
Faraday's observations can be stated as follows:

1. The amount of voltage induced in a coil is directly proportional to the rate of change of the magnetic field with respect to the coil ($d\phi/dt$)

2. The amount of voltage induced in a coil is directly proportional to the number of turns of wire in the coil (N)

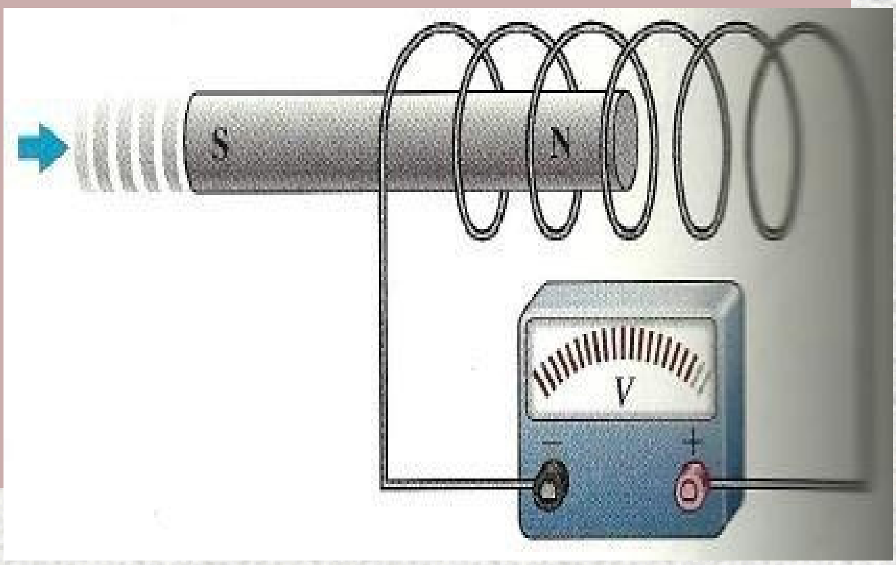
FARADAY'S FIRST OBSERVATION

As the magnet moves slowly to the right, its magnetic field is changing with respect to the coil, and a voltage is induced.



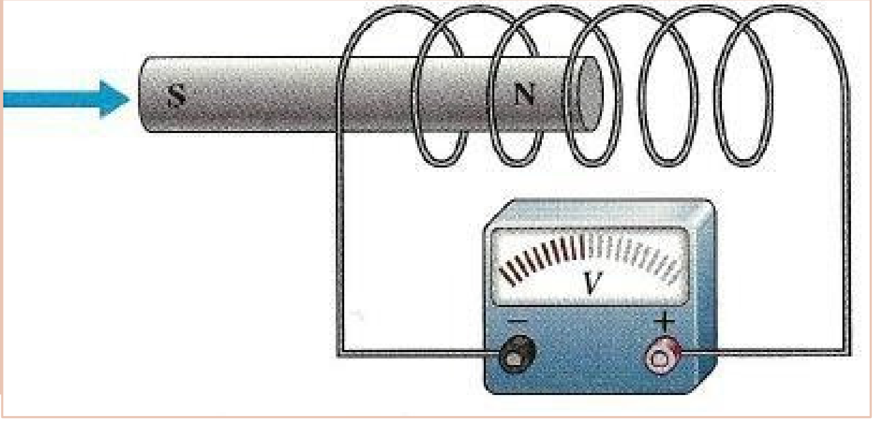
Faraday's Law is stated :

The voltage induced across a coil equals the number of turns in the coil times the rate of change of the magnetic flux

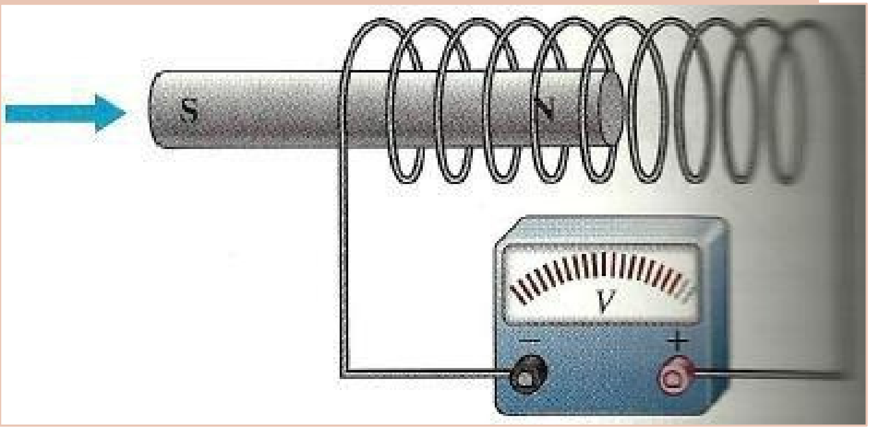


As the magnet moves more rapidly to the right, its magnetic field is changing more rapidly with respect to the coil, and a greater voltage is induced.

Faraday’s second observation



a) Magnet moves a coil and induces a voltage



b) Magnet moves at same rate through a coil with more turns (loops) and induces a greater voltage.

GENERATION OF AN ALTERNATING CURRENT

LENZ'S LAW

Overview

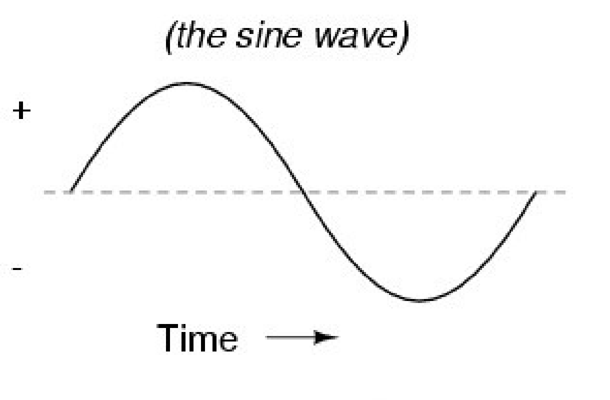
Lenz's law is stated as follows :

When the current through a coil changes, an induced voltage is created as a result of the changing electromagnetic field and the polarity of the induced voltage is such that it always opposes the change in current

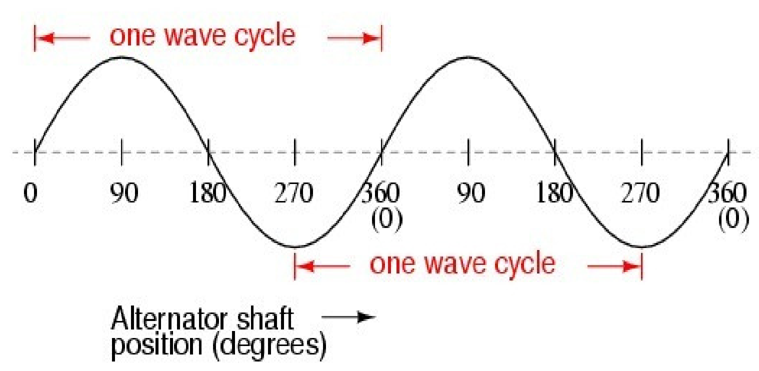
GENERATION OF AN ALTERNATING CURRENT

AC WAVEFORM

Overview



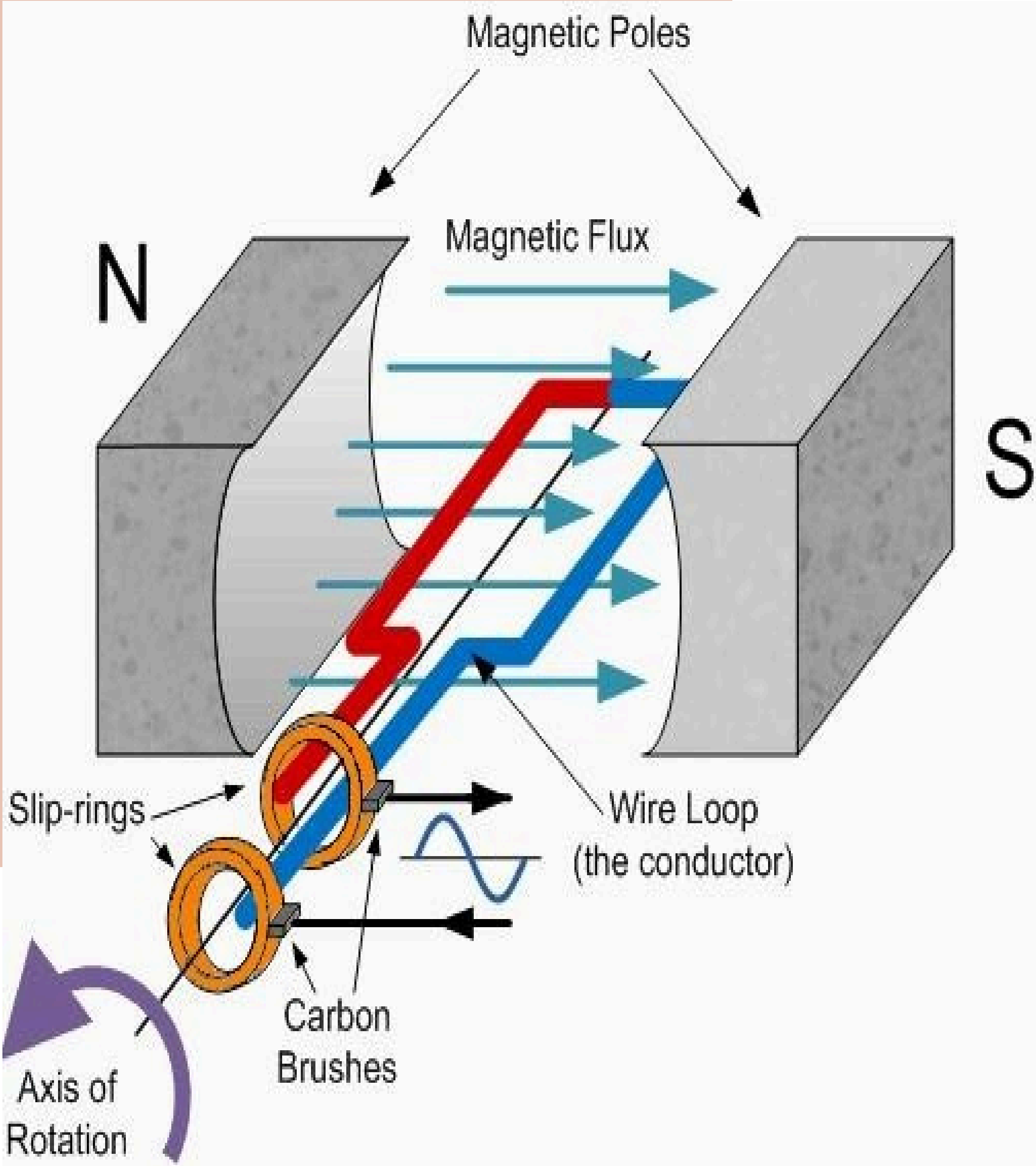
Graph of AC voltage over time (the sine wave).



Alternator voltage as function of shaft position (time).

If values of quantities which vary with time t are plotted to a base of time, the resulting graph is called a waveform.

BASIC SINGLE COIL AC GENERATOR



A single turn coil be free to rotate at constant angular velocity symmetrically between the poles of a magnet system.

Simple alternating-current generator

Figures (1) and (2) show a suspended loop of wire (conductor) being rotated (moved) in a clockwise direction through the magnetic field between the poles of a permanent magnet.

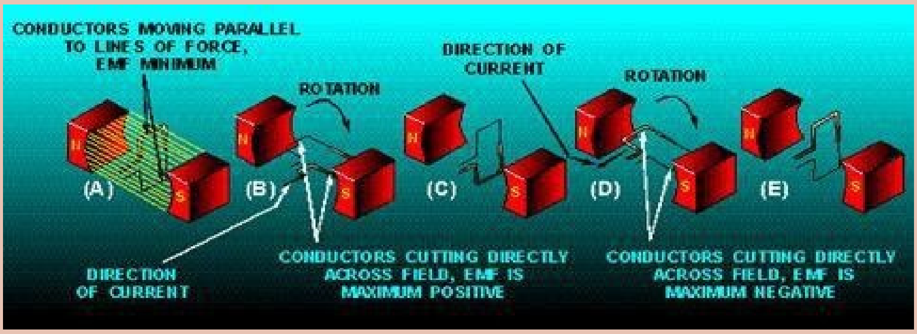


Figure 1

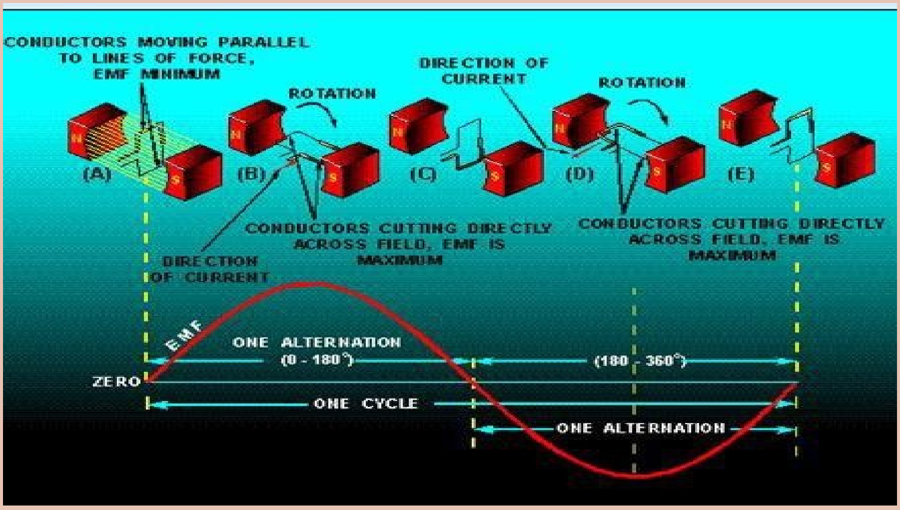


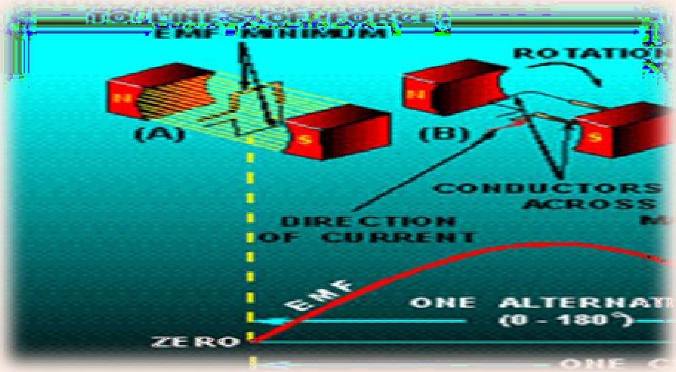
Figure 2

For ease of explanation, the loop has been divided into a dark half and light half

Simple alternating-current generator

CONDITION (A)

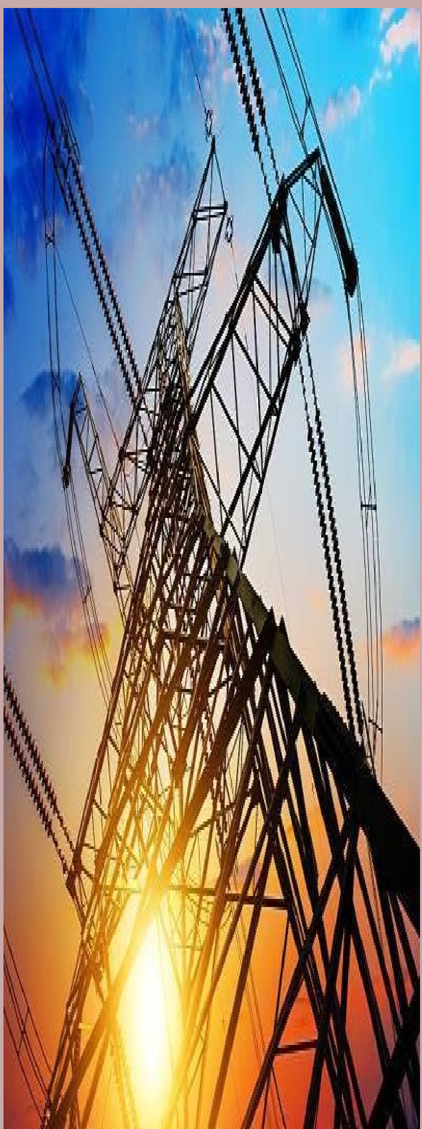
- the dark half is moving along (parallel to) the lines of force.
- the light half also moving in the opposite direction
- Consequently, it is cutting NO lines of force, so no EMF is induced



CONDITION(B)

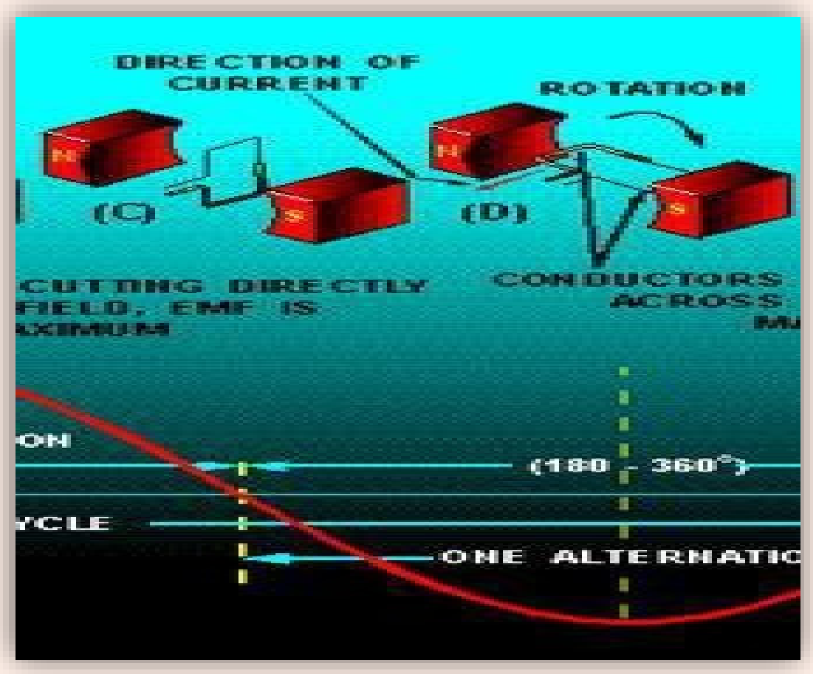
- the loop rotates toward the position, its cuts more line of force per second (inducing an ever-increasing voltage) because it is cutting more directly across the field (lines of force)
- the conductor is shown completing one-quarter of a complete revolution, or 90°, of a complete circle
- he conductor is cutting directly across the field, the voltage induced in the conductor is maximum
- the value of induced voltage at various points during the rotation from the (A) to (B) is plotted on a graph

Simple alternating-current generator



CONDITION (C)

- the loop rotates toward the position, its cuts fewer line of force
- the induced voltage decreases from its peak value and the loop is once again moving in a plane parallel to the magnetic field, so no EMF is induced in the conductor
- the loop is now rotated through half a circle (180°)



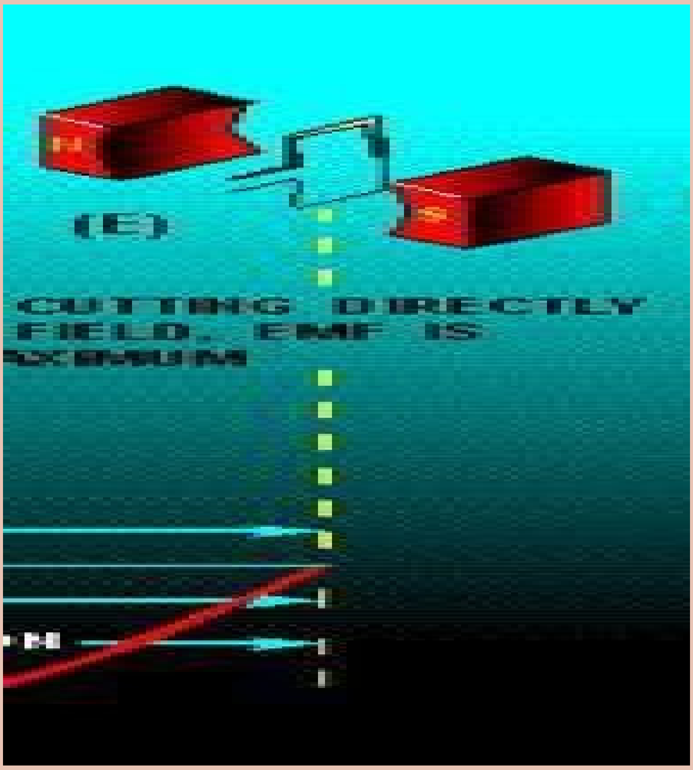
CONDITION(D)

- When the loop rotates to the position
- shown in (D), the action reverses
  The dark half is moving up and the light half is moving down, so that the total induced EMF and its current have reversed direction
- The voltage builds up to maximum in reversed direction, as shown in the graph.

CONDITION (E)

- The loop finally returns to its original position, at which point voltage is again zero
- The sine curve represents on complete cycle of voltage generated by the rotating loop
- Continuous rotation of the loop will produce a series of sine-wave voltage cycles (an AC voltage)

Simple alternating-current generator



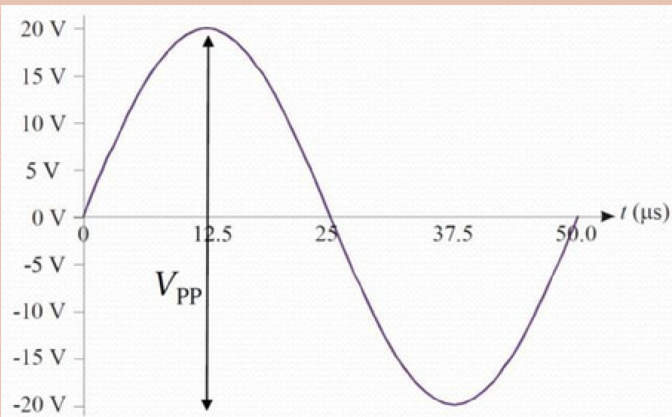


Figure : Sinusoidal waveform

An equation of a sinusoidal waveform is :

$$a = Am \sin \omega t \pm \theta$$

Where :

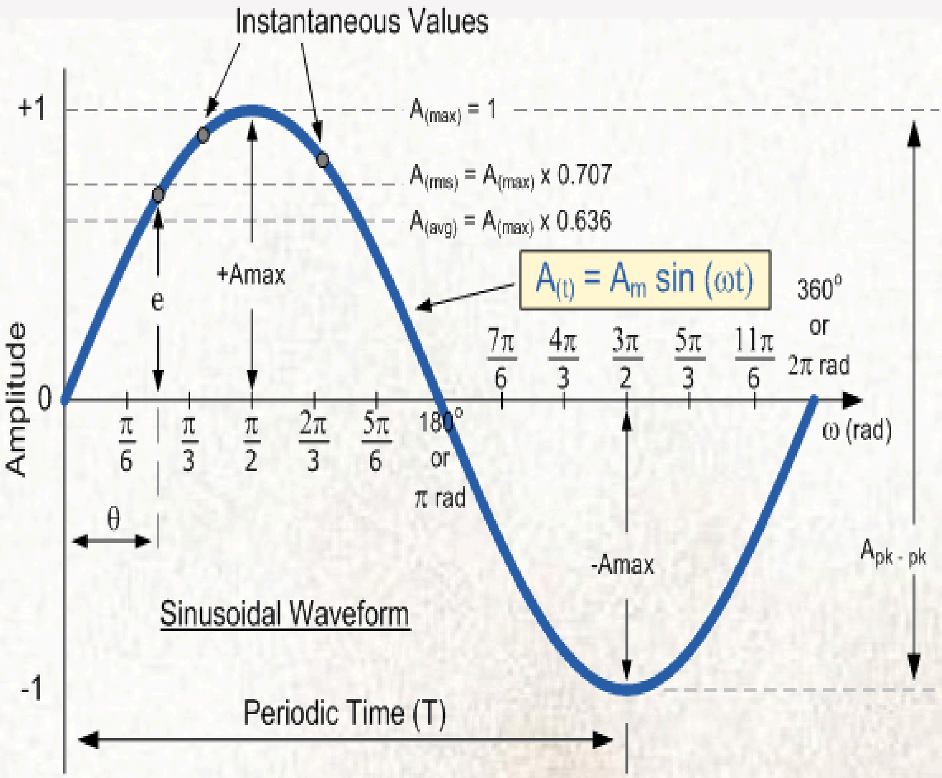
a = amplitude (ie, v, e.m.f)

Am = maximum amplitude @ peak amplitude

ω = angular velocity

θ = phase angle

SINUSOIDAL WAVEFORM



It is the shape of the waveform of e.m.f. produced by an alternator and thus the mains electricity supply is of 'sinusoidal' form. One complete series of values is called a cycle.

A sinusoidal voltage and current values

F R E Q U E N C Y (f)

The number of cycles per second.

It is measured in hertz(Hz)

$f = 1/T$

P E R I O D (T)

The time taken for the signal to complete one cycle. It is measured in seconds(s)

$T = 1/f$

P E A K V A L U E o r A M P L I T U D E

The maximum value of a waveform

INSTANTANEOUS VALUE

The value of voltage at one particular instant (any point)

EFFECTIVE (rms) VALUE

The value of alternating voltage that will have the same effect on a resistance as a comparable value of direct voltage will have on the same resistance

rms value = 0.707 × maximum value

$$rms\ value = \frac{1}{\sqrt{2}} \times peak\ value$$
$$= 0.707\ peak\ value$$

AVERAGE VALUE

The average of all instantaneous values during one alternation

average value (Vavg) = 0.637Em

FORM FACTOR

The ratio between rms value and average value

$$\frac{V_{rms}}{V_{avg}} = 1.11$$

PEAK FACTOR

peak value / 0.707 peak value = 1.414

SELF ASSESSMENT 1

Equation for an alternating current is : I = 70.71 sin 520t

Determine :

i) Peak current value
ii) Rms current value
iii) Average current value
iv) frequency

ANSWER

i) Peak value (Ip) = 70.71 A

ii) Rms value (Irms) = 0.707Ip

= 0.707 x 70.71 = 50 A

iii) Average value (Iavg) = 0.637Ip

= 0.637 x 70.71

= 45 A

iv) $f = \frac{\omega}{2\pi} = \frac{520}{2\pi} = 82.76Hz$

Self Assessment 2

A sinusoidal voltage has an RMS value of 240 V. What is the peak value of the voltage?

Answer : $rms\ value = 0.707\ peak\ value$

$$V_p = \frac{V_{rms}}{0.707} = \frac{240}{0.707} = 339.46\ V$$

Self Assessment 3

Q2 : Calculate the RMS voltage of an average value voltage of 12 V

Answer :

$$V_{avg} = 0.637 \times V_p$$

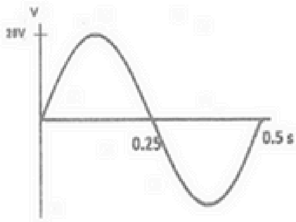
$$V_p = \frac{V_{avg}}{0.637} = \frac{12}{0.637} = 18.84\ V$$

$$V_{rms} = 0.707 \times V_p = 0.707 \times 18.84 = 13.32\ V$$

Self Assessment 4

Q3 : From to the waveform, calculate :

a) the peak voltage
b) the mean voltage
c) the RMS voltage
d) the peak factor



$$f = \frac{\omega}{2\pi} = \frac{520}{2\pi} = 82.76Hz$$

Answer :

Peak voltage

$$V_p = 20V$$

Mean voltage

$$V_{mean} = 0.637 \times 20 = 12.74\ V$$

RMS voltage

$$V_{rms} = 0.707 \times V_p = 0.707 \times 20 = 14.14\ V$$

Peak factor

$$peak\ factor = \frac{peak\ value}{rms\ value} = \frac{20}{14.14} = 1.414$$

Self Assessment 5

Q4 : Equation for an alternating current is :

$I = 70.71 \sin 520t$. Determine :

a) Peak current value

b) Rms current value

c) Average current value

d) frequency

Answer :

i) Peak value (I_p) = 70.71 A

ii) Rms value (I_{rms}) = $0.707I_p$

= 0.707 x 70.71 = 50 A

iii) Average value (I_{avg}) = $0.637I_p$

= 0.637 x 70.71

= 45 A

iv) $\omega = 2\pi f$ = 82.76 Hz

The phase angle of a sine wave

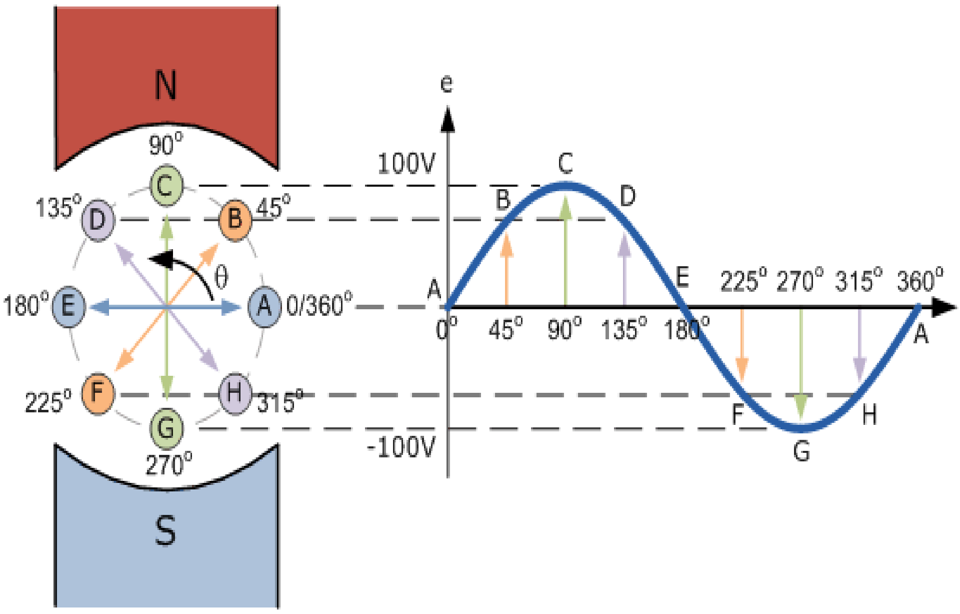


Figure : Sinusoidal waveform construction

Angle (θ)	0	45	90	135	180	225	270	315	360
emf = Vmax*sinθ	0	70.71	100	70.71	0	-70.71	-100	-70.71	-0

By plotting the graph at various positions of the rotating coil within the magnetic field, from 0° to 360°, we can generate a sine wave pattern. At the phases 0°, 180°, and 360°, the amplitude of the sine wave is zero, indicating that no EMF is induced in the rotating coil.

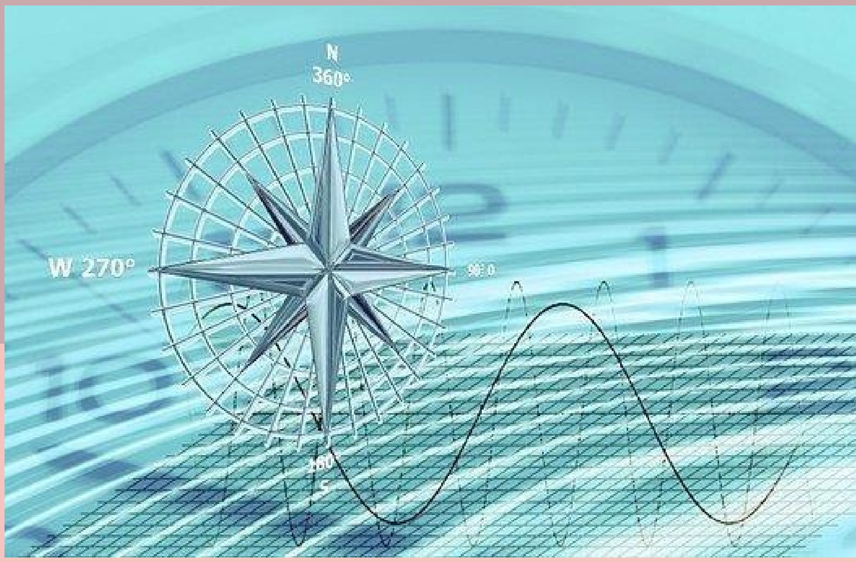
This is because no part of the moving coil is affected by magnetic flux lines at these positions. Zero EMF is induced at positions A and E. Conversely, at 90° and 270° phases, the sine wave reaches its maximum amplitude, occurring at positions C and G.

- But when θ is equal to 90^o and 270^o the generated EMF is at its maximum value as the maximum amount of flux is cut
- The sinusoidal waveform has a positive peak at 90^o and a negative peak at 270^o
- The waveform shape produced by our simple single loop generator is commonly referred to as a **Sine Wave** as it is said to be sinusoidal in its shape
- When dealing with sine waves in the time domain and especially current related sine waves the unit of measurement used along the horizontal axis of the waveform can be either time, degrees or radians
- In electrical engineering it is more common to use the **Radian** as the angular measurement of the angle along the horizontal axis rather than degrees
- For example, $\omega = 100$ rad/s, or 500 rad/s.
- Phase Difference Equation :

$$A_{(t)} = A_{max} \times \sin(\omega t \pm \Phi)$$

- A_m - is the amplitude of the waveform.
- ωt - is the angular frequency of the waveform in radian/sec.
- Φ (phi) - is the phase angle in degrees or radians that the waveform has shifted either left or right from the reference point.

Phase relationship of a sinusoidal waveform



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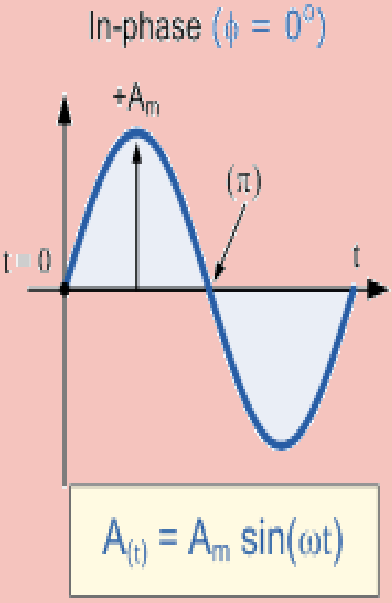

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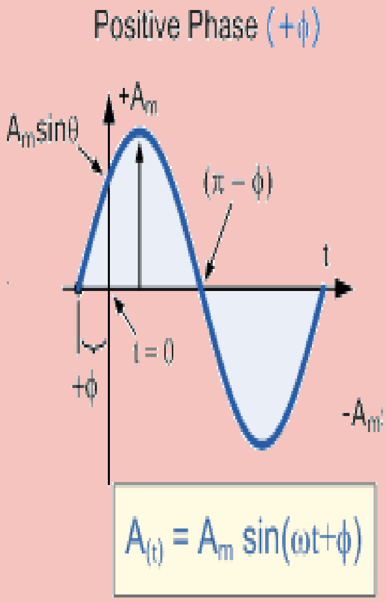

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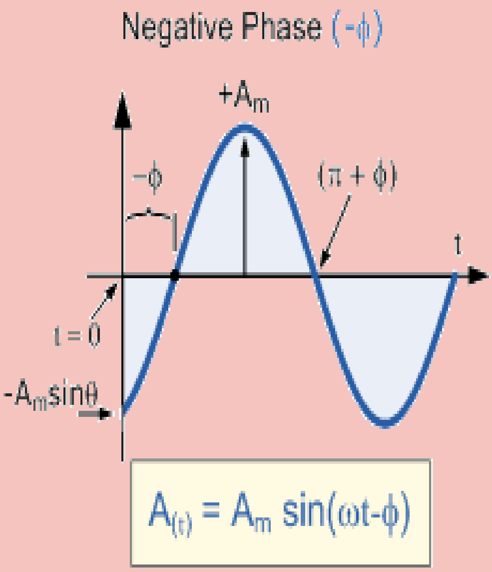
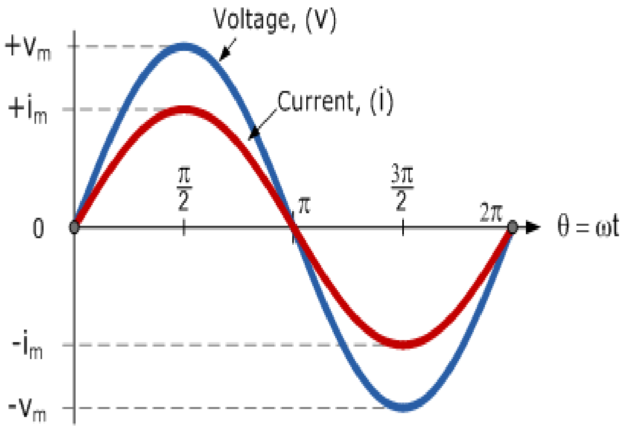




- Two waveform are said to be in phase when they have the same frequency.
- There is no phase difference between them.

Figure : Two sinusoidal waveforms 'in phase'

$$\text{Voltage}, (v_t) = V_m \sin \omega t$$

$$\text{Current}, (i_t) = I_m \sin(\omega t - \theta)$$

where, i lags v by angle Φ

The current, i is lagging the voltage, v by angle θ and in our example this is 30°

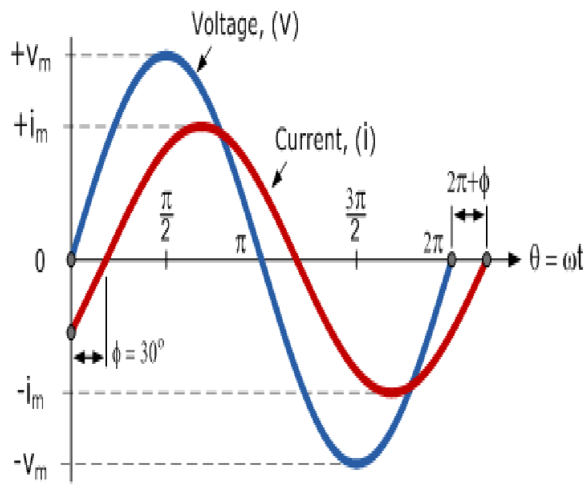


Figure : Phase difference of a sinusoidal waveform

SELF ASSESSMENT

Q1 : What is the phase relationship between the sinusoidal waveforms of each of the following sets?

a. $V = 10\sin(\omega t + 30^{\circ})$
$I = 5\sin(\omega t + 70^{\circ})$

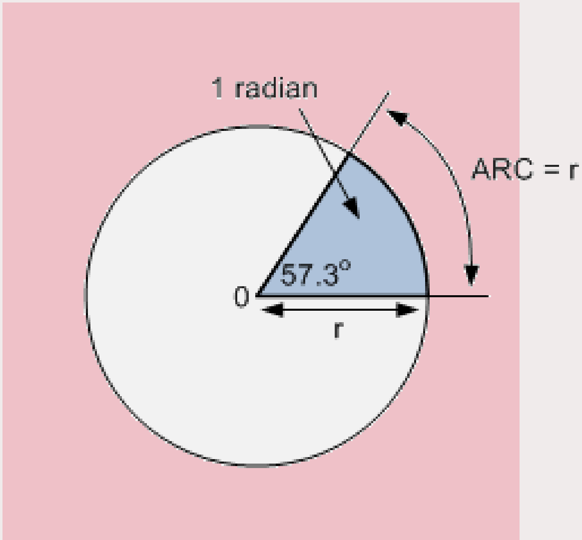
b. $I = 15\sin(\omega t + 60^{\circ})$
$v = 10\sin(\omega t - 20^{\circ})$

RADIANS

- The **Radian**, (rad) is defined mathematically as a quadrant of a circle where the distance subtended on the circumference equals the radius (r) of the circle.
- Since the circumference of a circle is equal to 2 π x radius, there must be 2 π radians around a 360° circle, so 1 radian = 360°/2 π = 57.3°

$$2\pi \text{ rads} = 360^{\circ}$$

$$\therefore 1 \text{rad} = 57.3^{\circ}$$



Using radians as the unit of measurement for a sinusoidal waveform would give 2 π radians for one full cycle of 360°. Then half a sinusoidal waveform must be equal to 1 π radians or just π (pi). Then knowing that pi, π is equal to 3.142 or 22÷7

RELATIONSHIP BETWEENDEGREES AND RADIANS

$$\text{Radians} = \left(\frac{\pi}{180^\circ}\right) \times \text{degrees}$$

$$\text{Degrees} = \left(\frac{180^\circ}{\pi}\right) \times \text{radians}$$

SELF ASSESSMENT

A sinusoidal waveform is defined as: $V_m = 169.8 \sin(377t)$ volts. Calculate the RMS voltage of the waveform, its frequency and the instantaneous value of the voltage after a time of 6mS.

Solution:

$$V_{(t)} = V_m \sin(\omega t)$$

Then comparing this to our given expression for a sinusoidal waveform above of $V_m = 169.8 \sin(377t)$ will give us the peak voltage value of 169.8 volts for the waveform.

$$V_{(rms)} = 0.707 \times \text{maximum peak value}$$

$$V_{(rms)} = 0.707 \times 169.8 = 120\text{volts}$$

The angular velocity (ω) is given as 377 rad/s.

Then $2\pi f = 377$. So the frequency of the waveform is calculated as:

$$\text{Frequency}, (f) = \frac{377}{2\pi} = 60\text{Hz}$$

The instantaneous voltage V_i value after a time of 6mS is given as:

$$V_{(i)} = 169.8 \sin(377 \times 6\text{mS})$$

$$V_{(i)} = 169.8 \sin(2.262)$$

$$\therefore V_{(i)} = 130.8 \text{ volts}$$

Note that the phase angle at time t = 6mS is given in radians. We could quite easily convert this to degrees if we wanted to and use this value instead to calculate the instantaneous voltage value. The angle in degrees will therefore be given as:

$$\text{Degrees} = \left(\frac{180^\circ}{\pi}\right) \times \text{radians}$$

$$\therefore \frac{180}{\pi} \times 2.262 = 130^\circ$$

Combination of waveforms

PHASOR

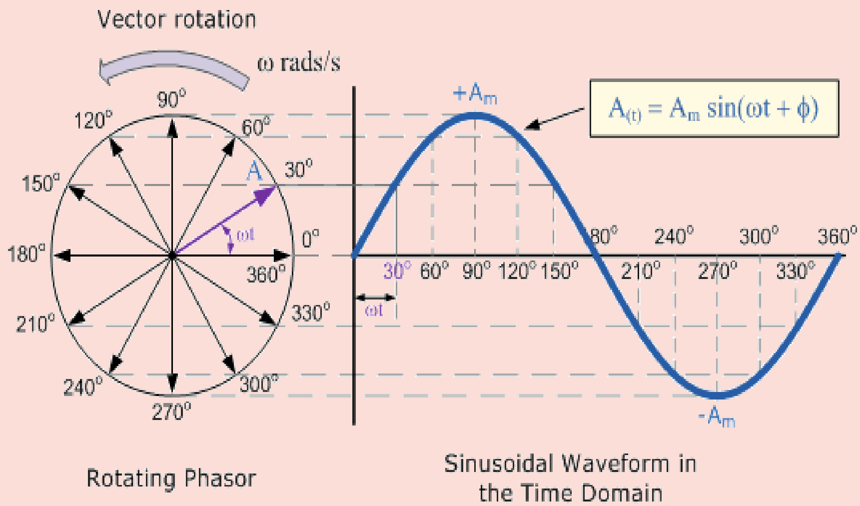
VECTOR

- Basically a rotating vector, simply called a "**Phasor**" is a scaled line whose length represents an AC quantity that has both magnitude ("peak amplitude") and direction ("phase") which is "frozen" at some point in time
- A phasor is a vector that has an arrow head at one end which signifies partly the maximum value of the vector quantity (V or I) and partly the end of the vector that rotates

PHASOR

- Generally, vectors are assumed to pivot at one end around a fixed zero point known as the "point of origin" while the arrowed end representing the quantity, freely rotates in an **anti-clockwise** direction at an angular velocity, (ω) of one full revolution for every cycle
- This anti-clockwise rotation of the vector is considered to be a positive rotation. Likewise, a clockwise rotation is considered to be a negative rotation

Phasor Diagram Of A Sinusoidal Waveform



The single vector rotates in an anti clockwise direction, its tip at point A will rotate one complete revolution of 360° or 2 π representing one complete cycle

The phasor diagram is drawn corresponding to time zero ($t = 0$) on the horizontal axis

The lengths of the phasors are proportional to the values of the voltage, (V) and the current, (I) at the instant in time that the phasor diagram is drawn.

The current phasor lags the voltage phasor by the angle, Φ, as the two phasors rotate in an *anticlockwise* direction as stated earlier, therefore the angle, Φ is also measured in the same anticlockwise direction.

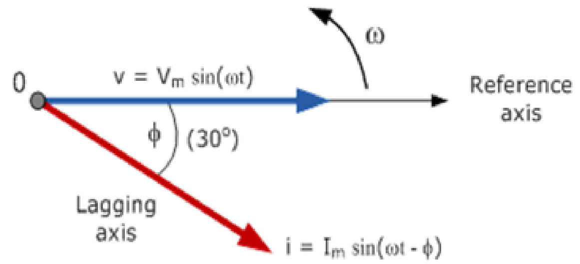


Figure: Phasor diagram current, i lagging voltage,V

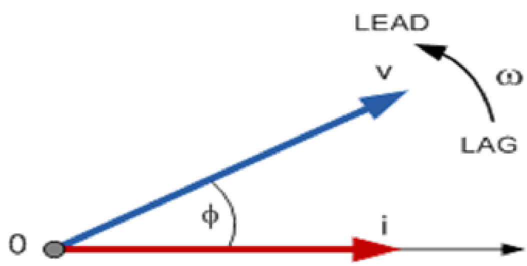
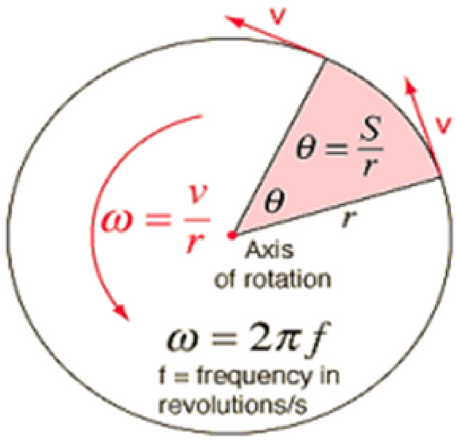


Figure: Phasor diagram voltage,V lead current, i

Angular Velocity

$$v = \omega r \ \ or \ \omega = \frac{v}{r}$$



- Angular velocity can be considered to be a vector quantity, with direction along the axis of rotation in the right-hand rule sense.
- For an object rotating about an axis, every point on the object has the same angular velocity.
- The tangential velocity of any point is proportional to its distance from the axis of rotation. Angular velocity has the units rad/s.

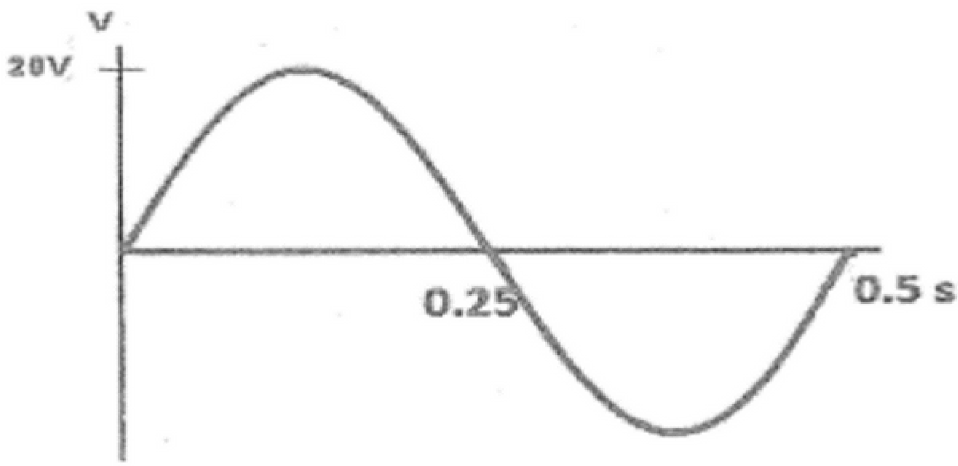
QUESTIONS

1. A sinusoidal waveform is defined as: Vm = 169.8 sin(377t) volts. Calculate the RMS voltage of the waveform, its frequency and the instantaneous value of the voltage after a time of 6mS.

2. The current in an AC circuit at any time t seconds is given by i = 40 sin $60\pi t$ + 0.36 A. Calculate:

i.The peak-to-peak current value
ii.The mean current value
iii.The period and frequency
iv.The value of the current when t = 2ms

3. Based on figure, write the sinusoidal waveform equation.



ANSWER - Q1

$$V_{(t)} = V_m \sin(\omega t)$$

Then comparing this to our given expression for a sinusoidal waveform above of Vm = 169.8 sin(377t) will give us the peak voltage value of 169.8 volts for the waveform.

$$V_{(rms)} = 0.707 \times \text{maximum peak value}$$

$$V_{(rms)} = 0.707 \times 169.8 = 120\text{volts}$$

The angular velocity (ω) is given as 377 rad/s. Then $2\pi f = 377$. So the frequency of the waveform is calculated as:

$$\text{Frequency}, (f) = \frac{377}{2\pi} = 60\text{Hz}$$

The instantaneous voltage Vi value after a time of 6ms is given as:

$$V_{(i)} = 169.8\sin(377 \times 6\text{mS})$$

$$V_{(i)} = 169.8\sin(2.262)$$

$$\therefore V_{(i)} = 130.8 \text{ volts}$$

The angle in degrees will therefore be given as:

$$\text{Degrees} = \left(\frac{180^\circ}{\pi}\right) \times \text{radians}$$

$$\therefore \frac{180}{\pi} \times 2.262 = 130^\circ$$

ANSWER - Q2

i) The peak-to-peak current value :

$$I_{P-P} = 2 \text{ x } I_m = 2 \times 40 = 80\ A$$

ii)The mean current value

$$I_{mean} = 0.637 I_m$$
$$= 0.637 \times 40$$
$$= 25.48\ A$$

iii)The period and frequency

$Period,\ T = 2\pi\ \omega$

$$= 2\pi\ 60\pi = 33.33\ ms$$

$Frequency,\ f = 1/\ T$

$$= 1/\ 33.33m = 30\ Hz$$

iv) The value of the current when $t = 2ms$

Convert calculator mode to rad (radians)

$i = 40 \sin (60\pi t + 0.36)$
$i = 40 \sin (60\pi \times 2m + 0.36)$
$i = 40 \sin (0.737)$
$i = 26.88$ A

ANSWER - Q3

The sinusoidal waveform equation

$$A_t = A_{max} \sin(\omega t \pm \theta)$$

$Amax$ = $peak\ value$ = Vp = 20V

$\omega = 2\pi/T$

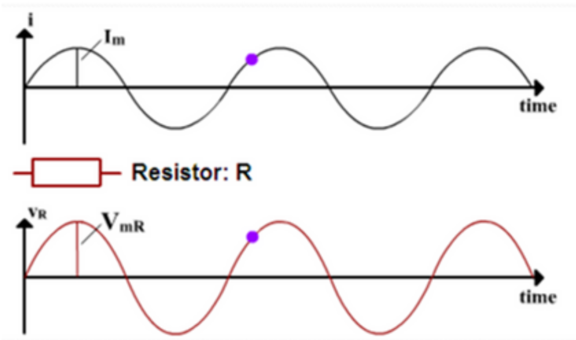
$= 2\pi/0.5$

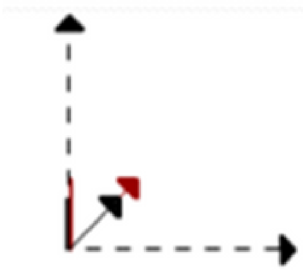
$= 12.57$

$\theta = 0°$ $\therefore v = 20 \sin 12.57$

Apply Ohm's Law to resistive circuits with AC sources

- The voltage V across a resistor is proportional to the current I travelling through it.
- This is true at all times: V = RI.





Sinusoidal equation: $e = E_m \sin\omega t$

So; If $I = I_m \sin\omega t$ and $V = V_m \sin\omega t$

ohm's law; $V = RI$

So replace in ohm's law equation;

$$V_m \sin\omega t = RI_m \sin\omega t$$

Simplify the equation;

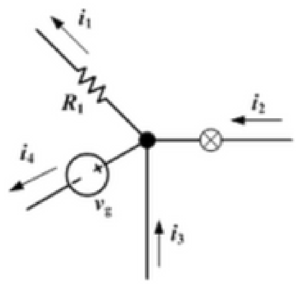
Ohm's law

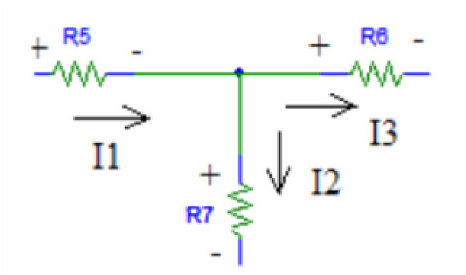
$$\boldsymbol{V_m = RI_m}$$

Kirchhoff's Voltage and Current Laws apply to all AC circuits as well as DC circuits

Kirchhoff's Current Law:

- The sum of current into a junction equals the sum of current out of the junction.
- $i_2 + i_3 = i_1 + i_4$
- The sum of all currents at a node must equal to zero

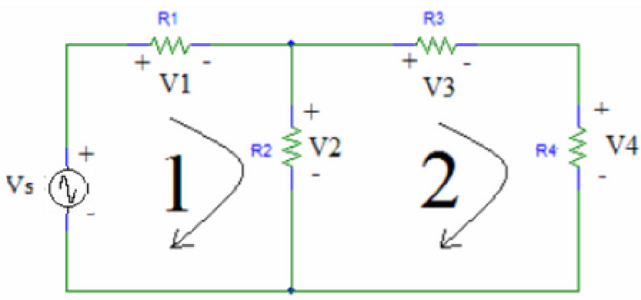




$$i_1 - i_2 - i_3 = 0$$

Kirchhoff's Voltage Law:

- The algebraic sum of the voltage (potential) differences in any loop must equal zero
- Example:



$$V1 + V2 – Vs = 0$$

Power In Resistive AC Circuits

1) In a direct current circuit the power is equal to the voltage times the current, or P = V x I.

2) The **true power** depends upon the phase angle between the current and voltage.

3) True power of a circuit is the power actually used in the circuit.

4) Measured in watts.

$$\mathbf{P} = \text{true power} \qquad P = I^2R \qquad P = \frac{E^2}{R}$$

*Measured in units of **Watts***

5) Note that the waveform for power is always positive, never negative for this resistive circuit.

6) This means that power is always being dissipated by the resistive load, and never returned to the source as it is with reactive loads.

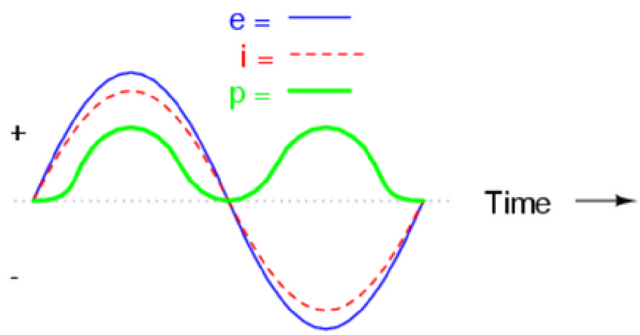
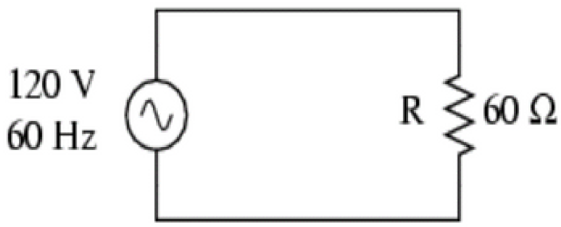


Figure : Current is in phase with voltage in a resistive circuit

Power In Resistive AC Circuits

Self assessment:



$$Z_R = 60 + j0\ \Omega \quad or \quad 60\ \Omega \angle 0^\circ$$

$$I = \frac{E}{Z}$$

$$I = \frac{120\ V}{60\ \Omega}$$

$$I = 2\ A$$

In this example, the current to the load would be 2 amps.

$$P = \text{true power} = I^2R = 240\ W$$

The power dissipated at the load would be 240 watts.

Because this load is purely resistive (no reactance), the current is in phase with the voltage, and calculations look similar to that in an equivalent dc circuit.

QUESTIONS

4. Calculate the current and power consumed in a single phase 240V AC circuit by a heating element which has an impedance of 60 Ohms. Also draw the corresponding phasor diagram.

5. A sinusoidal voltage supply defined as:

$$V(t) = 100 \times \cos(\omega t + 30^\circ)$$

is connected to a pure resistance of 50Ω. Determine its impedance and the value of the current flowing through the circuit. Draw the corresponding phasor diagram.

ANSWER - Q4

$$I = \frac{V}{R} = \frac{240}{60} = 4.0\ A$$

The Active power consumed by the AC resistance is calculated as:
$P = I^2R = 4^2(60) = 960W$

The corresponding phasor diagram is given as:

ANSWER Q5

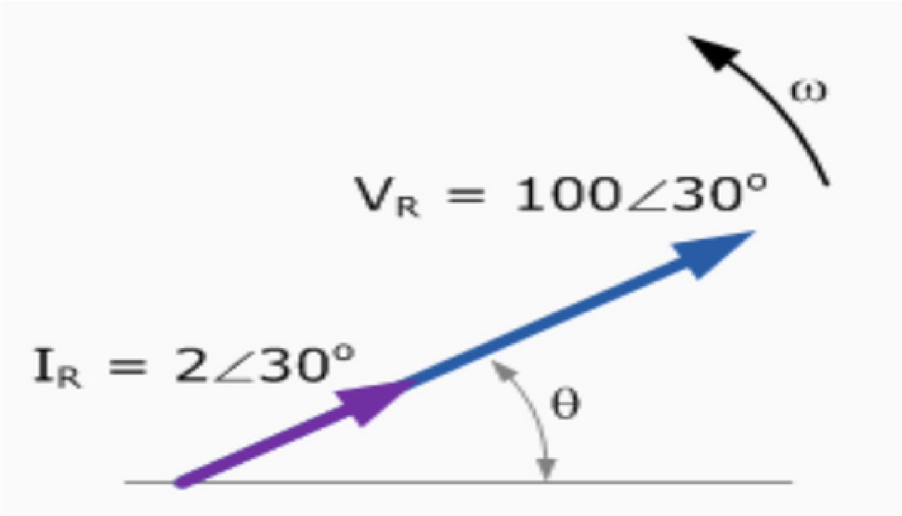
Converting this voltage from the time-domain expression into the phasor-domain expression gives us:

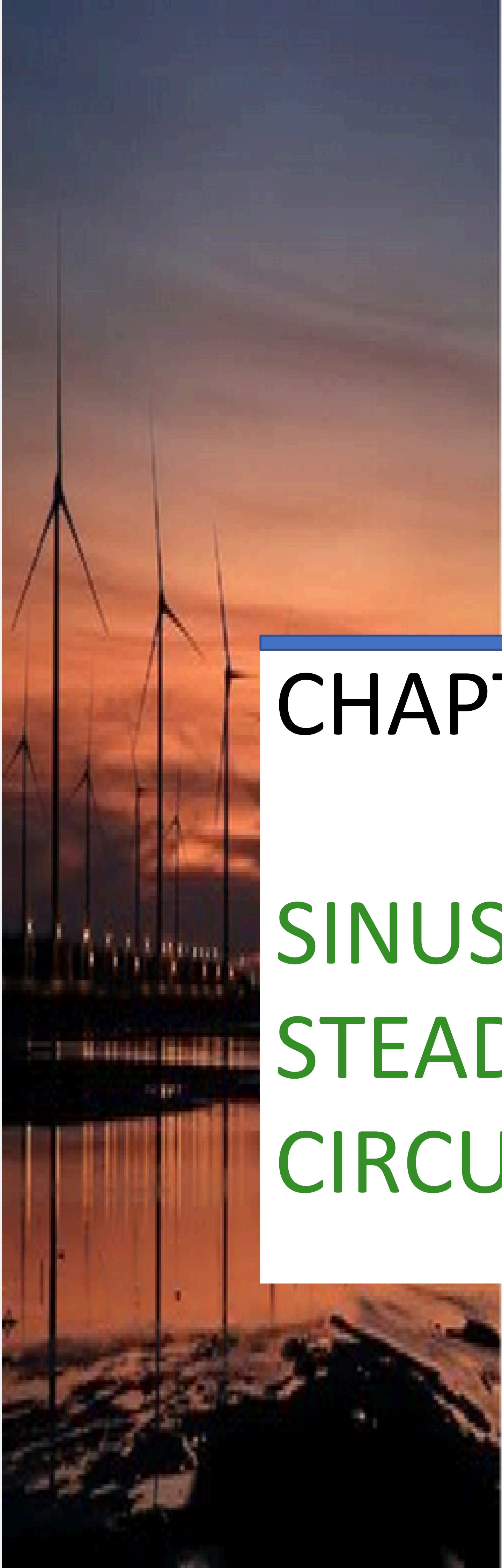
$$V_{R(t)} = 100\cos(\omega t + 30^o) \Rightarrow V_R = 100\angle 30° \; volts$$

Applying Ohms Law gives us:

$$V_R = I_R R \Rightarrow I_R = \frac{V_R}{R} = \frac{100\angle 30°}{50} = 2\angle 30° \; Amps$$

The corresponding phasor diagram will be;





CHAPTER 2

SINUSOIDAL STEADY-STATE CIRCUIT ANALYSIS

COURSE LEARNING OUTCOME

1. Explain AC circuit concept and their analysis using AC circuit law.
2. Apply the knowledge of AC circuit in solving problem related to AC electrical circuit.

OBJECTIVE

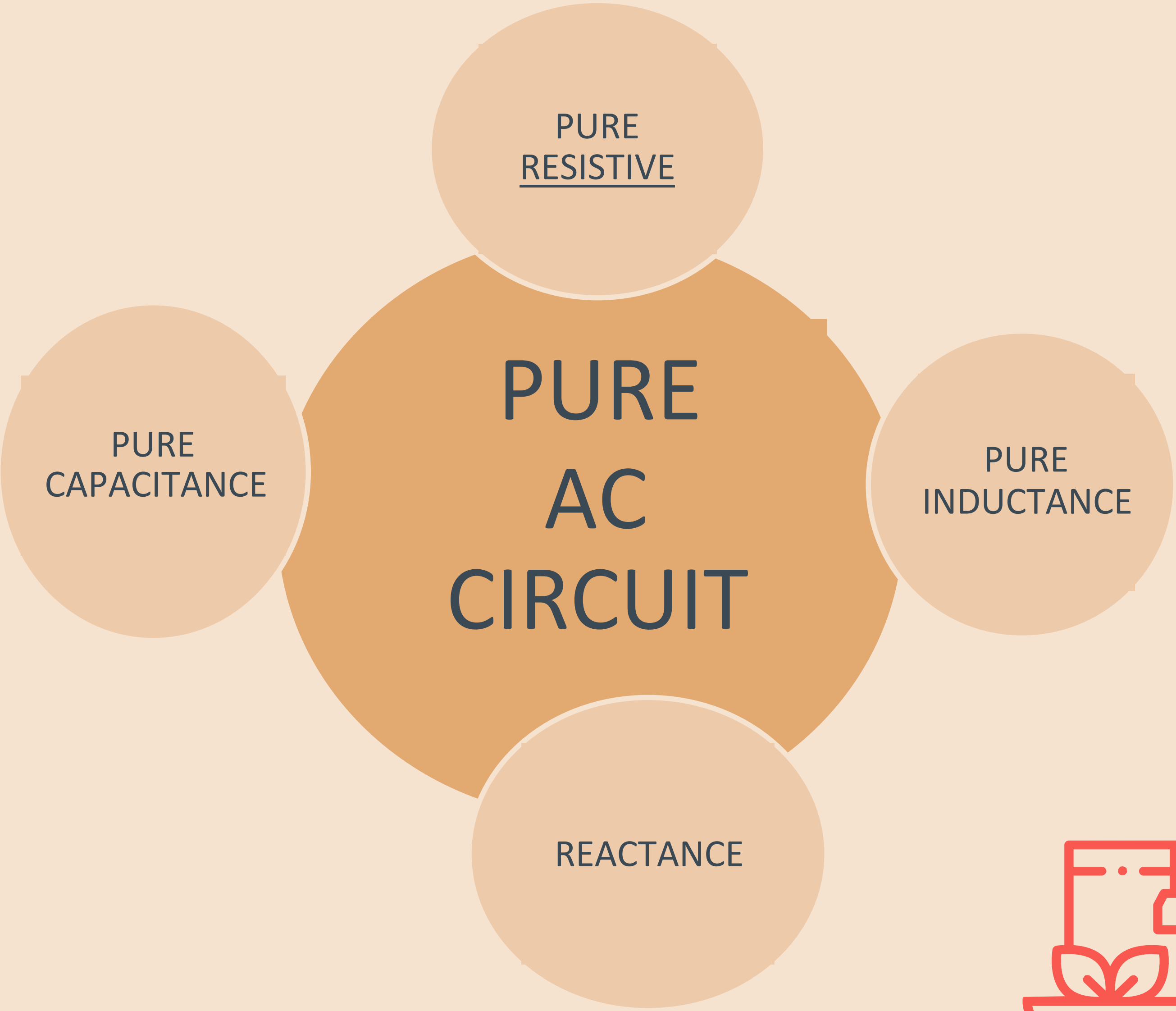
2.1 Understand the AC basic circuits

a. Explain how the voltage and current are in phase in a purely resistive circuit, i.e. the phase angle difference between the voltage and current is zero.
b. Explain how the current lags voltage by 90° in a pure inductive circuit and inductive reactance is directly proportional to frequency, $XL = 2\pi fL$.
c. Explain how the current leads voltage by 90° in a purely capacitive circuits and capacitive reactance is inversely proportional to frequency as shown by the equation $XC = 1/2\pi fC$.
d. Illustrate the vector diagram and waveforms of voltage and current for (a),(b) and (c).
e. List differences between resistance and impedance.
f. Sketch the impedance triangle.

.

INTRODUCTION

In this topic, we will suppress an electrical circuit consisting of three passive elements These passive elements are known as resistor R, inductor L and capacitor C. In this topic we will also explain about the current-voltage relationship produced while providing alternating current (AC) when connected to a circuit. In addition in this topic will also determine the voltage-current relationship of inductive and pure resistive capacitive circuit networks, and the resulting results using phasor diagrams. determining the voltage-current relationship and phasor diagrams will also be discussed inthistopic



PURE RESISTIVE AC CIRCUIT

In a purely resistive a.c. circuit, the current (I_R) and applied voltage (V_R) are **in phase**.

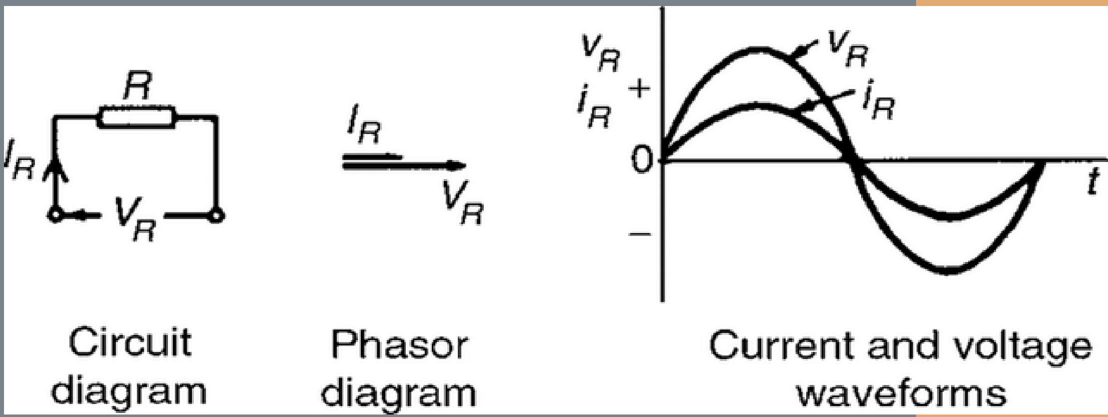


Figure : Circuit diagram, phasor diagram and current and voltage waveforms

AC value that flows at any point in the pure resistive circuit does not affected by frequency of that circuit.

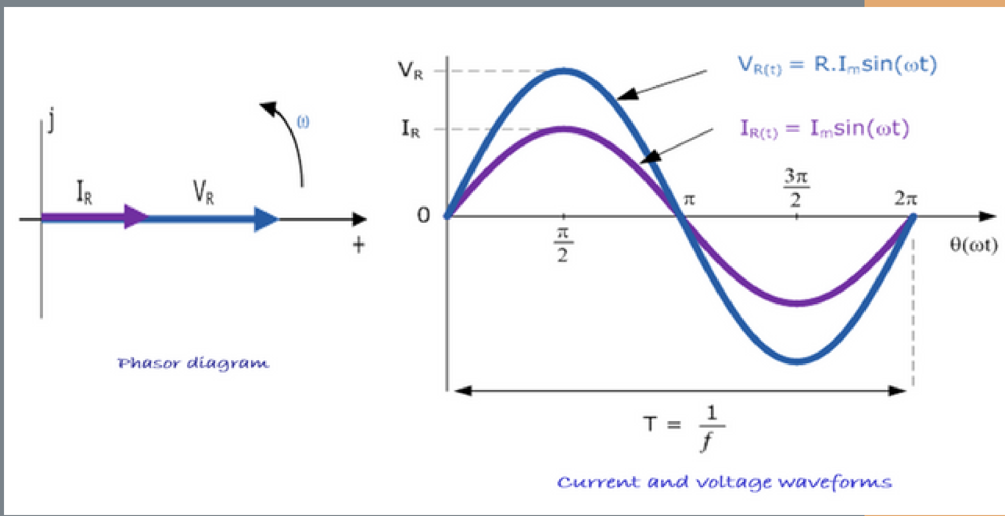


Figure : Phasor diagram and current and voltage waveforms

$$R = \frac{V}{I} = \frac{V<0°}{I<0°} = R\angle 0°\ \Omega$$

Phase angle for resistance

PURE INDUCTIVE AC CIRCUIT

In a purely inductive a.c. circuit, the current (I_L) **lags** the applied voltage (V_L) by 90°

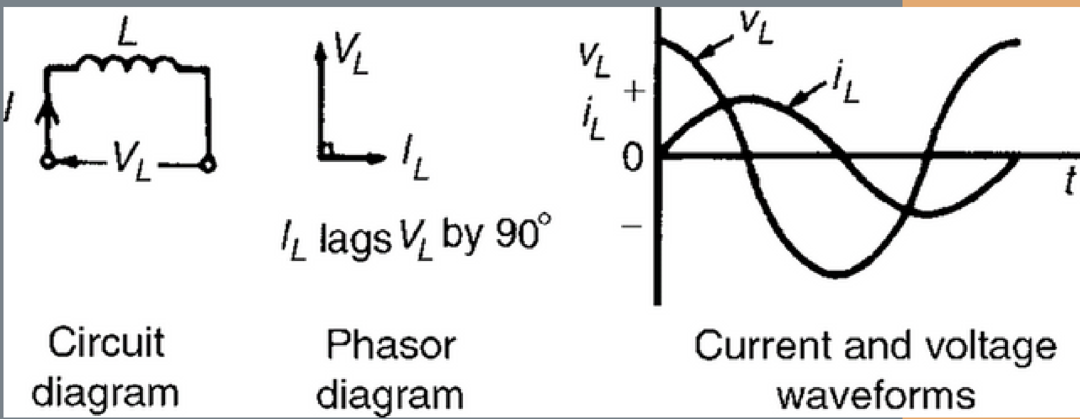


Figure : Circuit diagram, phasor diagram and current and voltage waveforms

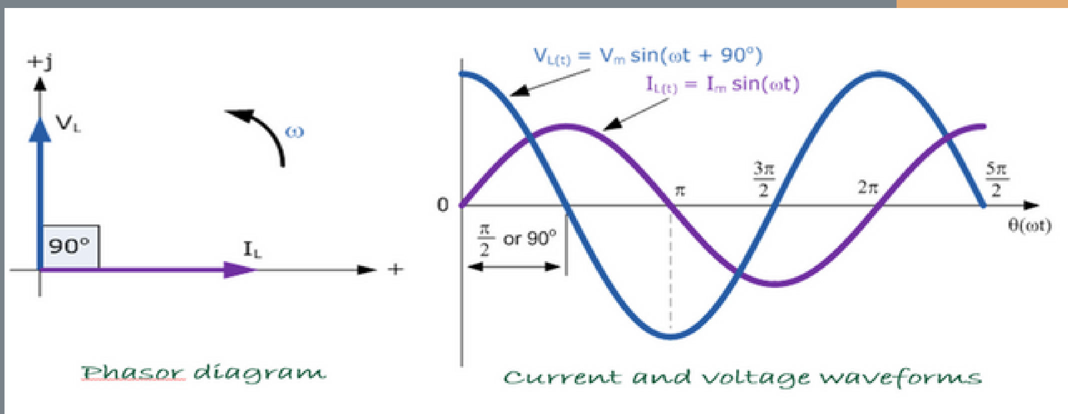


Figure : Phasor diagram and current and voltage waveforms

$$X_L = \frac{V_L}{I_L} = \frac{V<90°}{I<0°} = X_L\angle 90°\ \Omega$$

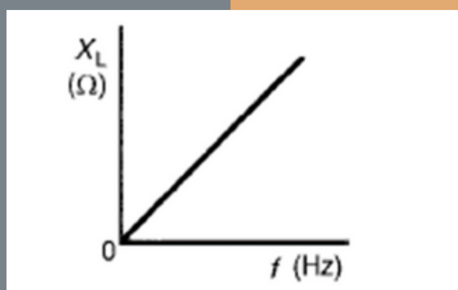
Phase angle for inductive reactance

The opposition to the flow of alternating current is called the **inductive reactance, X_L.**

$$X_L = 2\pi fL = \omega L = X_L\angle 90°\ \Omega$$

ω = 2πf - angular velocity

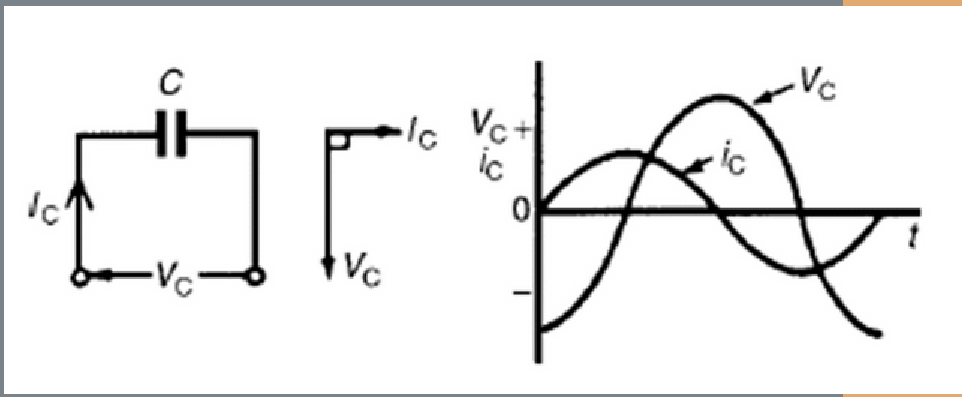
Inductive reactance vs frequency

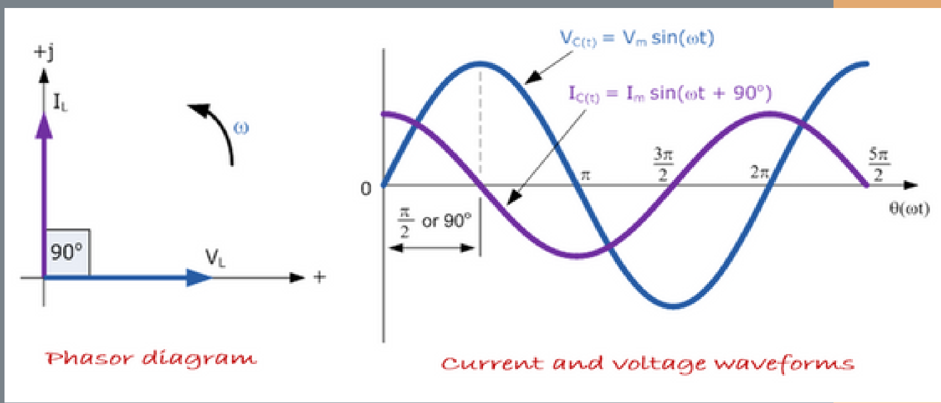


F ↑ X_L ↑

PURE CAPACITIVE AC CIRCUIT

In a purely capacitive a.c. circuit, the current (I_C) **leads** the applied voltage (V_C) by 90°.





$$X_C = \frac{V_C}{I_c} = \frac{V<0°}{I<90°} = X_C\angle-90°\ \Omega$$

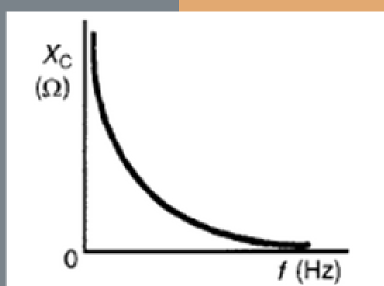
Phase angle for capacitive reactance

The opposition to the flow of alternating current is called the **capacitive reactance, X_C.**

$$X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C} = X_C\angle-90°\ \Omega$$

ω = 2πf - angular velocity

Capacitive reactance vs frequency



F ↑ X_C ↓

REACTANCE

Inductive reactance is present energy is stored in the form of a changing magnetic field and the current waveform lags the voltage waveform by 90degrees. Inductive reactance is caused by devices in which wire is wound circularly.

REACTANCE INDUCTIVE

$$X_L = 2\pi f L \qquad \boxed{V = I X_L}$$

- We will define inductive reactance, X_L, as the opposition to current in an inductor.

REACTANCE CAPACITIVE

$$X_c = \frac{1}{2\pi f C} \qquad \boxed{V = I X_C}$$

- In resistor, the Ohm's Law is V = IR, where R is the opposition to current.
- We will define Capacitive Reactance, X_C, as the opposition to current in a capacitor.

Impedance in the combination of resistance and reactance (both inductive and capacitive) and is a complex number, containing both real and imaginary parts. The real part of impedance is resistance, while the imaginary part is reactance. Impedance gas both magnitude a phase.

SELF ASSESMENT 1

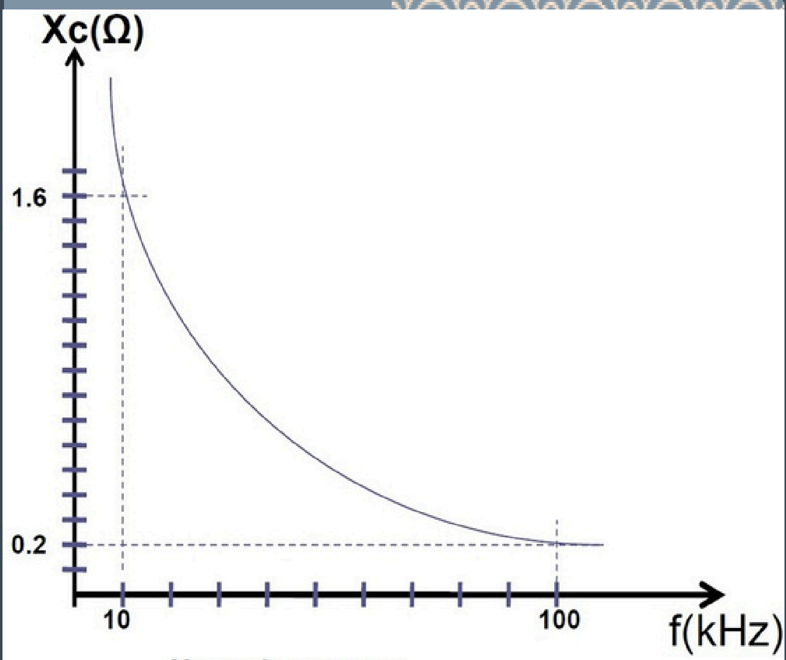
Determine the capacitive reactance of a capacitor 10µF when connected to a circuit with input frequency 100kHz and 10kHz. Plot a graph XC vs frequency base to the XC value calculated above and discuss the relationship.

ANSWER:

$$X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C} = X_C\angle -90°\ \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(100K)(10u)} = 0.159\Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(10k)(10u)} = 1.59\Omega$$



Xc vs frequency

QUESTION

1. Calculate the reactance of a coil of inductance 0.32 H when it is connected to a 50 Hz supply.

2. A coil has a reactance of 124Ω in a circuit with a supply of frequency 5 kHz. Determine the inductance of the coil.

3. A coil has an inductance of 40 mH and negligible resistance. Calculate its inductive reactance and the resulting current if connected to :

(a) a 240V, 50 Hz supply

(b) a 100V, 1 kHz supply

4. Determine the capacitive reactance of a capacitor of 10 μF when connected to a circuit of frequency;
(a) 50 Hz
(b) 20 kHz

5. A capacitor has a reactance of 40Ω when operated on a 50 Hz supply. Determine the value of its capacitance.

6. Calculate the current taken by a 23μF capacitor when connected to a 240V, 50 Hz supply.

ANSWER

Q1) $X_L = 2\pi fL = \omega L = X_L\angle 90°\,\Omega$

XL = 2πfL = 2π(50)(0.32) = 100.5Ω

Q2) $X_L = 2\pi fL = \omega L = X_L\angle 90°\,\Omega$

L = XL / 2πf
= 124 / 2π(5000)
= 3.95 mH

Q3) a. Inductive reactance, $X_L = 2\pi fL = \omega L = X_L\angle 90°\,\Omega$

= 2π(50)(40m)
= 12.57Ω

Current, I = V / XL
= 240 /12.57
= 19.09 A

b. Inductive reactance, $X_L = 2\pi fL = \omega L = X_L\angle 90°\,\Omega$

XL = 2π(1000)(40m)
= 251.3Ω

Current, I = V / XL
= 100 / 251.3
= 0.398 A

Q4) a. 50 Hz

Capacitive reactance $X_C = \frac{1}{2\pi fC} = \frac{1}{\omega C} = X_C\angle -90°\,\Omega$

XC = 1/ 2πfC
= 1/ 2π(50)(10μ)
= 318.3 Ω

Q4) b. 20 kHz

Capacitive reactance $X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C} = X_C\angle -90°\ \Omega$

XC = 1/2πfC
= 1/ 2π(20k)(10μ)
= 0.796 Ω

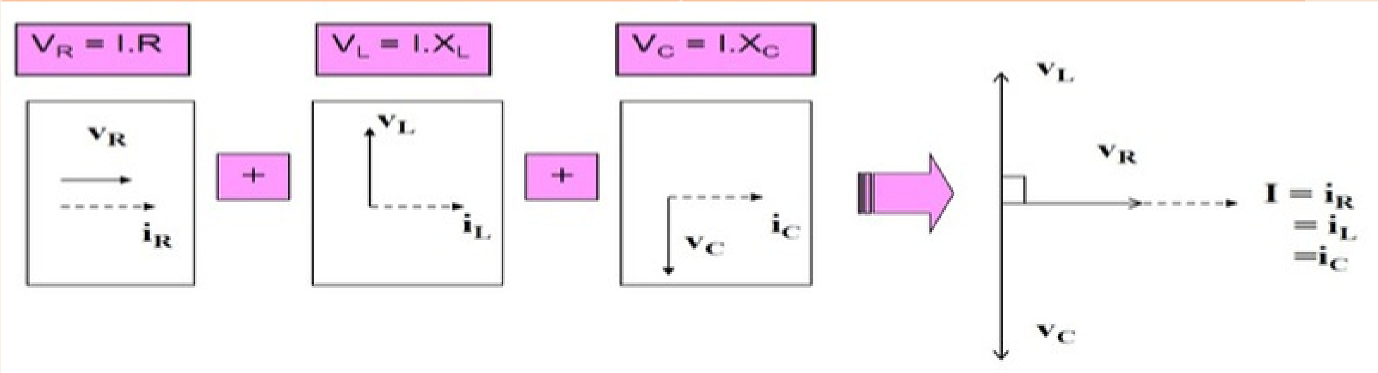
Q5) Since $X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C} = X_C\angle -90°\ \Omega$

capacitance , C = 1/2πfXC
= 1/2π(50)(40)
= 79.58 μF

Q6) Current , I = V/XC
= V /1/2πfC
= 2πfCV
= 2π(50)(23μ)(240)
= 1.73 A

CIVIL

In a capacitor (C) the current (I) is ahead of the voltage (V) and the voltage is ahead of the current (I) for the inductor (L). In this, the summary of the relationship between voltage and current for inductive and capacitive circuit



DIFFERENCES BETWEEN RESISTANCE & IMPEDANCE

BASIS	RESISTANCE	IMPEDANCE
Definition	The opposition offered to the flow of current in an electric circuit is known as the Resistance	The opposition to the flow of current in an AC circuit because of resistance, capacitance and inductance is known as Impedance
Circuit	occurs in both AC and DC circuit	Occurs only in an AC circuit
Phase angle	Does not have any phase angle	Have magnitude and phase angle
Symbol	It is denoted by R	It is denoted by Z
Elements	It is contribution of the resistive element in the circuit	It is contribution of both resistance and reactance
Frequency	Resistance is constant in a circuit and does not vary according to the frequency of AC or DC	Impedance varies according to the frequency of AC current

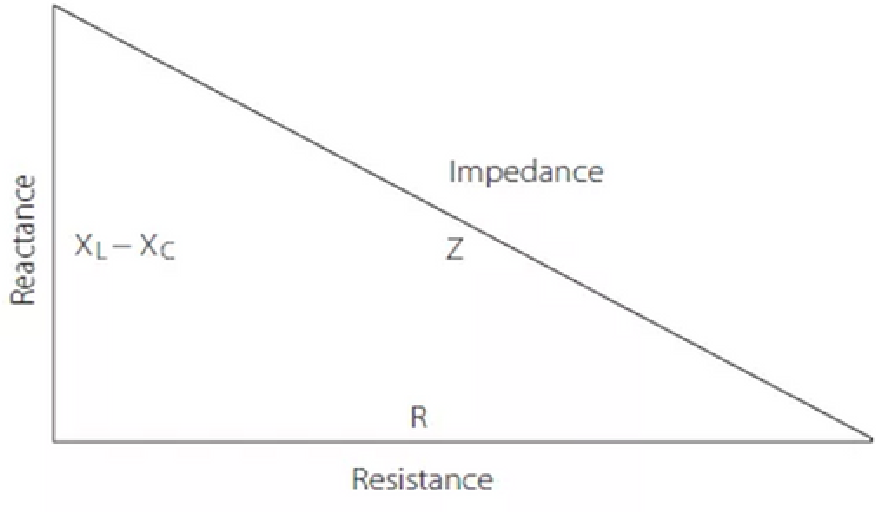


Figure : Impedance triangle

Components of the Impedance Triangle

1. **Resistance (R):**
   - This represents the real part of impedance.
   - It's the opposition to the flow of AC current due to resistors.
   - Measured in ohms (Ω).
2. **Reactance (X):**
   - This represents the imaginary part of impedance.
   - It is the opposition to the flow of AC current due to inductors and capacitors.
   - Reactance can be inductive (X_L) or capacitive (X_C).
   - Inductive reactance $X_L = 2\pi f L$ (where f is the frequency and L is inductance).
   - Capacitive reactance $X_C = \frac{1}{2\pi f C}$ (where f is the frequency and C is capacitance).
3. **Impedance (Z):**
   - The total opposition to the flow of AC current in the circuit.
   - It is a complex quantity having both magnitude and phase.
   - $Z = R + jX$ (where j is the imaginary unit).
   - The magnitude of impedance is given by $|Z| = \sqrt{R^2 + X^2}$.

EXERCISE

1. Calculate the reactance of a coil of inductance 0.2 H when it is connected to
   (a) a 50 Hz,
   (b) a 600 Hz
   (c) a 40 kHz supply

   [(a) 62.83Ω (b) 754Ω (c) 50.27 kΩ]

2. A supply of 240V, 50 Hz is connected across a pure inductance and the resulting current is 1.2A. Calculate the inductance of the coil.

   [0.637 H]

3. Calculate the capacitive reactance of a capacitor of 20 μF when connected to an a.c. circuit of frequency
   (a) 20 Hz,
   (b) 500 Hz,
   (c) 4 kHz

   [(a) 397.9 (b) 15.92 (c) 1.989]

4. A capacitor has a capacitive reactance of 400Ω when connected to a 100V, 25 Hz supply. Determine its capacitance and the current taken from the supply.

   [15.92 μF, 0.25A]

OBJECTIVE

2.2 Apply the circuit with inductive and capacitive load

1. Calculate the current and voltage in R-L, R-C and R-L-C:
   a. series circuits
   b. parallel circuits

2 Construct the phasor diagram to show.
   a. current is taken as reference in series circuit.
   b. voltage is taken as reference in parallel circuit.

2.3 Apply the series and parallel R-L-C circuits.

1. Show that the supply voltage in a series circuit is equivalent to the total vector of voltages across each component.
2. Construct the phasor diagram to explain (1)
3. Calculate that the total current in a parallel circuit is equivalent to the total vector of current in each branch.
4. Construct the phasor diagram to explain (3)

2.4 Apply the combination of series-parallel R-L-C circuits

1 Calculate the total impedance in a series-parallel circuit.
2 Calculate the total current and the branch current.
3 Calculate the voltage drop across each resistor in a series-parallel circuit.

R-L SERIES AC CIRCUIT

In an a.c circuit containing inductance L and resistance R, the applied voltage V is the phasor sum of V_R and V_L

Current I lags the applied voltage V by an angle lying between 0◦ and 90◦ (depending on the values of V_R and V_L), shown as angle ϕ

In any a.c. series circuit the current is common to each component and is thus taken as the reference phasor

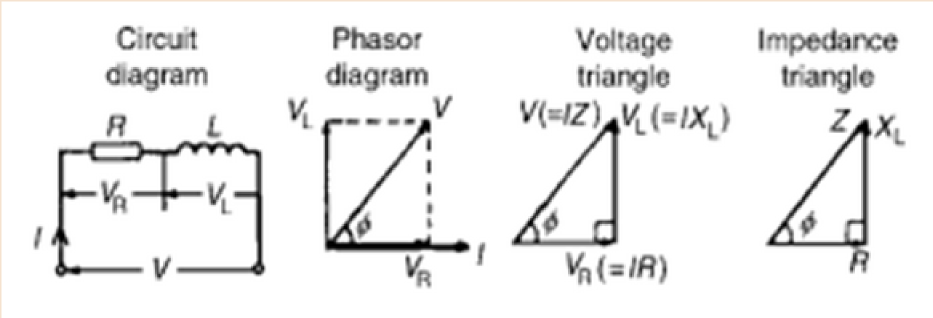


Figure : Phasor diagram R-L series ac circuits

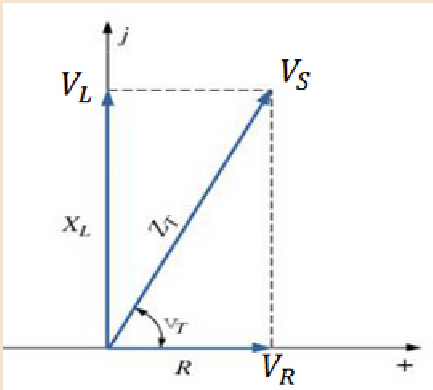


Figure : Graph R-L series ac circuits

Table: Equation R-L series ac circuits

$X_L = 2\pi fL$	$Z_T = \|Z_T\|\angle\theta$
$Z = R + iX_L$	$\Theta = tan^{-1}\frac{X_L}{R}$
$\|Z_T\| = \sqrt{(R^2 + X_L{}^2)}$	$I = \frac{V}{Z_T}$
$V = \sqrt{(V_R{}^2 + V_L{}^2)}$	$V_L = IX_L$
$V_R = IR$	

R-C SERIES AC CIRCUIT

In an a.c circuit containing capacitance C and resistance R, the applied voltage V is the phasor sum of V_R and V_C

Current I leads the applied voltage V by an angle lying between 0◦ and 90◦ (depending on the values of V_R and V_C), shown as angle ϕ

In any a.c. series circuit the current is common to each component and is thus taken as the reference phasor

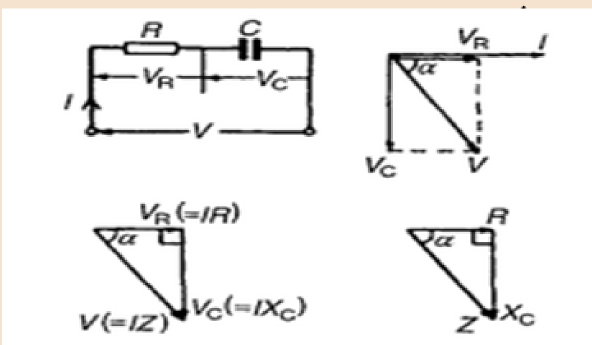


Figure : Phasor diagram R-C series ac circuits

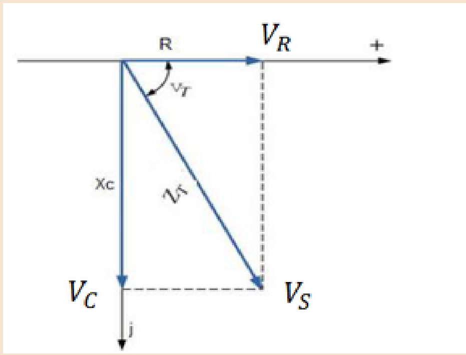
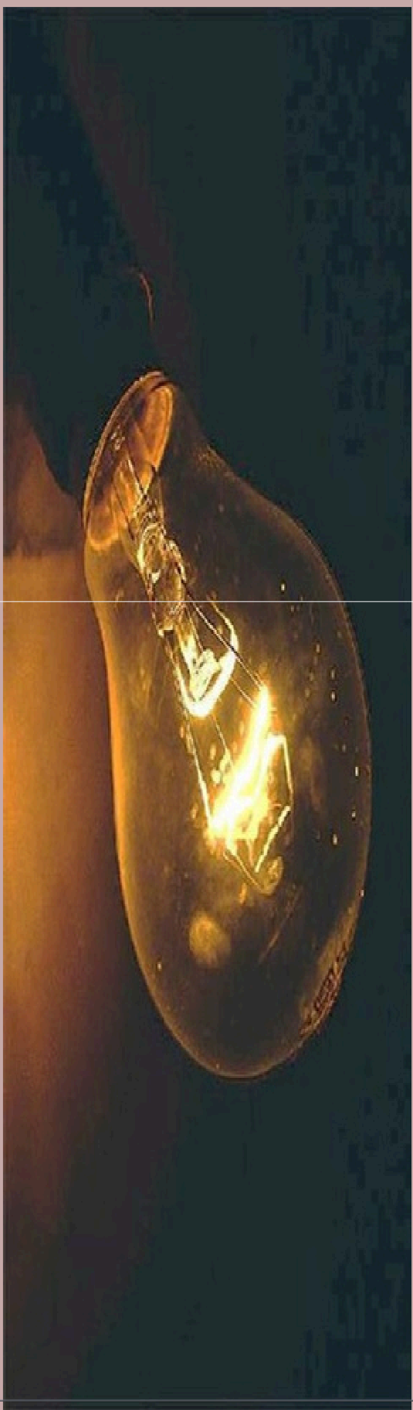


Figure : Graph R-C series ac circuits

Table: Equation R-C series ac circuits

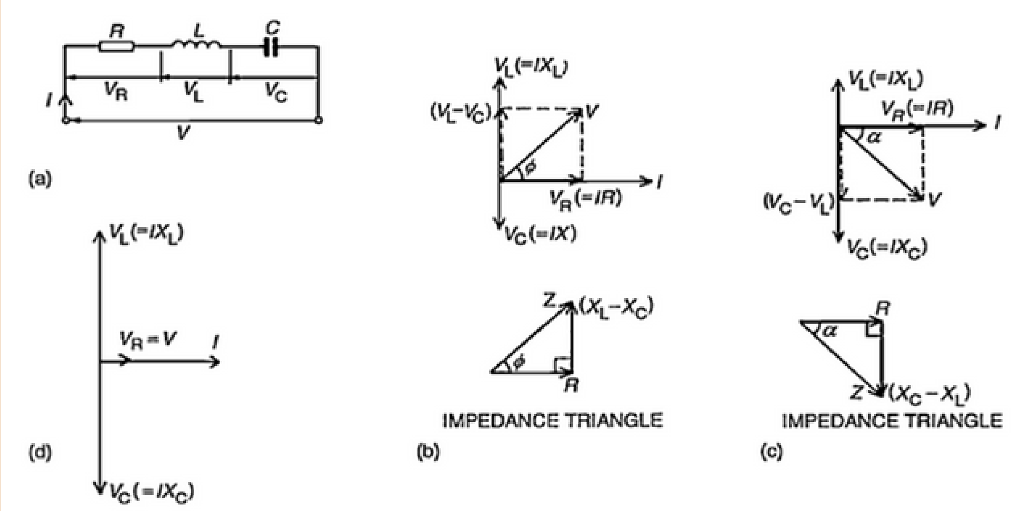
$X_C = \frac{1}{2\pi fC}$	$Z_T = \lvert Z_T \rvert \angle\theta$
$Z = R + (-iX_C)$	$\theta = tan^{-1} (\frac{-X_C}{R})$
$\lvert Z_T \rvert = \sqrt{(R^2 + (-X_C)^2)}$	$I = \frac{V}{Z_T}$
$V = \sqrt{(V_R{}^2 + V_C{}^2)}$	$V_C = IX_C$
$V_R = IR$	

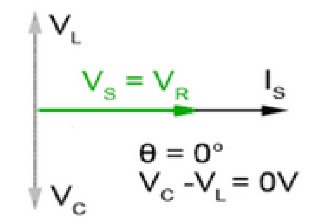
R-L-C SERIES AC CIRCUIT



In an a.c series circuit containing resistance R, inductance L and capacitance C, the applied voltage V is the phasor sum of V_R, V_L and V_C

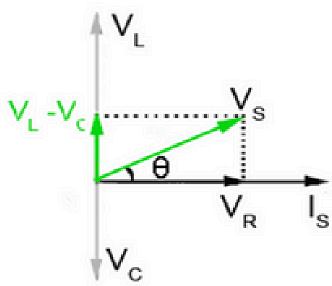
V_L and V_C are anti-phase (displaced by 180°) and there are three phasor diagrams possible — each depending on the relative values of V_L and V_C





When V_L and V_C are equal the circuit is purely resistive.

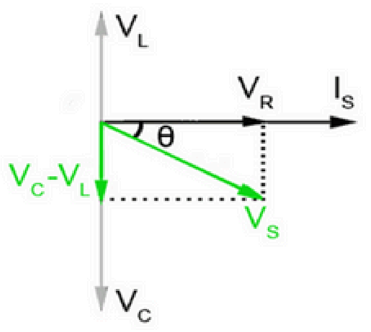
$X_L = 2\pi fL$	$X_C = \dfrac{1}{2\pi fC}$
IF $X_L > X_C$	
$Z = R + j(X_L - X_C)$	$\theta = \tan^{-1}\left(\dfrac{(X_L - X_C)}{R}\right)$
$\lvert Z_T\rvert = \sqrt{R^2 + (X_L - X_C)^2}$	$Z = \lvert Z_T\rvert \angle\theta$
$I_T = \dfrac{V}{Z_T}$	$V_S = \sqrt{V_R^2 + (V_L - V_C)^2}$



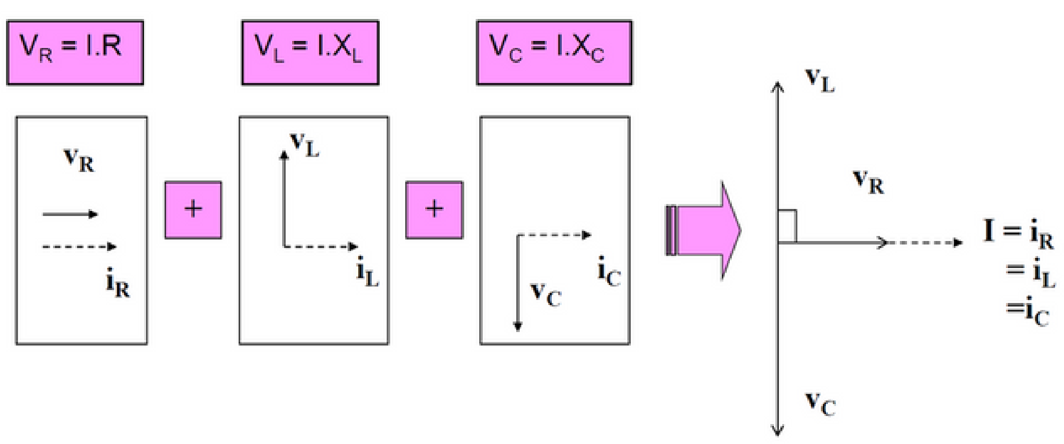
V_L is greater than V_C so the circuit behaves like an inductor

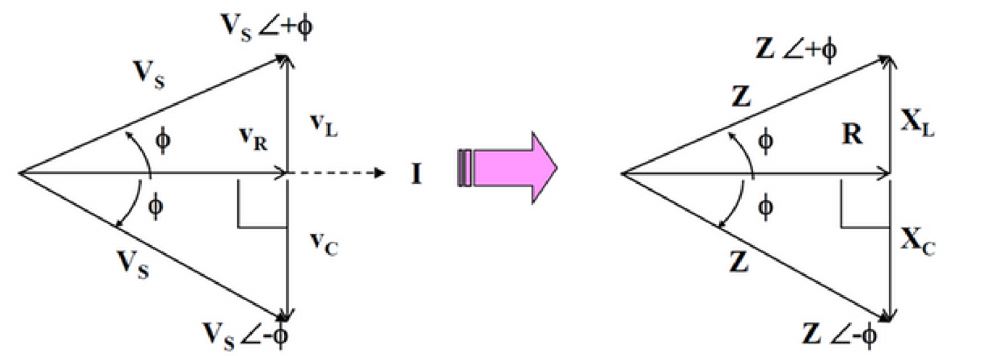
R-L-C SERIES AC CIRCUIT

$X_L = 2\pi fL$	$X_C = \frac{1}{2\pi fC}$
IF $X_C > X_L$	
$Z = R - j(X_C - X_L)$	$\theta = \tan^{-1} -\left(\frac{(X_C - X_L)}{R}\right)$
$\lvert Z_T \rvert = \sqrt{R^2 + (X_C - X_L)^2}$	$Z = \lvert Z_T \rvert \angle \theta$
$I = \frac{V}{Z_T}$	$V_S = \sqrt{V_R^2 + (V_C - V_L)^2}$



When VC is larger than VL the circuit is capacitive.

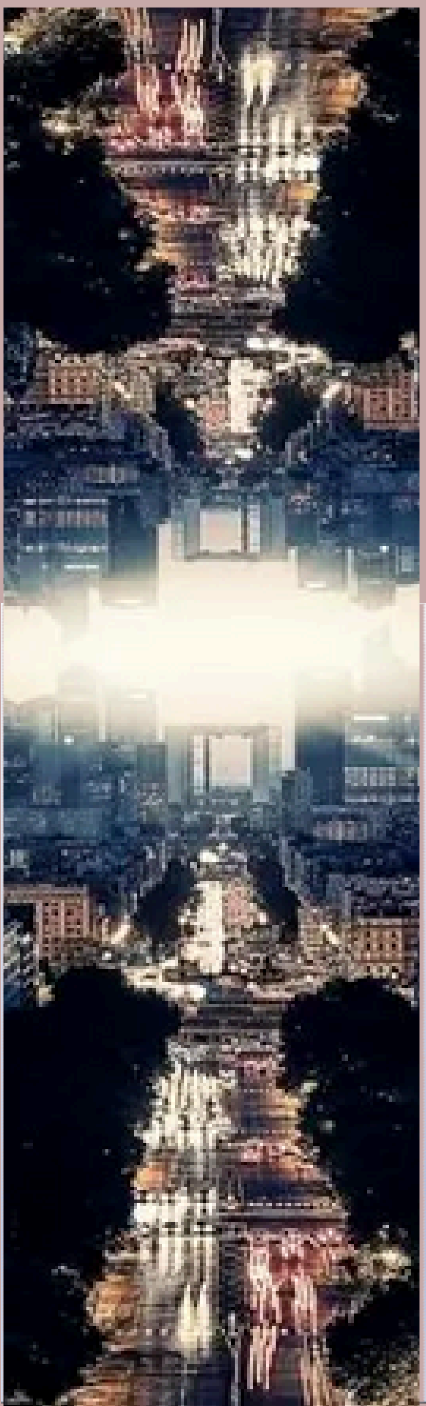




Voltage vector diagram

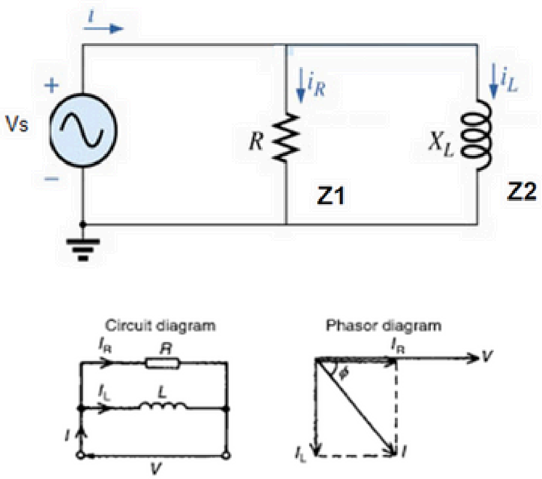
Impedance Triangle

R-L PARALLEL AC CIRCUIT



In the two branch parallel circuit containing resistance R and inductance L the current flowing in the resistance, I_R, is in-phase with the supply voltage V and the current flowing in the inductance, I_L, lags the supply voltage by 90°

The supply current I is the phasor sum of I_R and I_L and thus the current I lags the applied voltage V by an angle lying between 0° and 90° (depending on the values of I_R and I_L), shown as angle ϕ in the phasor diagram



$$i_T = \frac{V_T}{Z_T}$$

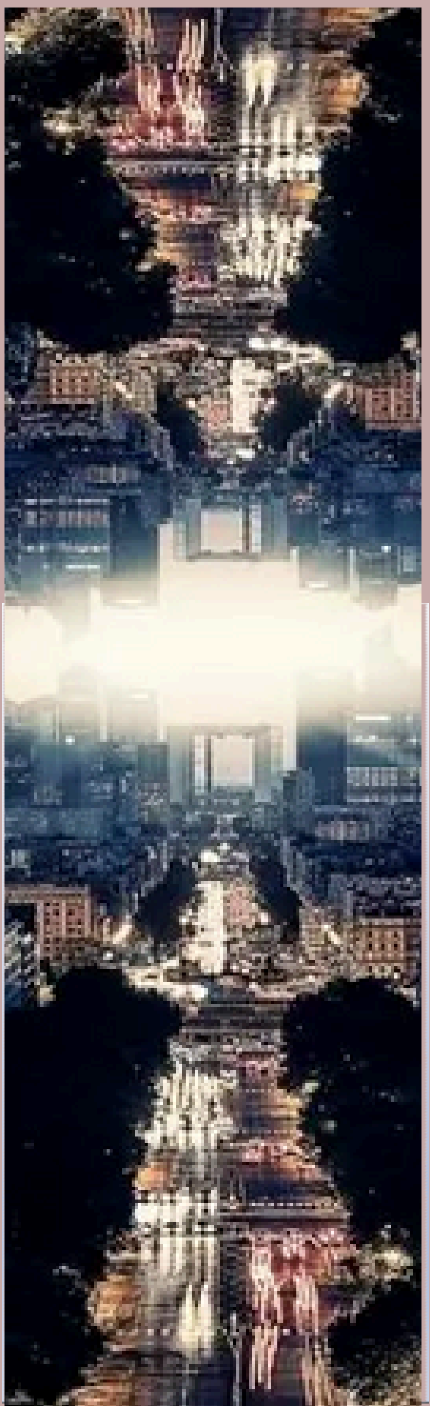
$$i_T = i_R + i_L$$

$$i_R = \frac{V_T}{Z_1} \qquad i_L = \frac{V_T}{Z_2}$$

$$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2} @ Z_T = \frac{(Z_1 x Z_2)}{Z_1 + Z_2}$$

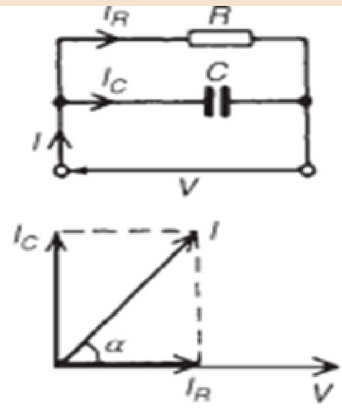
$$|Z| = R + j\varpi L = R + jX_L$$

R-C PARALLEL AC CIRCUIT



In the two branch parallel circuit containing resistance R and capacitance C the current flowing in the resistance, I_R, is in-phase with the supply voltage V and the current flowing in the capacitance, I_C, leads the supply voltage by 90°

The supply current I is the phasor sum of I_R and I_C and thus the current I leads the applied voltage V by an angle lying between 0° and 90° (depending on the values of I_R and I_C), shown as angle ϕ in the phasor diagram



$$I = \sqrt{(I_R^2 + I_C^2)},$$

$$I_R = \frac{V}{R}\ ; Z = \frac{V}{I}$$

$$I_C = \frac{V}{X_C}$$

EXAMPLE

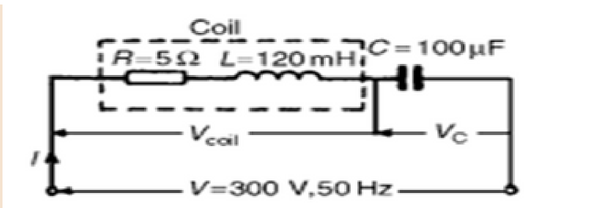
1. In a series R–L circuit the voltage across the resistance R is 12V and the voltage across the inductance L is 5V. Find the supply voltage and the phase angle between current and voltage.

2. A coil has a resistance of 4Ω and an inductance of 9.55mH from a 240V, 50Hz supply. Calculate :
   (a) the reactance,
   (b) the impedance,
   (c) the current
   Determine also the phase angle between the supply voltage and current

3. A resistor of 25Ω is connected in series with a capacitor of 45μF from a 240V, 50Hz supply. Calculate
   (a) the impedance,
   (b) the current.
   Find also the phase angle between the supply voltage and the current.

4. A capacitor C is connected in series with a 40Ω resistor across a supply of frequency 60 Hz. A current of 3A flows and the circuit impedance is 50 . Calculate
   (a) the value of capacitance, C,
   (b) the supply voltage,
   (c) the phase angle between the supply voltage and current,
   (d) the p.d. across the resistor, and
   (e) the p.d. across the capacitor.
   Draw the phasor diagram.

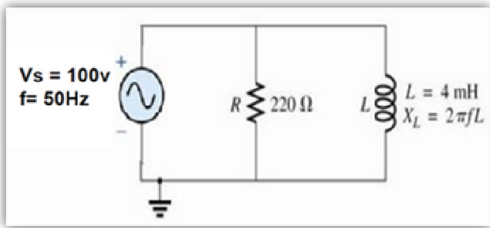
5. A coil of resistance 5Ω and inductance 120mH in series with a 100μF capacitor, is connected to a 300V, 50 Hz supply. Calculate
   (a) the current flowing,
   (b) the phase difference between the supply voltage and current,
   (c) the voltage across the coil and
   (d) the voltage across the capacitor.



EXAMPLE

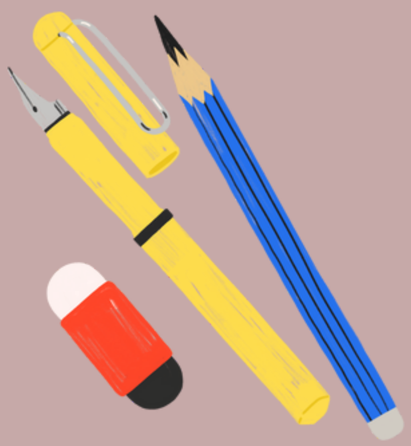
6. A 20Ω resistor is connected in parallel with an inductance of 2.387 mH across a 60V, 1 kHz supply. Calculate :
   (a) the current in each branch,
   (b) the supply current,
   (c) the circuit phase angle,
   (d) the circuit impedance

7. Refer to the figure below, calculate :
   (a) the total impedance
   (b) the current in each branch
   (c) the supply current,
   Draw the phasor diagram



8. A 30µF capacitor is connected in parallel with an 80Ω resistor across a 240V, 50Hz supply. Calculate :
   (a) the current in each branch,
   (b) the supply current,
   (c) the circuit phase angle,
   (d) the circuit impedance

ANSWER



1. In a series R–L circuit the voltage across the resistance R is 12V and the voltage across the inductance L is 5V. Find the supply voltage and the phase angle between current and voltage

$$V = \sqrt{({V_R}^2 + {V_L}^2)} \;; \qquad \Theta = tan^{-1}\frac{V_L}{V_R}$$

$$V = \sqrt{(12^2 + 5^2)} \;; \qquad \Theta = tan^{-1}\frac{5}{12}$$

$$V = 13V \;; \qquad \Theta = 22.62°$$

2. A coil has a resistance of 4Ω and an inductance of 9.55mH from a 240V, 50Hz supply. Calculate :

(a) the reactance

$$X_L = 2\pi fL$$
$$X_L = 2\pi(50)(9.55m)$$
$$X_L = 3\Omega$$

(b) the impedance,

$$Z = R + iX_L$$
$$Z = 4 + i3$$
$$Z = 5\Omega\angle 36.87°$$

(c) the current

$$I = \frac{V}{Z_T}$$

$$I = \frac{240\angle 0°}{Z_T\angle 36.87°}$$

$$I = 48A\angle -36.87°$$

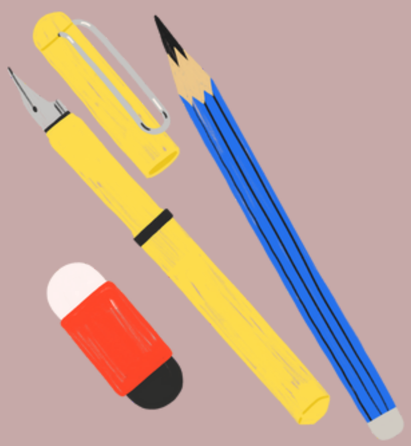
Determine also the phase angle between the supply voltage and current

V=240V

I=48A

$$I \; lags \; V \; by \; \text{-} \; 36.87°$$

ANSWER



3. A resistor of 25Ω is connected in series with a capacitor of 45μF from a 240V, 50Hz supply. Calculate
   (a) the impedance,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi 50(45\mu)}$$

$$X_C = 70.74\Omega$$

$$Z = R + (-iX_C) = 25 + (-i70.74)$$

$$Z = 75.03\Omega \angle -70.54°$$

(b) the current
Find also the phase angle between the supply voltage and the current.

$$I = \frac{V}{Z_T}$$

$$I = \frac{240\angle 0°}{75.03\Omega \angle -70.54°}$$

$$I = 3.2A \angle 70.54° \qquad I \text{ leads } V \text{ by } 70.54°$$

4. A capacitor C is connected in series with a 40Ω resistor across a supply of frequency 60 Hz. A current of 3A flows and the circuit impedance is 50 . Calculate
   (a) the value of capacitance, C,

Impedance, $Z = R + (-iX_C) = \sqrt{(R^2 + (-X_C)^2)}$

$$50 = \sqrt{(40^2 + (-X_C)^2}, \; X_C = 30\Omega \quad \theta = tan^{-1}\left(\frac{-X_C}{R}\right)$$

$$\theta = tan^{-1}\left(\frac{-30}{40}\right) = -36.87°$$

$$X_C = \frac{1}{2\pi fC}, \; C = \frac{1}{2\pi f X_C}$$

$$C = \frac{1}{2\pi(60)(30)} = 88.42\mu F$$

(b) the supply voltage,

$$I = \frac{V}{Z_T}, \; V = IZ_T$$

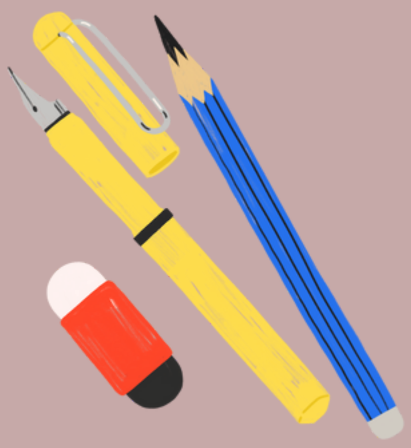
$$V = (3\angle 0°)(50\angle -36.87°)$$

$$V = 150V\angle -36.87°$$

(c) the phase angle between the supply voltage and current,

$$I \text{ leads } V \text{ by } 36.87°$$

ANSWER



(d) the p.d. across the resistor

$$V = IR = (3\angle 0°)(40\angle 0°) = 120V\angle 0°$$

(e) the p.d. across the capacitor.

$$V = IX_C = (3\angle 0°)(30\angle -90°) = 90V\angle\text{-}90°$$

$$V = V_R + jV_C = (120\angle 0°) + j(90\angle -90°) = 150V\angle\text{-}36.87°$$

Draw the phasor diagram.

5. A coil of resistance 5Ω and inductance 120mH in series with a 100μF capacitor, is connected to a 300V, 50 Hz supply. Calculate
(a) the current flowing,

$$X_L = 2\pi fL = 2\pi(50)(120m) = 37.70\Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi 50(100\mu)} = 31.83\Omega$$

$$Z = R + iX_L + (-iX_C) = 5 + i37.70 + (-i31.83)$$
$$= 7.71\Omega\angle 49.58°$$

$$I = \frac{V}{Z_T} = \frac{300<0°}{7.71\Omega<49.58°} = 38.91A\angle -49.58°$$

(b) the phase difference between the supply voltage and current,

$$V \text{ leads } I \text{ by } 49.58°$$

(c) the voltage across the coil and

$$Z_{COIL} = R + iX_L = 5 + i37.70 = 38.03\Omega\angle 82.45°$$

$$V_{COIL} = IZ_{COIL} = 38.91A\angle -49.58°(38.03\Omega\angle 82.45°)$$

$$V_{COIL} = 1480V\angle 32.87°$$

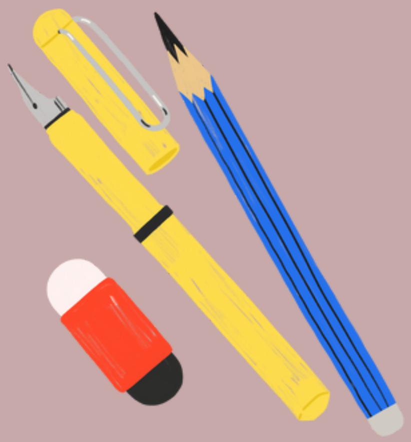
(d) the voltage across the capacitor.

$$V_C = IX_C = 38.91A\angle -49.58°(31.83\Omega\angle -90°)$$

$$V_C = 1239V\angle -139.58°$$

$$V_T = V_{COIL} + (-jV_C) = 1480\angle 32.87° + j(1239\angle -139.58°)$$
$$= 300V\angle 0°$$

ANSWER



6. A 20Ω resistor is connected in parallel with an inductance of 2.387 mH across a 60V, 1 kHz supply. Calculate :

(a) the current in each branch,

$$I_R = \frac{V}{R} = \frac{60}{20} = 3A$$

$$X_L = 2\pi f L = 2\pi(1000)(2.387m) = 15\Omega$$

$$I_L = \frac{V}{X_L} = \frac{60}{15} = 4A$$

(b) the supply current,

$$I_T = I_R + iI_L$$

$$I_T = 3 + \text{i}4 = 5\text{A}\angle 53.13°$$

(c) the circuit phase angle,

$$\theta = tan^{-1}\frac{I_L}{I_R}$$

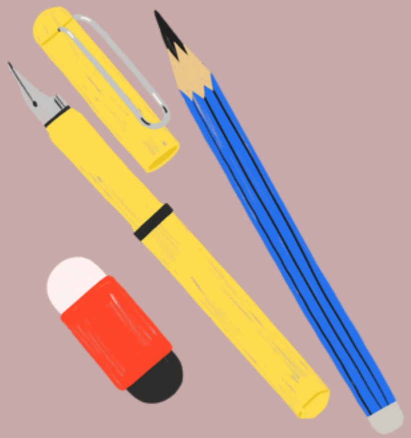
$$\theta = tan^{-1}\frac{4}{3}$$

$$\theta = 53.13°$$

(d) the circuit impedance

$$\text{Z} = \frac{V}{I_T} = \frac{60<0°}{5<53.13°} = 12\Omega\angle - 53.13°$$

ANSWER



7(a) the total impedance

Z1=R+i0 = $220\Omega\angle 0°$

Z2=0+iXL , $X_L = 2\pi f L = 2\pi(50)(4\text{m}) = 1.257\Omega$

Z2=0+i1.257 $= 1.257\Omega\angle 90°$

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$=(220\Omega\angle 0°)(1.257\Omega\angle 90°)/((220\Omega\angle 0°)+(1.257\Omega\angle 90°))$$

$$=1.257\Omega\angle 89.67°$$

(b) the current in each branch

$$I_1 = \frac{V}{Z_1} = \frac{100\Omega\angle 0°}{220\Omega\angle 0°} = 0.455A\angle 0°$$

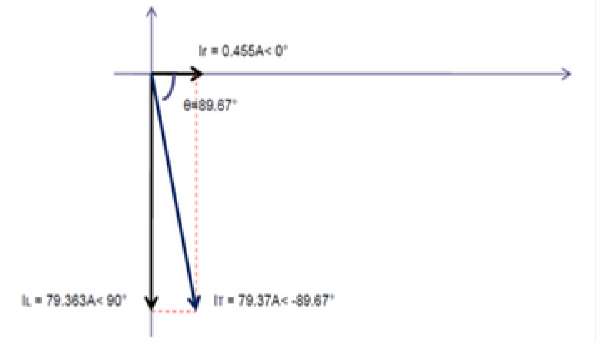
$$I_2 = \frac{V}{Z_2} = \frac{100\Omega\angle 0°}{1.257\Omega\angle 90°} = 79.365A\angle -90°$$

(c) the supply current,

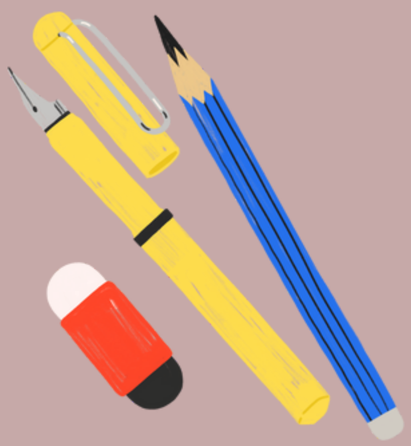
$$I_T = I_1 + I_2 = 0.455A\angle 0° + 79.365A\angle -90°$$

$$I_T = 7.97A\angle -89.67°$$

Draw the phasor diagram



ANSWER



8. A 30µF capacitor is connected in parallel with an 80Ω resistor across a 240V, 50Hz supply. Calculate :

(a) the current in each branch,

$$I_R = \frac{V}{R} = \frac{240}{80} = 3A$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(50)(30\mu)} = 106.1\Omega$$

$$I_C = \frac{V}{X_C} = \frac{240}{106.1} = 2.262A$$

(b) the supply current,

$$I_T = I_R + iI_C$$

$$I_T = 3 + \mathrm{i}2.2.62 = 3.757\mathrm{A}\angle 37.02°$$

(c) the circuit phase angle,

$$\theta = tan^{-1}\frac{I_C}{I_R}$$

$$\theta = tan^{-1}\frac{2.262}{3}$$

$$\theta = 35.02°$$

(d) the circuit impedance

$$\mathrm{Z} = \frac{V}{I_T} = \frac{240<0°}{3.757<35.02°} = 63.88\Omega\angle - 53.13°$$

EXERCISE

1. A coil of inductance 80 mH and resistance 60Ω is connected to a 200V, 100 Hz supply. Calculate the circuit impedance and the current taken from the supply. Find also the phase angle between the current and the supply voltage.

[78.27 , 2.555A, 39.95° lagging]

2. A coil takes a current of 5A from a 20V d.c supply. When connected to a 200V, 50 Hz a.c supply the current is 25A. Calculate the (a) resistance, (b) impedance and (c) inductance of the coil.

[(a) 4 (b) 8 (c) 22.05 mH]

3. A resistance of 50Ω is connected in series with a capacitance of 20 μF. If a supply of 200V, 100 Hz is connected across the arrangement find
(a) the circuit impedance,
(b) the current flowing, and
(c) the phase angle between voltage and current.

[(a) 93.98 (b) 2.128A (c) 57.86° leading]

4. A 24.87 μF capacitor and a 30Ω resistor are connected in series across a 150V supply. If the current flowing is 3A find
(a) the frequency of the supply,
(b) the p.d. across the resistor and
(c) the p.d. across the capacitor.

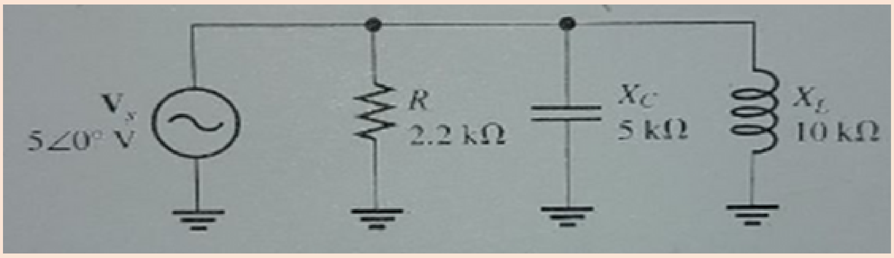
[(a) 160 Hz (b) 90V (c) 120V]

5. The following three impedances are connected in series across a 40V, 20 kHz supply:
(i) a resistance of 8 ,
(ii) a coil of inductance 130 μH and 5 resistance, and
(iii) a 10 resistor in series with a 0.25 μF capacitor.
Calculate :
(a) the circuit current, I=1.442A
(b) the circuit phase angle and θ=33.96°
(c) the voltage drop across each impedance 11.54V, 24.64V, 48.11V

6. A 40 μF capacitor in series with a coil of resistance 8 and inductance 80 mH is connected to a 200V, 100 Hz supply. Calculate :
(a) the circuit impedance,
(b) the current flowing,
(c) the phase angle between voltage and current,
(d) the voltage across the coil, and
(e) the voltage across the capacitor.

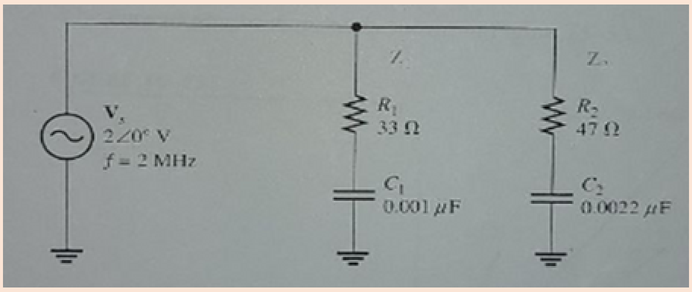
[(a) 13.18 (b) 15.17A (c) 52.63° lagging (d) 772.1V (e) 603.6V]

EXERCISE



7. Refer to the circuit above, find:
(a) each branch current,
(b) total current,
(c) current phasor diagram,

$I_R = 2.27\angle 0°\text{mA}$
$I_C = 1\angle 90°\text{mA}$
$I_L = 0.5\angle -90°\text{mA}$



8. Determine all currents in the figure above.

$Z_1 = 86.2\angle -67.5°\Omega$
$Z_2 = 59.3\angle -37.6°\Omega$
$I_1 = 23.2\angle 67.5°\text{mA}$
$I_2 = 33.7\angle 37.6°\text{mA}$

COURSE LEARNING OUTCOME

1. Explain AC circuit concept and their analysis using AC circuit law.
2. Apply the knowledge of AC circuit in solving problem related to AC electrical circuit.

OBJECTIVE

2.5 Understand power in AC circuits.

1. Explain power in resistive and reactive AC circuits.
2. Explain true, reactive and apparent power.
3. Explain power factor and power factor correction

2.6 Apply the understanding of the power consumption in AC circuits.

1. Calculate power consumption in an AC circuit with pure resistance.
2. Show no energy is consumed in a pure capacitance and inductance circuit.
3. Draw the power triangle (phasor diagram).
4. Sketch the following terms based on the power triangle.

    a. apparent power (VA)
    b. reactive power (VAR)
    c. actual power (Watt)

5. Calculate power factor.
6. Solve problem related to reactance, impedance and power factor correction.

CONCEPT OF WORK, POWER, ENERGY AND EFFICIENCY

WORK

The product of force and the distance moved in the direction of the force.

WORK → Symbol : W → Units : Nm or Joule (J)

Work (W) = Force x Distance

ENERGY

Energy and work are very closely related. Energy is the ability to do work

ENERGY → Units: kWh

ENERGY = Power x Time(hour)

POWER

Power is the rate at which energy is used.

POWER → Symbol : P → Units : J/s or Watt

POWER (P) = $\frac{Work\ (Joule)}{Time\ (second)}$ [J/s] or [Watt]

Power (P) = Force [N] x Speed [v] [J/s] or [Watt]

Speed = $\frac{Distance}{Time}$

EFFICIENCY

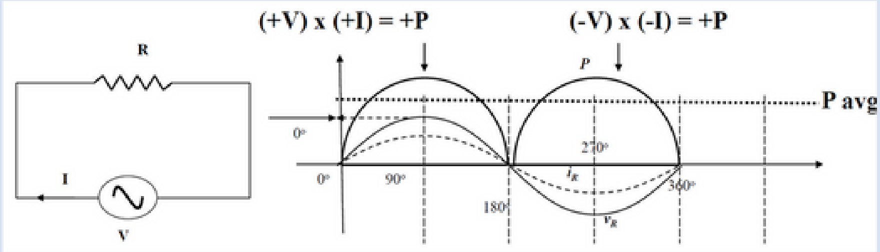
Defined as useful power output divided by the total power consumed.

Efficiency → Symbol : η → Unit : none

$$\text{Efficiency } (\eta) = \frac{Power\ Output}{Total\ Power\ Input} = \frac{P_o}{P_{in}}$$

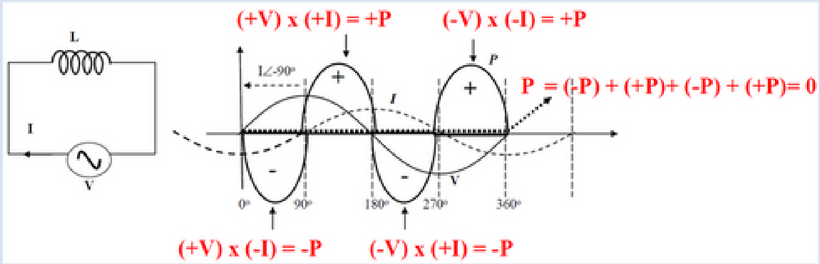
POWER CONSUMPTION IN AC CIRCUIT

PURE RESISTANCE CIRCUIT



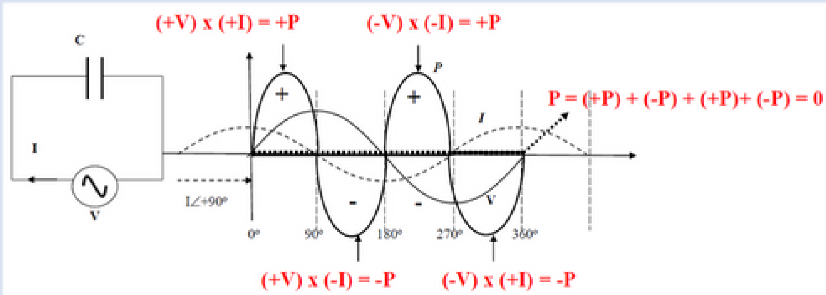
- Resistance (R) = current and voltage (in phase)
- Waveform for power is always positive, never negative for this resistive circuit
- Power is always being dissipated by the resistive load

PURE INDUCTIVE CIRCUIT

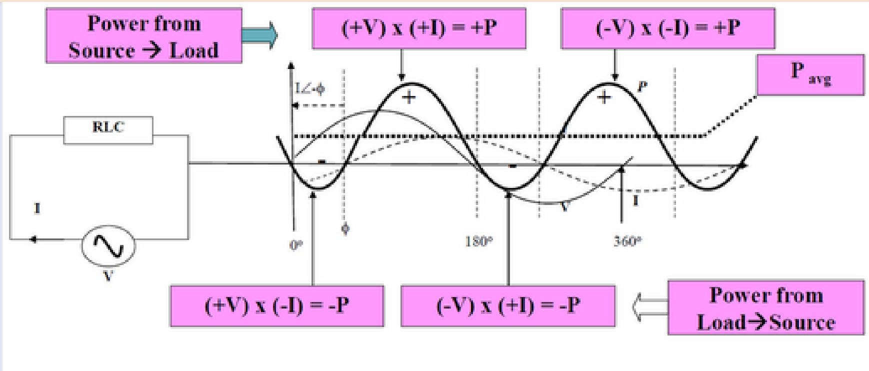


- Inductance (L) = current lagging voltage by 90°
- Waveform for power is alternates equally between cycles of positive and negative
- Power is being alternately absorbed from and returned to the source.

PURE CAPACITIVE CIRCUIT



- Capacitance (C) = current leading voltage by 90°
- Waveform for power is alternates equally between cycles of positive and negative
- Power is being alternately absorbed from and returned to the source.



Power for practical circuit

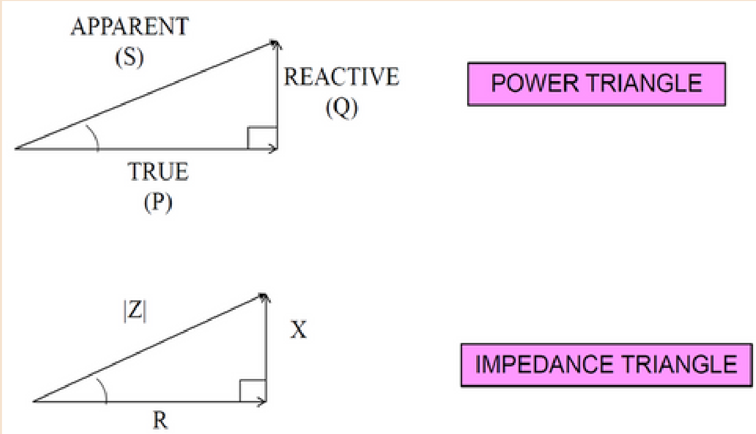
- Therefore, any reactance in this load will likewise dissipate zero power. The only thing left to dissipate power here is the resistive portion of the load impedance.

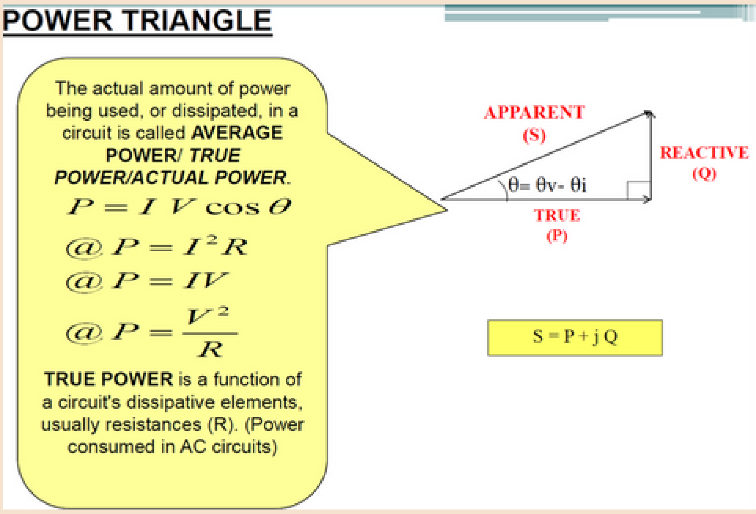
POWER CONSUMPTION IN AC CIRCUIT

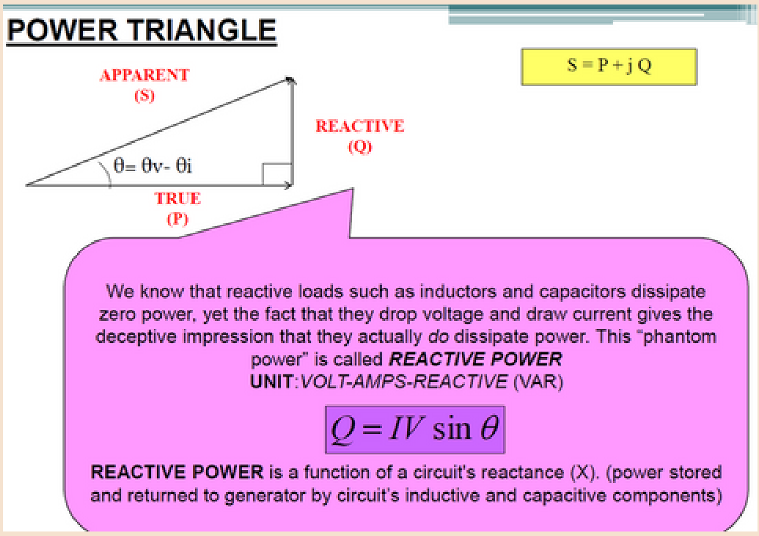
SUMMARY

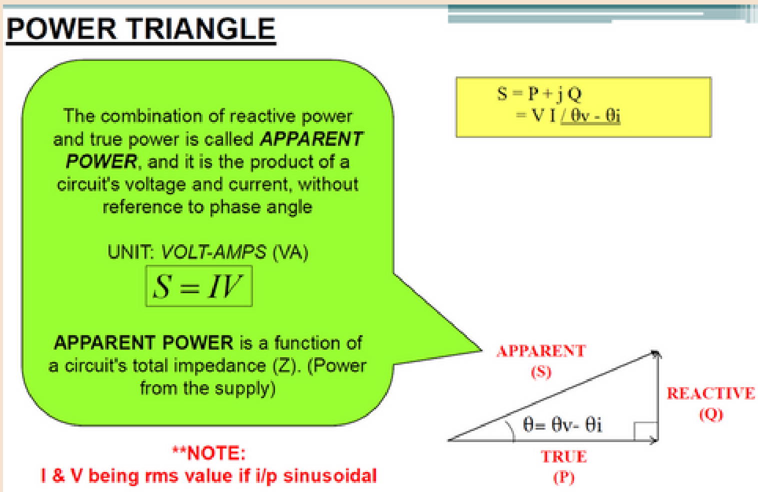
1. In any reactive circuit, the power alternates between positive and negative instantaneous values over time. In a purely reactive circuit that alternation between positive and negative power is equally divided, resulting in a net power dissipation of zero.
2. In circuits with mixed resistance and reactance, the power waveform will still alternate between positive and negative, but the amount of positive power will exceed the amount of negative power. In other words, the combined inductive/resistive load will consume more power than it returns back to the source.
3. From the waveform plot for power, it should be evident that the wave spends more time on the positive side of the centre line than on the negative, indicating that there is more power absorbed by the load than there is returned to the circuit.

POWER TRIANGLE









POWER FACTOR

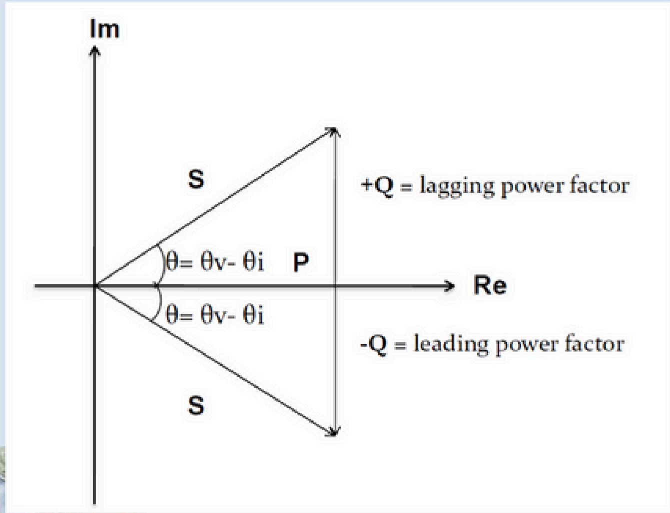
POWER FACTOR

$$P.F = \frac{\textit{True Power}}{\textit{Apparent Power}}$$

$$P.F = \cos\theta$$

- The **ratio** between true power and apparent power is called the **power factor**
- Power factor, like all ratio measurement, is a unitless quantity
- It is often desirable to adjust the power factor of a system to near 1.0.
- A high power factor is generally desirable in a transmission system to reduce transmission losses and improve voltage regulation at the load.

POWER FACTOR GRAPH



POWER FACTOR CORRECTION

- A high power factor reduces the current flowing in a supply system, therefore reduces the cost of cables, switch gear, transformers & generators
- Eg. of industrial load:
  - motors (inductive load)

METHOD OF POWER FACTOR CORRECTION

CAPACITOR

- power factor in an AC circuit may be "corrected", or re-established at a value close to 1, by adding a **parallel reactance opposite** the effect of the load's reactance.
- If the load's reactance is **inductive in nature** (which is almost always will be), **parallel *capacitance*** needed to correct poor power factor.

Why Capacitor?
- Its cheap
- Less space
- Low temperature
- Power losses ignorance

SYNCHRONOUS MOTOR

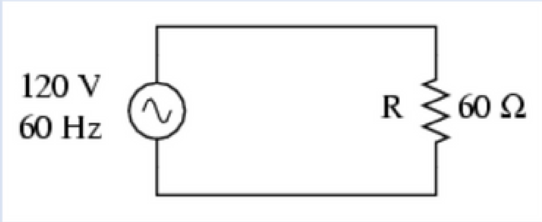
- It operates at a leading power factor and puts VAR onto the network as required to support a system's voltage or to maintain the system power factor at a specified level

Automatic power factor correction unit

- It consists of a number of capacitor that are switched by means of contactors.

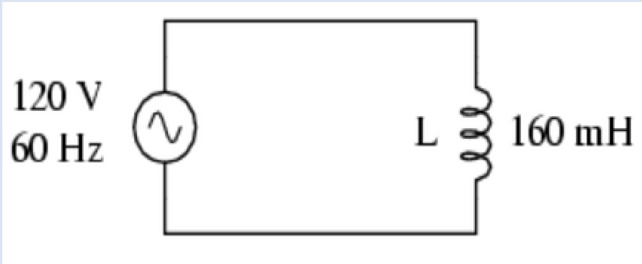
EXAMPLE

EXAMPLE 1



Find the power consumption?

EXAMPLE 2



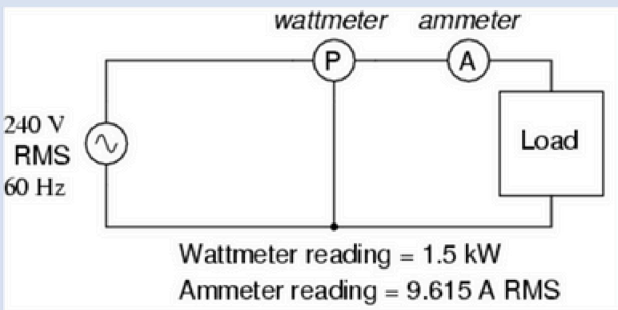
Find the power consumption?

EXAMPLE 3

A load Z draws 12kVA at a power factor of 0.856 lagging from a 120V rms sinusoidal source. Calculate:

a) The average & reactive powers delivered to the load
b) The peak current
c) Load impedance

EXAMPLE 4



Find the power consumption?

ANSWER

ANSWER (E1)

$$Z_R = 60 + j0 = 60\angle 0^\circ$$

$$I = \frac{V}{Z_R} = \frac{120}{60} = 2A$$

$$P = I^2 R = 2^2(60) = 240W$$

$$P = \frac{V^2}{R} = \frac{120^2}{60} = 240W$$

$$P = IV = 2(120) = 240W$$

** In this example, the current to the load would be 2A. The power dissipated at the load would be 240 Watts

ANSWER (E2)

$$X_L = 60.319\Omega$$

$$Z_L = 0 + j60.319 = 60.319\angle 0^\circ$$

$$I = \frac{V}{Z} = \frac{120}{60.319} = 1.989A$$

$$P = I^2 R = 1.989^2(0) = 0W$$

$$Q = I^2 X = 1.989^2(60.319) = 238.63VAR$$

$$S = I^2 Z = 1.989^2(60.319) = 238.63VA$$

ANSWER

ANSWER (E3)

A load Z draws 12kVA at a power factor of 0.856 lagging from a 120V rms sinusoidal source. Calculate:

a) The average & reactive powers delivered to the load

$$Pf = \cos\theta = 0.856, \text{ so } \theta = 31.13^o$$
$$S = IV = 12kVA,$$
$$P = IV\cos\theta = 10.272 \text{ kW}$$
$$Q = IV\sin\theta = 6.204 \text{ kVAR}$$

b) The peak current

$$S = P + jQ = 10.272kW + j6.204kVAR$$
$$= 12\angle 31.13^o$$
$$S = IV, \text{ so } I = \frac{S}{V}$$

$V_{RMS} = 0.707 \times V_P$; $\therefore V_P = \frac{V_{RMS}}{0.707} = \frac{120}{0.707} = 169.73$ V

$$= \frac{12\angle 31.13^o}{169.73} = 70.7\angle 31.13^o$$

c) Load impedance

$$V = IZ;$$
$$Z = \frac{V}{I} = \frac{169.7\angle 0°}{70.7\angle 31.13^o} = 2.4\angle -31.13°\Omega$$

ANSWER (E4)

<u>Solution 1:</u>

First, we need to calculate the apparent power in kVA.
S = IV = 9.615 x 240 = 2307.6 VA = 2.31kVA

Now, we figure the power factor of this load;
$$\text{Power factor} = \frac{true\ power}{apparent\ power} = \frac{1.5k}{2.31k} = 0.65$$

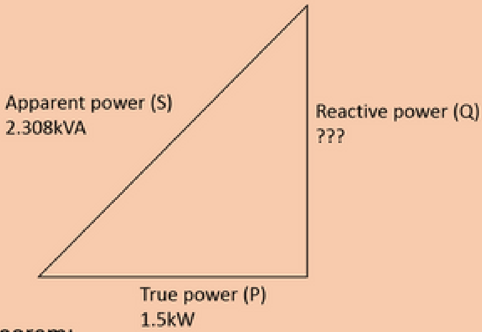
6
3

ANSWER

ANSWER (E4)

Solution 2:

Using this value for power factor, we can draw a power triangle, and from that determine the reactive power of this load:



Use the Pythagorean Theorem;

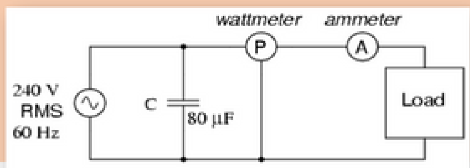
Q = $\sqrt{S^2 + P^2} = \sqrt{2.308k^2 + 1.5k^2} = 1.754kVAR$

Solution 3:

- If this load is an electric motor, or most any other industrial AC load, it will have a lagging (inductive) power factor, which means that we'll have to correct for it with a *capacitor* of appropriate size, wired in parallel.
- Now that we know the amount of reactive power (1.754 kVAR), we can calculate the size of capacitor needed to counteract its effects:

$$Q = \frac{V^2}{X}\ ;\ \therefore X = \frac{V^2}{Q} = \frac{240}{1.754k} = 32.845\Omega$$

- We know that $X_C = \frac{1}{2\pi f C}$ $\therefore$So, $C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \ x\ 60\ x\ 32.854} = 80.761\mu F$
- Rounding this answer off to $80\mu F$, we can place that size of capacitor in the circuit and calculate the results:



Solution 4:

- An 80 µF capacitor will have a capacitive reactance of 33.157 Ω, giving a current of 7.238 amps, and a corresponding reactive power of 1.737 kVAR (for the capacitor *only*).

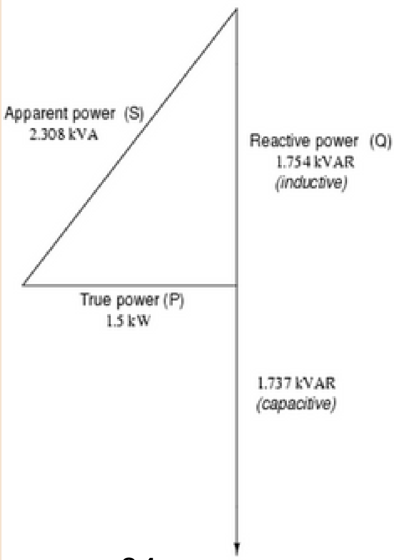
C = 80 µF; so;

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi\ x\ 60\ x\ 80\mu} = 33.157\Omega$$

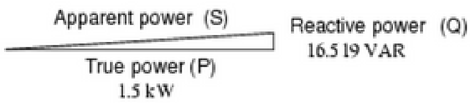
$$V = IX_C\ ;\ \therefore I = \frac{V}{X_C} = \frac{240}{33.157} = 7.238A$$

$$Q = \frac{V^2}{X_C} = \frac{240^2}{33.157} = 1.737kVAR$$

- This correction, of course, will not change the amount of true power consumed by the load, but it will result in a substantial reduction of apparent power, and of the total current drawn from the 240 Volt source:



Power triangle after adding capacitor



$$Inductive\ Q - Capacitive\ Q = Total\ Q$$

$$1.754kVAR - 1.737kVAR = 16.519VAR$$

The new apparent power can be found from the true and new reactive power values, using the standard form of the Pythagorean Theorem:

$$S = \sqrt{(Q)^2 + (P)^2} = \sqrt{(16.519)^2 + (1.5k)^2} = 1.50009kVA$$

So, new power factor is;

$$power\ factor = \frac{true\ power}{apparent\ power} = \frac{1.5kW}{1.50009kVA} = 0.99994$$

CHAPTER 3

RESONANCE

OBJECTIVE

At the end of this chapter, students will be able to:

1. Understand resonance in series and parallel circuits.
2. Apply resonance in series and parallel circuits.

Explain the resonance phenomenon and its functions

Resonance is a condition in RLC circuit in which the capacitive and inductive reactance are equal in magnitude, thereby resulting in a purely resistive impedance.

$$Z = R + j\left(\omega\, l - \frac{1}{\omega\, L}\right) \qquad ;\ \text{note} : \omega\, l - \frac{1}{\omega\, L} = 0$$

$$X\,L = X\,C$$

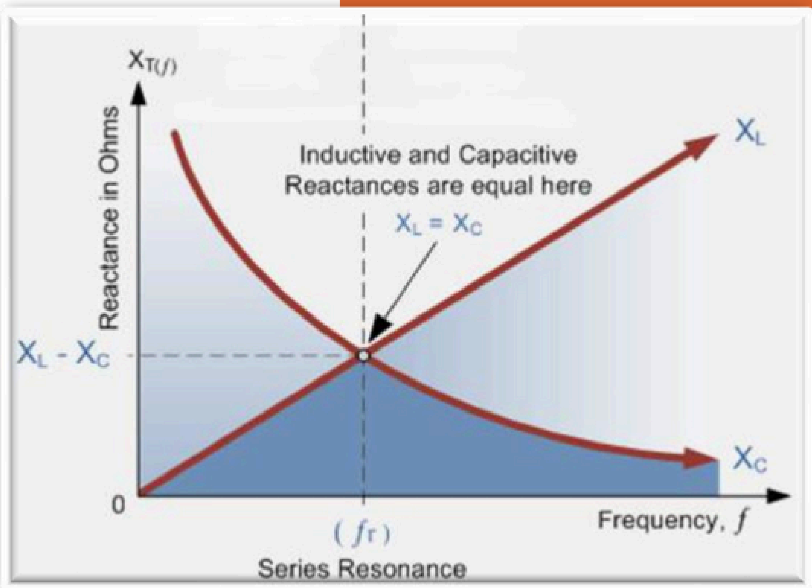


Figure 1: Resonant Circuit

$$X_L = 2\pi f L \qquad X_C = \frac{1}{2\pi f C}$$

. . setting the two equal to each other, representing a condition of equal reactance (resonance) . . .

$$2\pi f L = \frac{1}{2\pi f C}$$

SERIES RESONANCE

Current will be maximum & offering minimum impedance

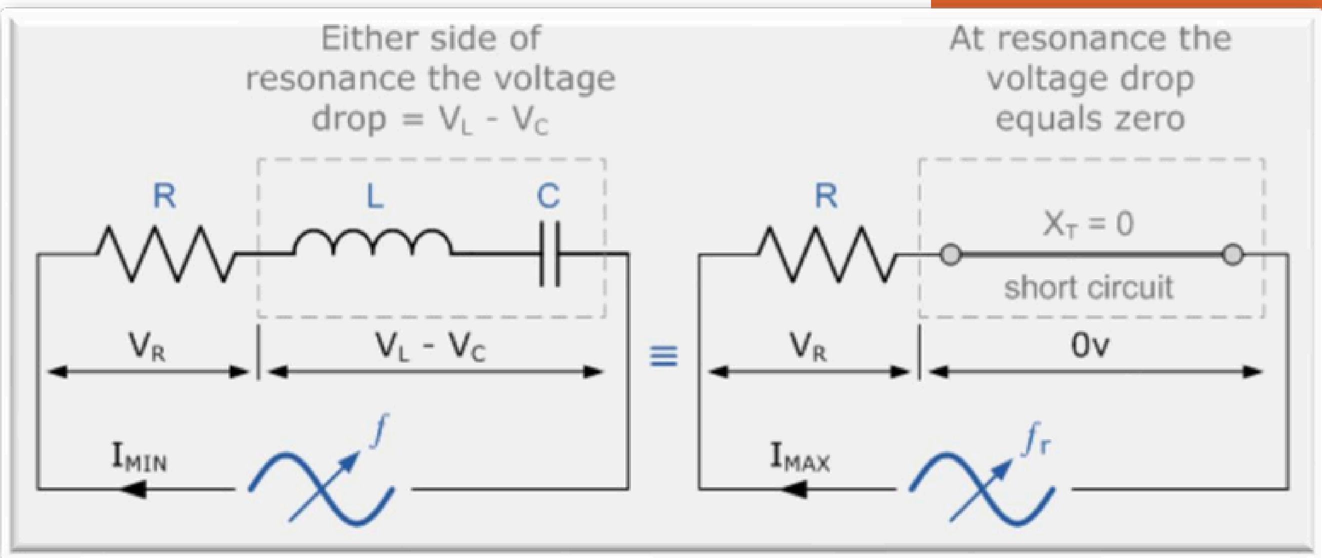
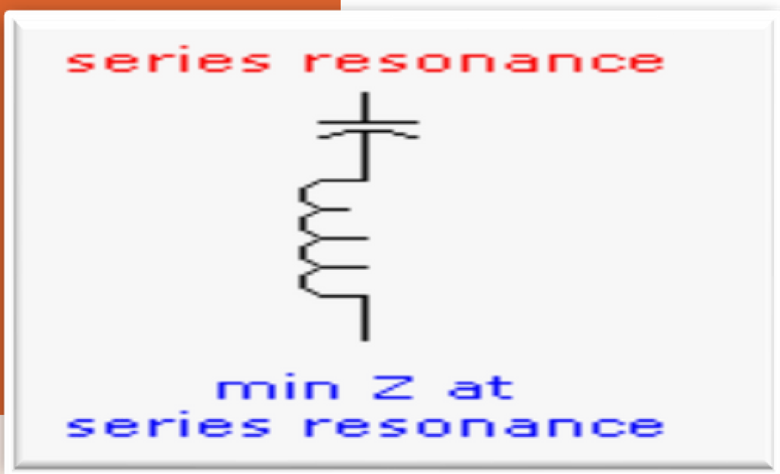
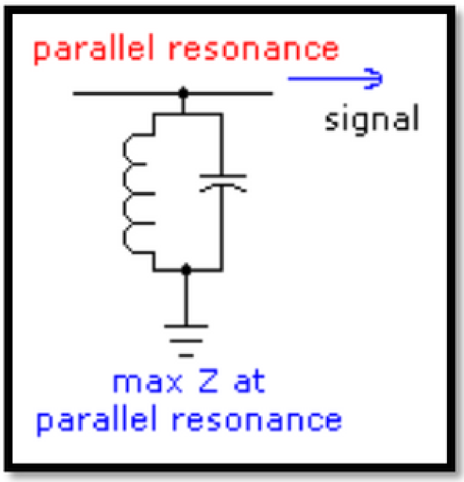


Figure 2: Series resonance circuits



PARALLEL RESONANCE

Current will be minimum & offering maximum impedance.



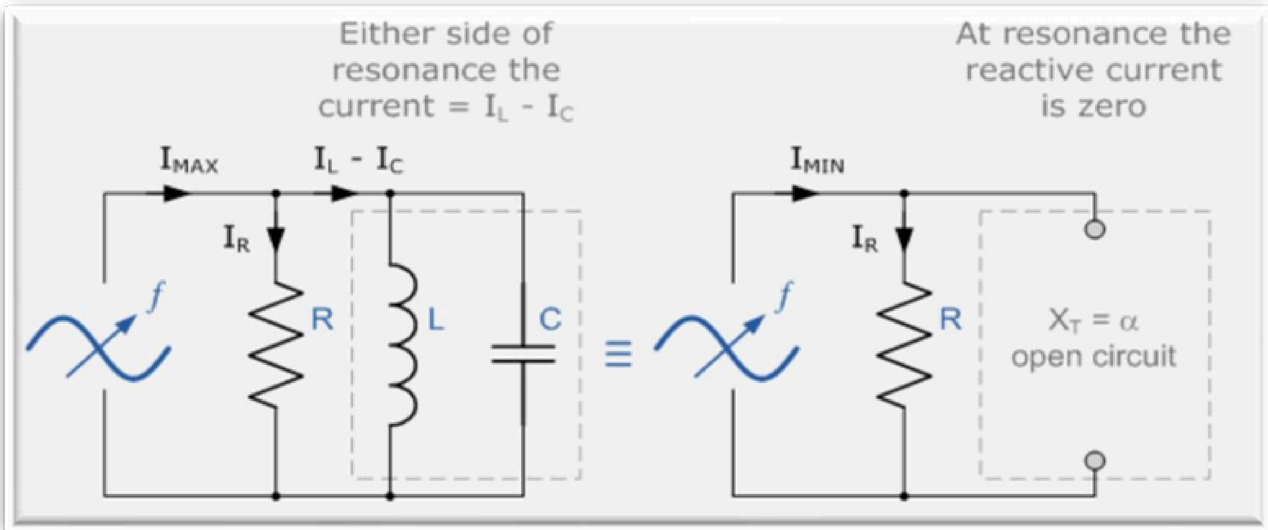
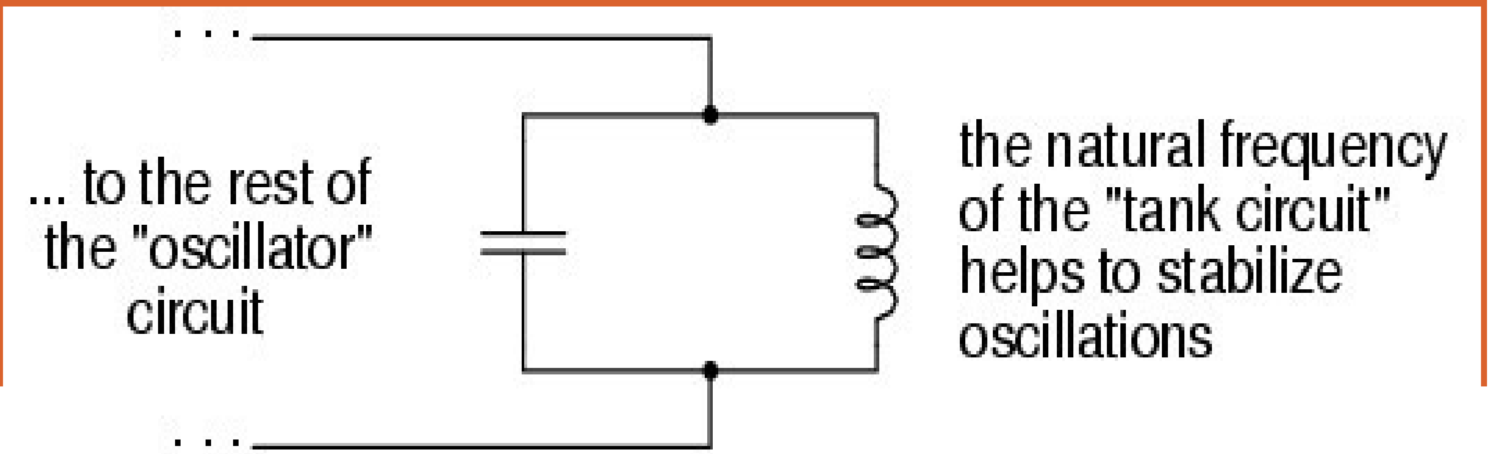
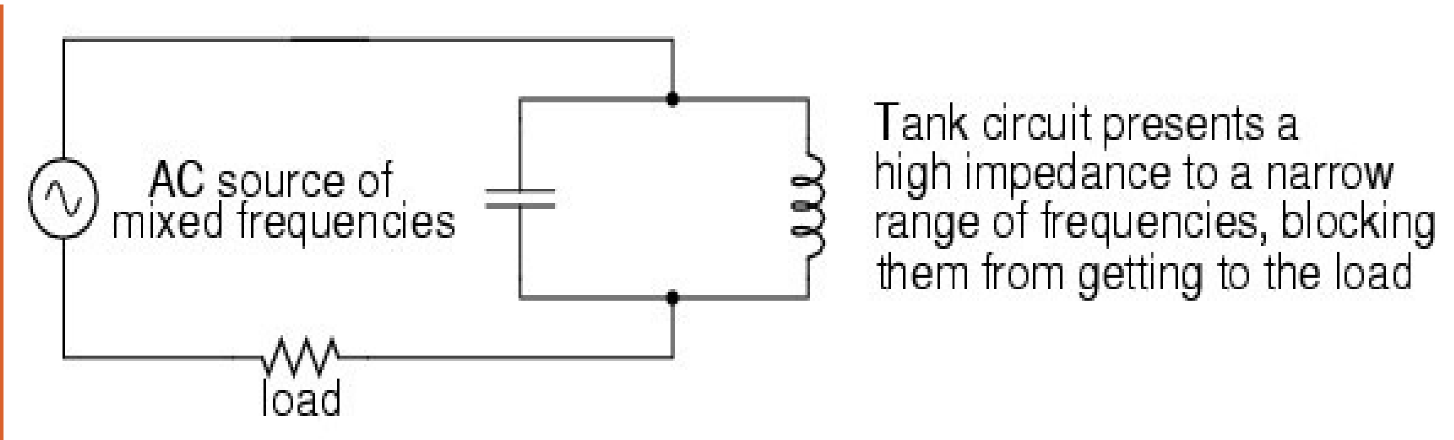


Figure 3: Parallel resonance circuits

RESONANCE FUNCTION

Resonance circuit serves as stable frequency source. In resonance, the frequency set by the tank circuit is safety dependent upon the value of L & C





Resonance circuit serves as filter. Acting as a short of frequency " filter" to strain certain frequencies out of a mix of others.

RLC Series Circuit

Overview

Explain the effect of changing the frequency o:

- A series RLC circuit's reactance changes as you change the voltage source's frequency.
- Its total impedance also changes.
- At low frequencies, $XC > XL$ and the circuit is primarily capacitive.
- At high frequencies, $XL > XC$ and the circuit is primarily inductive.

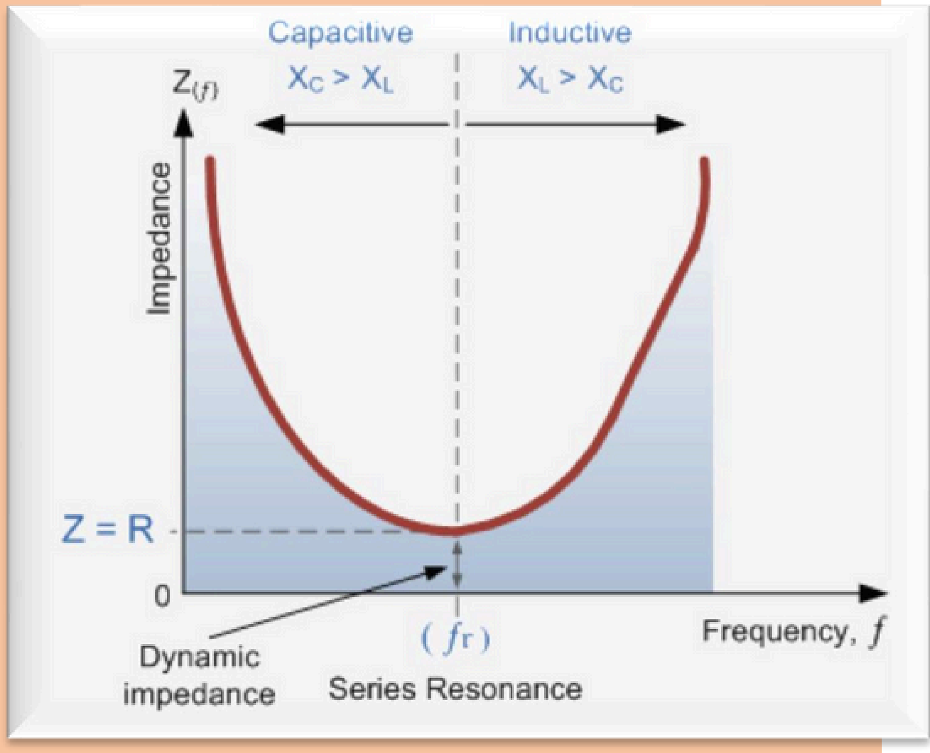


Figure 4: Series resonance graph

RLC Parallel

Circuit

Explain the effect of changing the frequency to:

- Reactance change as you change the voltage source's frequency.
- At low frequencies, $XL < XC$ and the circuit is primarily inductive.
- At high frequencies, $XC < XL$ and the circuit is primarily capacitive.

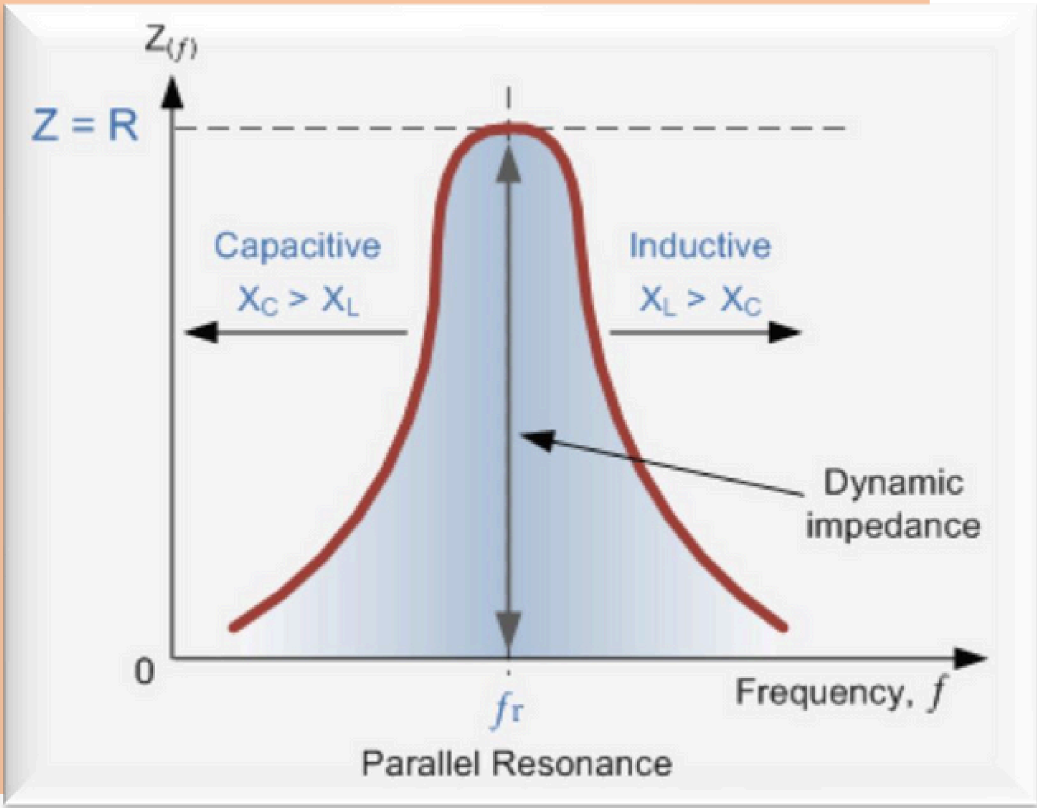
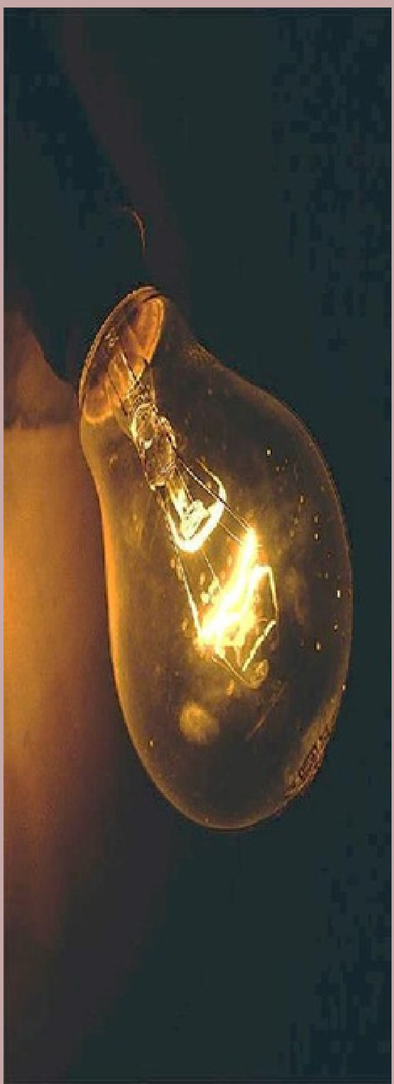


Figure 5: Parallel resonance graph

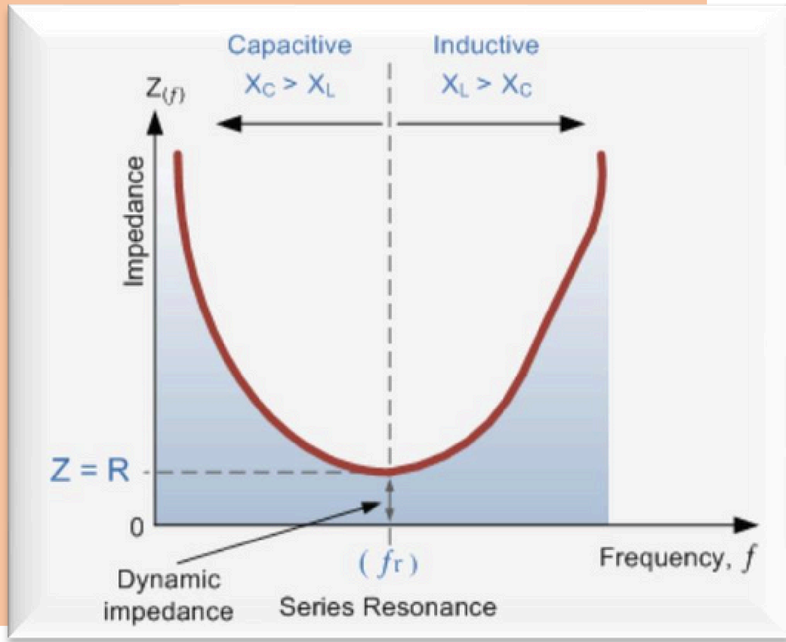
DRAW THE GRAPH OF IMPEDANCE vs FREQUENCY



SERIES

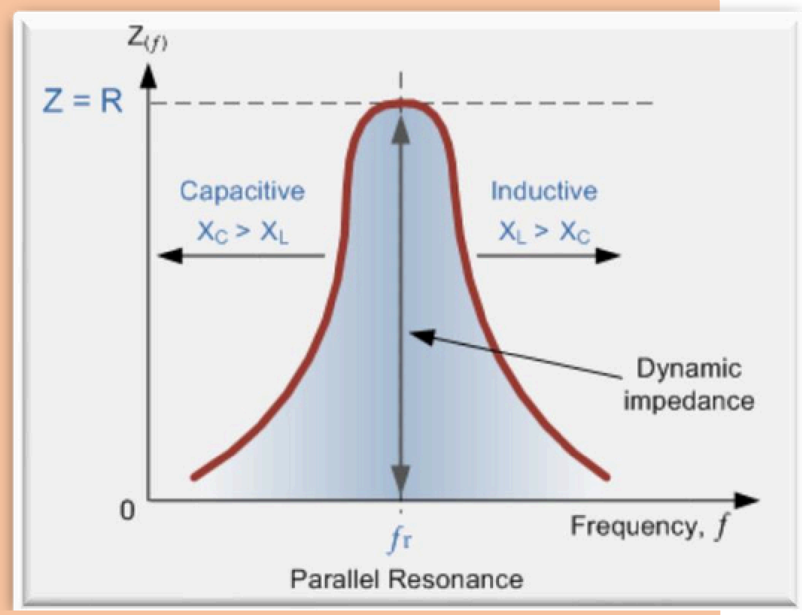
- A series RLC circuit contains both inductive reactance (XL) and capacitive reactance (XC).
- Since XL and XC have opposite phase angles, they tend to cancel each other out and the circuit's total reactance is smaller that either individual reactance:

$$XT < XL \ \& \ XT < XC$$



PARALLEL

- The smaller reactance dominates, since a smaller reactance results in a larger branch current



PRODUCE RESONANT FREQUENCY EQUATION FOR SERIES AND PARALLEL CIRCUITS

Resonance occurs when $XL = XC$ and the imaginary parts of Y become zero. Then in series and parallel:

$$X_L = X_C \Rightarrow 2\pi f L = \frac{1}{2\pi f C}$$

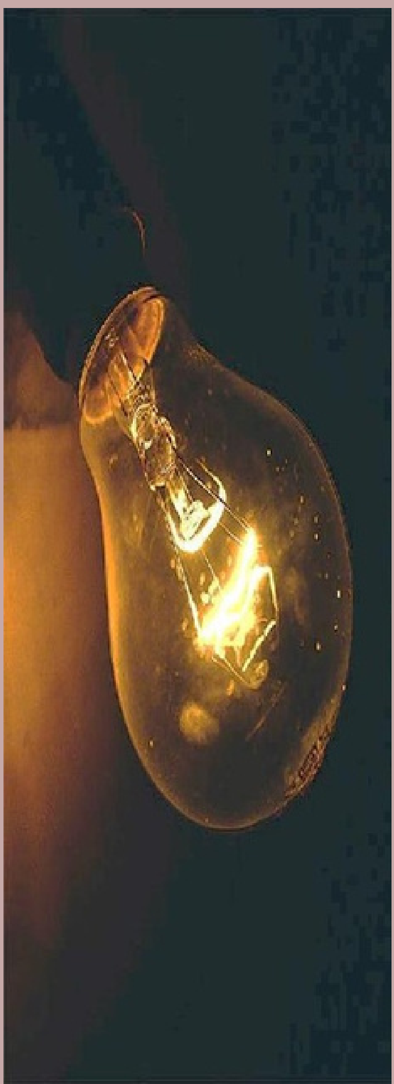
$$f^2 = \frac{1}{2\pi L \; x \; 2\pi C} = \frac{1}{4\pi^2 LC} = \sqrt{\frac{1}{4\pi^2 LC}}$$

$$\therefore f_r = \frac{1}{2\pi\sqrt{LC}}(Hz) \quad or \quad \omega_r = \frac{1}{\sqrt{LC}}(rads)$$

SELF ASSESSMENT

- A series circuit with R = 5 ohms, L = 20 mH and a variable capacitance C has an applied voltage with a frequency of 50 Hz. Find the value of C for series resonance.
- Answer :-
  C = 506.0 μF.

QUALITY FACTOR, Q



A) SERIES CIRCUIT:

Q factor:

- Q is the ratio of power stored to power dissipated in the circuit reactance and resistance.
- Q is the ratio of its resonant frequency to its bandwidth.

$$Q = \frac{X_L}{R} = \frac{2\pi f L}{R}$$

$$if:\ f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$Q = \frac{2\pi\left(\frac{1}{2\pi\sqrt{LC}}\right)L}{R}$$

$$Q = \frac{L}{R\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$Q = \frac{L}{R\sqrt{L\,C}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\therefore\ Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$L \uparrow R \downarrow = Q \uparrow \uparrow$$

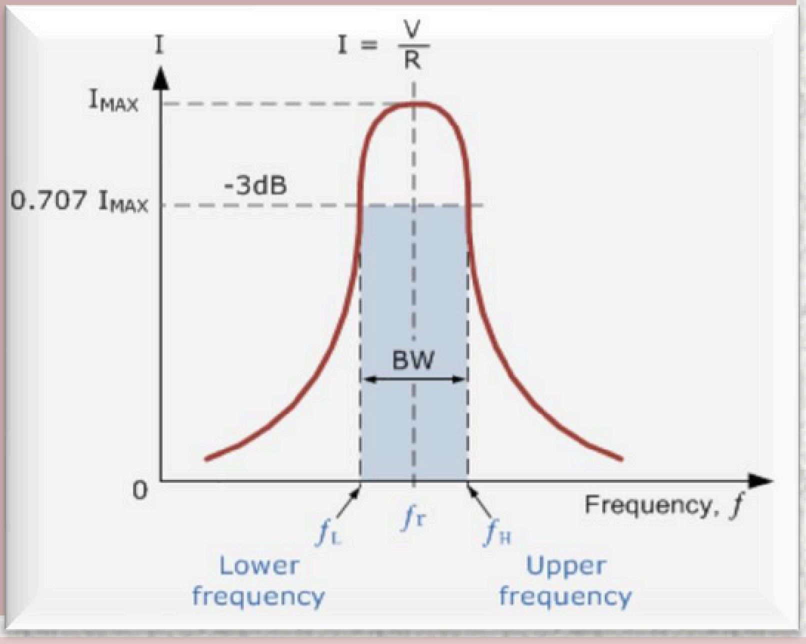
B) PARALLEL CIRCUIT:

Quality factor: the ratio of the circulating branch currents to the supply current.

Quality Factor, $Q = \frac{R}{2\pi f L} = 2\pi f C R = R\sqrt{\frac{C}{L}}$

Frequency Bandwidth, $B = f_2 - f_1$

- The difference between the two half-power frequencies.
- Bandwidth, Δfis measured between the 70.7% amplitude points of series resonant circuit.



$$BW = \frac{f_r}{Q},\quad f_H - f_L\,,\ \frac{R}{L}(rads) \quad or \quad \frac{R}{2\pi L}(Hz)$$

Low cut-off frequency, (f_L) :

$$f_L = f_r - \frac{BW}{2}$$

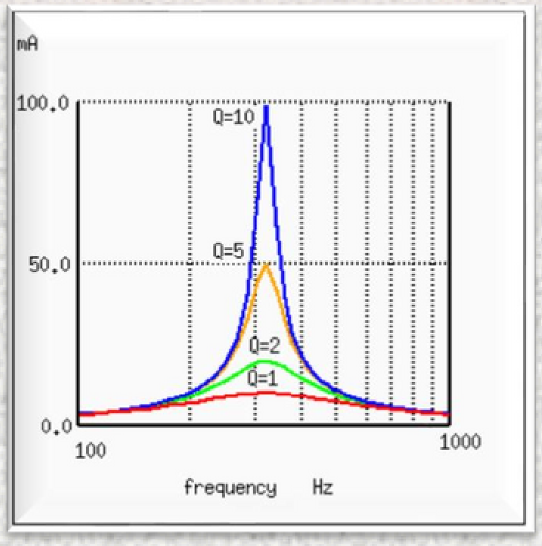
$$\omega_L = -\frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}}$$

Upper cut-off frequency, (f_H) :

$$f_H = f_r + \frac{BW}{2}$$

$$\omega_L = +\frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}}$$

A high Q resonant circuit has a narrow bandwidth as compared to a low Q



$$\text{BW} = \frac{f_c}{Q}$$

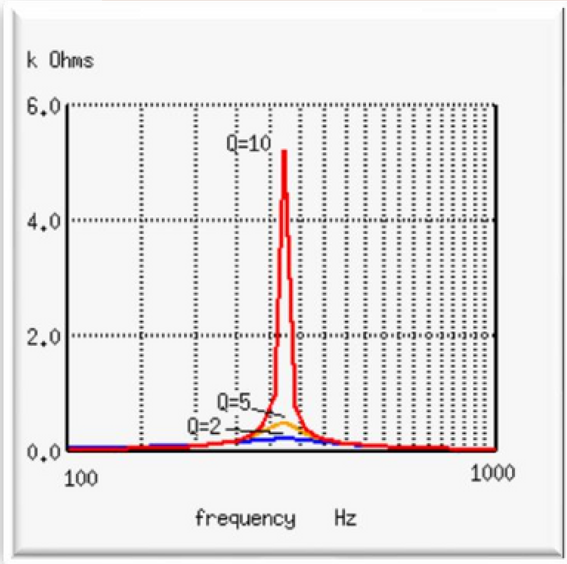
Where :

f_c = Resonant Frequency

Q = Quality Factor

PARALLEL RESONANT CIRCUIT

A low Q due to a high resistance in series with the inductor produces a low peak on a broad response curve for a parallel resonant circuit



THE DISSIPATION FACTOR, D

The ratio of the power loss in a dielectric material to the total power transmitted through the dielectric.

$$D = \frac{energy\,lost}{energy\,stored} = \frac{real\,part\,of\,Z}{-imaginary\,parts\,of\,Z}$$

SUMMARY OF THE CHARACTERUSTICS

CHARACTERISTIC	SERIES CIRCUIT	PARALLEL CIRCUIT
Resonant frequency, f_r	$f_r = \frac{1}{2\pi\sqrt{LC}}(Hz)$ or $\omega_r = \frac{1}{\sqrt{LC}}(rads)$	
Quality factor, Q	$Q = \frac{X_L}{R} = \frac{1}{RX_C} = \frac{1}{R}\sqrt{\frac{L}{C}}$	$Q = \frac{R}{2\pi f L} = 2\pi f CR = R\sqrt{\frac{C}{L}}$
Bandwidth, BW	$BW = \frac{f_r}{Q},\ f_H - f_L,\ \frac{R}{L}(rads)$ or $\frac{R}{2\pi L}(Hz)$	
Half power frequency, f_L & f_H	$f_L = f_r - \frac{BW}{2}$ & $f_H = f_r + \frac{BW}{2}$	

SELT ASSESSMENT 1

A series resonance network consisting of a resistor of 30Ω, a capacitor of 2µF and an inductor of 20mH is connected across a sinusoidal supply voltage which has a constant output of 9 Volts at all frequencies. Calculate, the resonant frequency, the current at resonance, the voltage across the inductor and capacitor at resonance, the quality factor and the bandwidth of the circuit. Also sketch the corresponding current waveform for all frequencies.

ANSWER

1) Resonant Frequency, f_r ; $f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(20m)(2\mu)}} = 796Hz$

2) Circuit current at resonance, I_m ; $I = \frac{V}{R} = \frac{9}{30} = 0.3A$

3) Inductive reactance at resonance, X_L ; $X_L = 2\pi f\mathrm{L} = 2\pi(796)(20\mathrm{m}) = 100\Omega$

4) Voltage across the inductor and the capacitor, V_L, V_C

$$V_L = V_C \;;\; V_L = IX_L = 0.3(100) = 30V$$

5) Quality Factor, Q ; $Q = \frac{X_L}{R} = \frac{100}{30} = 3.33$

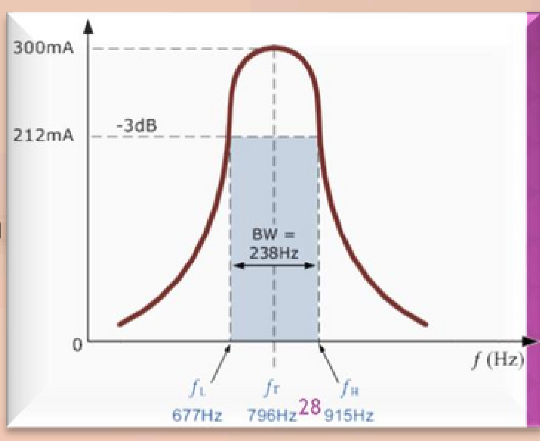
6) Bandwidth, BW ; $BW = \frac{f_r}{Q} = \frac{796}{3.33} = 238\ Hz$

7) The upper and lower frequency point, f_L & f_H

$$\text{lower frequency, } f_L = f_r - \frac{BW}{2} = 796 - \frac{238}{2} = 677\ Hz$$

$$\text{upper frequency, } f_H = f_r + \frac{BW}{2} = 796 + \frac{238}{2} = 915\ Hz$$

Figure: Current waveform



SELT ASSESSMENT 2

A series circuit consists of a resistance of 4Ω, an inductance of 500mH and a variable capacitance connected across a 100V, 50Hz supply.
Calculate the capacitance require to give series resonance and the voltages generated across both the inductor and the capacitor.

Answer:

Resonant frequency, f_r

$$X_L = 2\pi f L = 2\pi(50)(500m) = 157.1\Omega$$

$$\text{At resonance} ; X_C = X_L = 157.1\Omega$$

$$\therefore C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(50)(157.1)} = 20.3\mu F$$

Voltage across the inductor and the capacitor, V_L, V_C

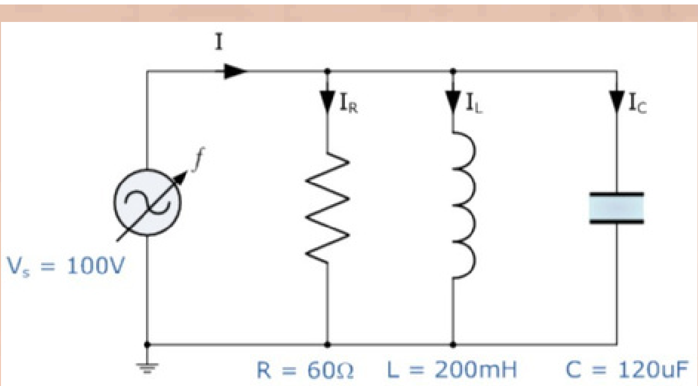
$$I = \frac{V}{R} = \frac{100}{4} = 25\ A$$

$$\text{at resonance} ; V_L = V_C$$

$$V_L = IX_L = 25 * 157.1 = 3927.5\ V$$

SELT ASSESSMENT 3

A parallel resonance network consisting of a resistor of 60Ω, a capacitor of 120uf and an inductor of 200mh is connected across a sinusoidal supply voltage which has a constant output of 100 volts at all frequencies. Calculate, the resonant frequency, the quality factor and the bandwidth of the circuit, the circuit current at resonance and current magnification.



Answer:

1) Resonant Frequency, f_r

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(200m)(120\mu)}} = 32.5Hz$$

2) Inductive reactance at resonance, X_L

$$X_L = 2\pi f\mathrm{L} = 2\pi(32.5)(200\mathrm{m}) = 40.8\Omega$$

3) Quality Factor, Q

$$Q = \frac{R}{X_L} = \frac{R}{2\pi fL} = \frac{60}{40.8} = 1.47$$

4) Bandwidth, BW

$$BW = \frac{f_r}{Q} = \frac{32.5}{1.47} = 22\ Hz$$

5) The upper and lower frequency point, f_L & f_H

$$\text{lower frequency, } f_L = f_r - \frac{BW}{2} = 32.5 - \frac{22}{2} = 21.5\ Hz$$

$$\text{upper frequency, } f_H = f_r + \frac{BW}{2} = 32.5 + \frac{22}{2} = 43.5\ Hz$$

6) Circuit current at resonance, I_T

At resonance the dynamic impedance of the circuit is equal to R

$$I_T = I_R = \frac{V}{R} = \frac{100}{60} = 1.67\ A$$

7) Current Magnification, I_{mag}

$$I_{mag} = Q * I_T = 1.47 * 1.67 = 2.45\ \mathrm{A}$$

we can check this value by calculating the current flowing through the inductor (or capacitor) at resonance.

$$I_L = \frac{V}{X_L} = \frac{V}{2\pi fL} = \frac{100}{2\pi(32.5)(200m)} = 2.45\ A$$

SUMMARY

- For resonance to occur in any circuit it must have at least one inductor and one capacitor.
- Resonance is the result of oscillations in a circuit as stored energy is passed from the inductor to the capacitor.
- Resonance occurs when $VL = VC$ and the imaginary part of the transfer function is zero.
- At resonance the impedance of the circuit is equal to the resistance value as Z = R.

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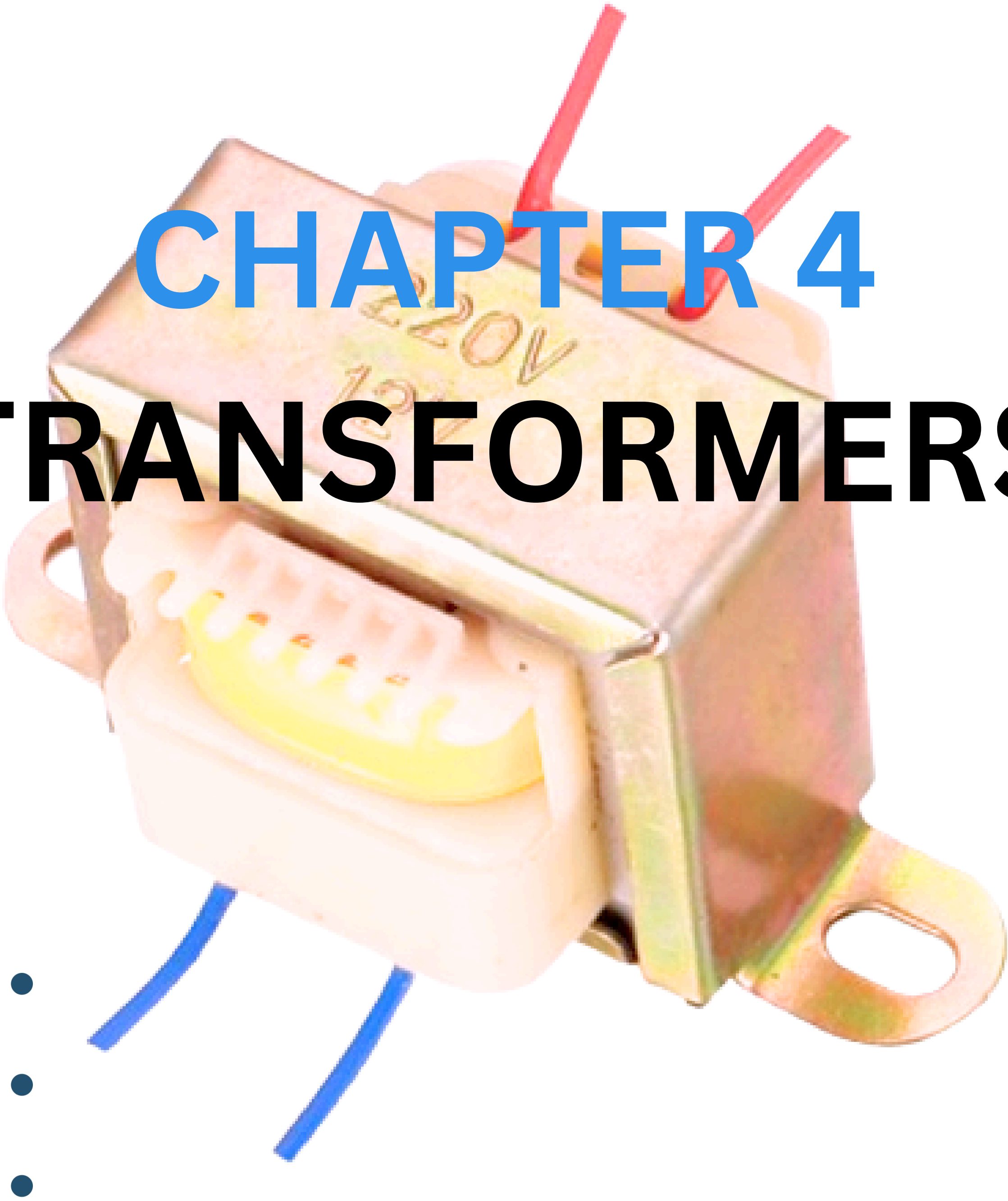
CHAPTER 4
TRANSFORMERS



COURSE LEARNING OUTCOME

1. Apply the concept and principle in solving problems of electrical circuits using the appropriate AC electrical laws and theorem.
2. Construct of an AC electrical circuit and measured related electrical parameter using appropriate electrical equipment's.
3. Demonstrate ability to work in team to complete assigned tasks within the stipulated time frame.

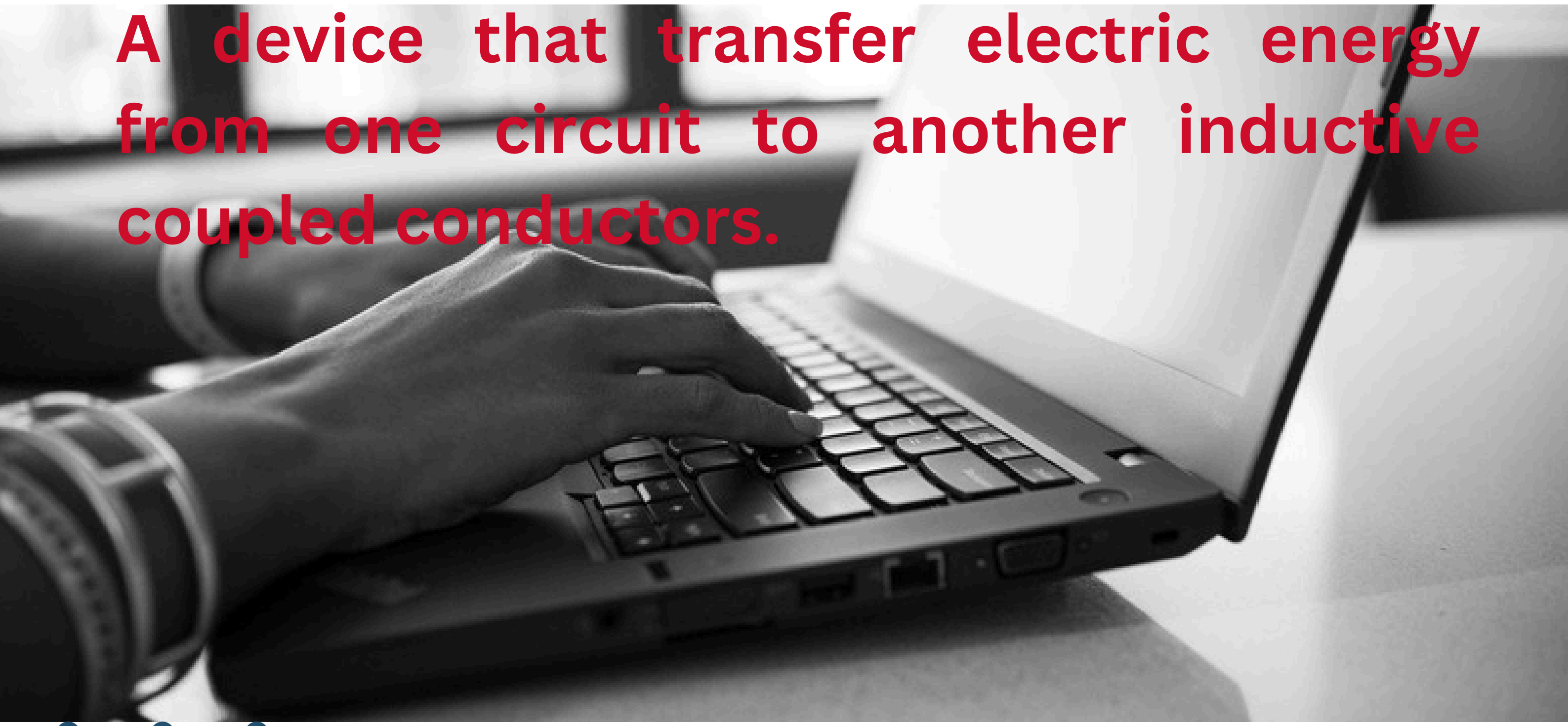
OBJECTIVE

1. Remember several types of transformers.
2. Remember a non-ideal transformer.
3. Understand the construction and the operation of a transformer.
4. Apply transformer increases and decreases voltage.
5. Apply the understanding of the effect of a resistive load across the secondary winding.

INTRODUCTION

TRANSFORMERS:

A device that transfer electric energy from one circuit to another inductive coupled conductors.

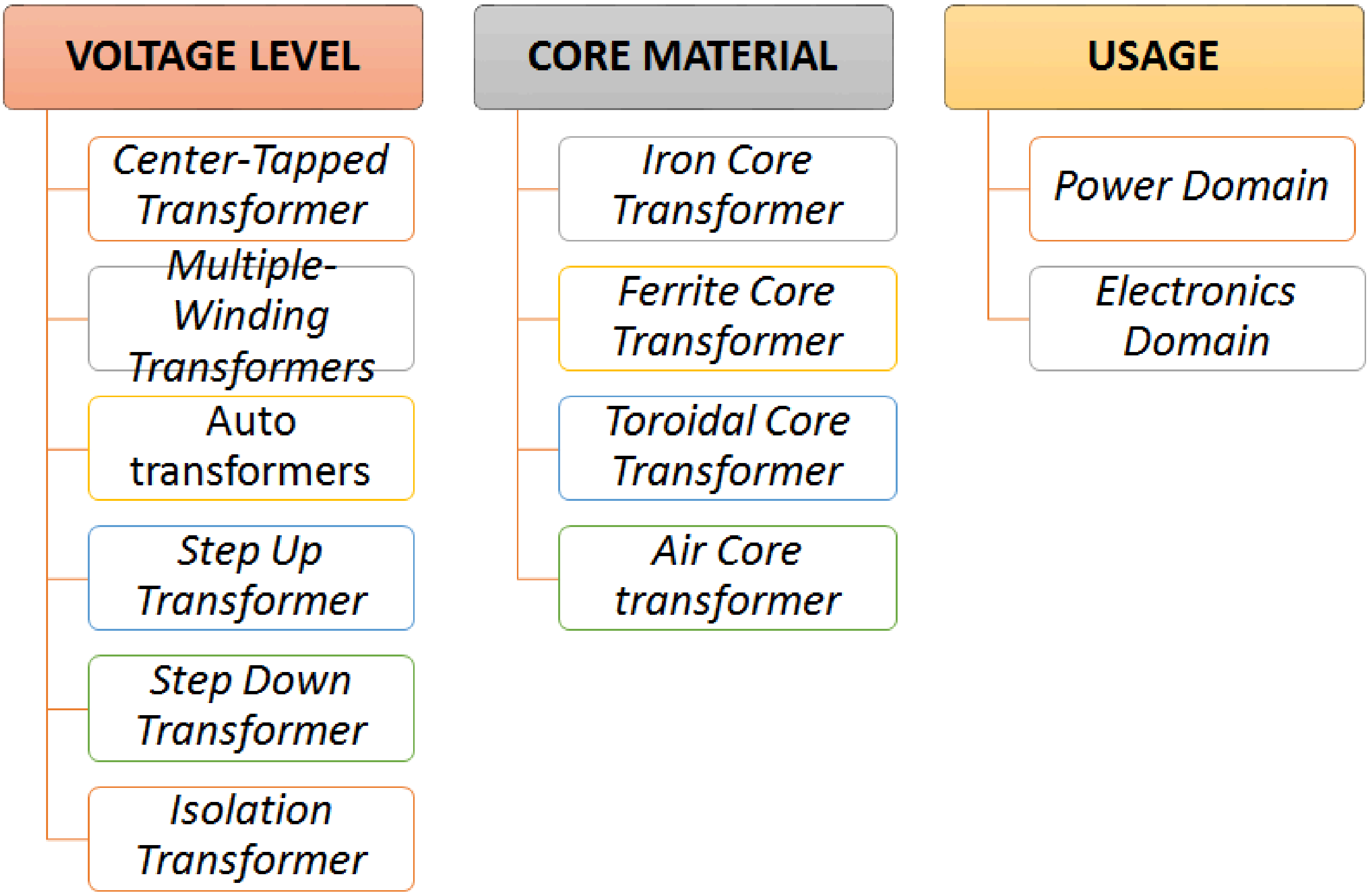


INTRODUCTION

PRINCIPLES OF TRANSFORMERS

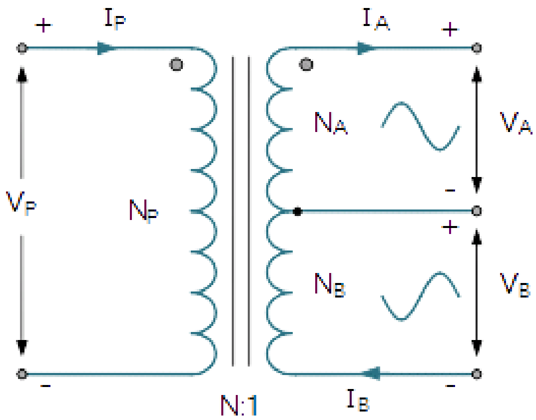
- An electric current can produce a magnetic field (electromagnetism).
- A changing magnetic field within a coil of wire induces a voltage across the ends of the coil (electromagnetic induction).
- Changing the current in the primary coil changes the magnetic flux that is developed.
- The changing magnetic flux induces a voltage in the secondary coil.

DIFFERENT TYPES OF TRANSFORMERS AND THEIR APPLICATIONS

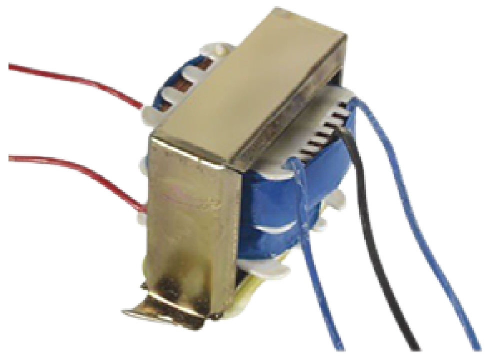


CENTER-TAPPED TRANSFORMER

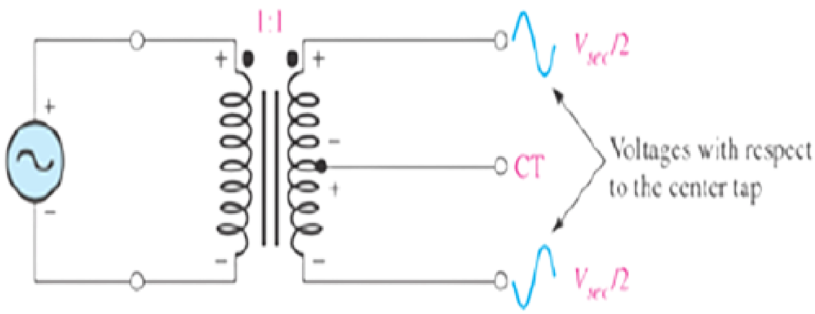
- **The center tap (CT) transformer is equivalent to two secondary windings with half the voltage across each.**
- **Center-tapped windings are used for rectifier supplies and impedance-matching transformers.**



SYMBOL



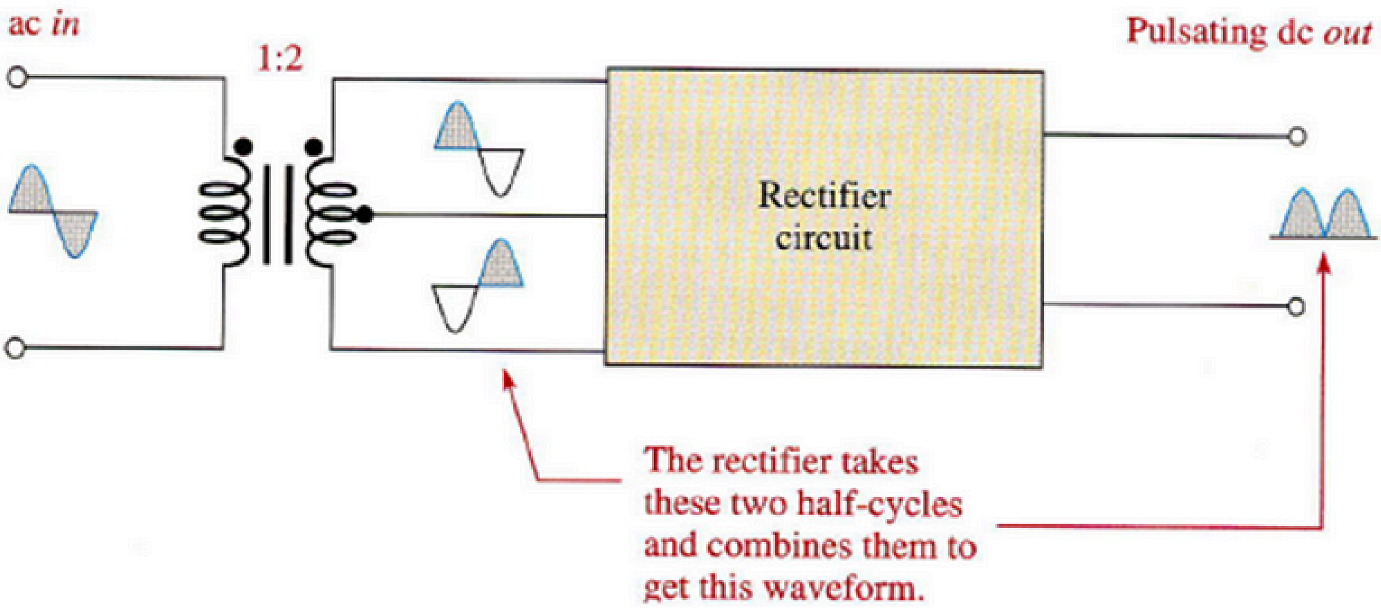
PHYSICAL VIEW



Output voltages with respect to the center tap are 180° out of phase with each other and are one-half the magnitude of the secondary voltage.

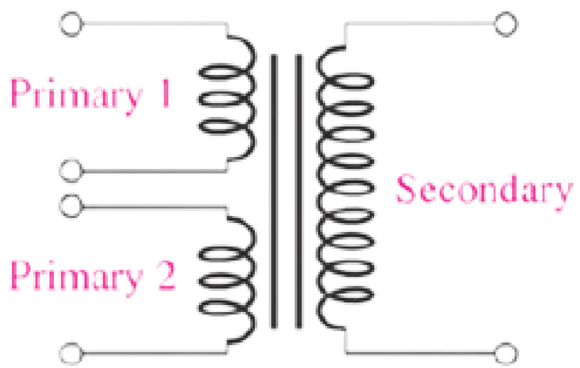
CENTER-TAPPED TRANSFORMER

- **Application of a center-tapped transformer in ac-to-dc conversion.**

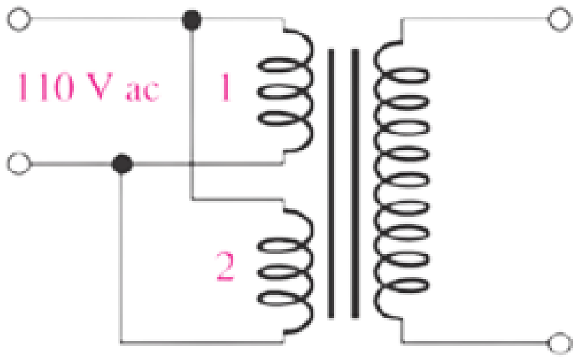


MULTIPLE-WINDING TRANSFORMER

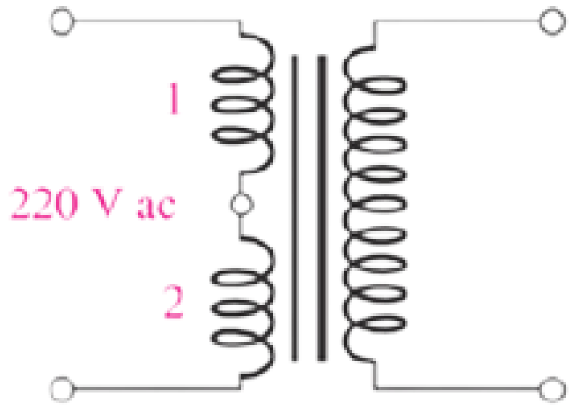
- **Multiple-winding transformers have more than one winding on a common core.**
- **They are used to operate on, or provide different operating voltages.**



Two Primary Windings



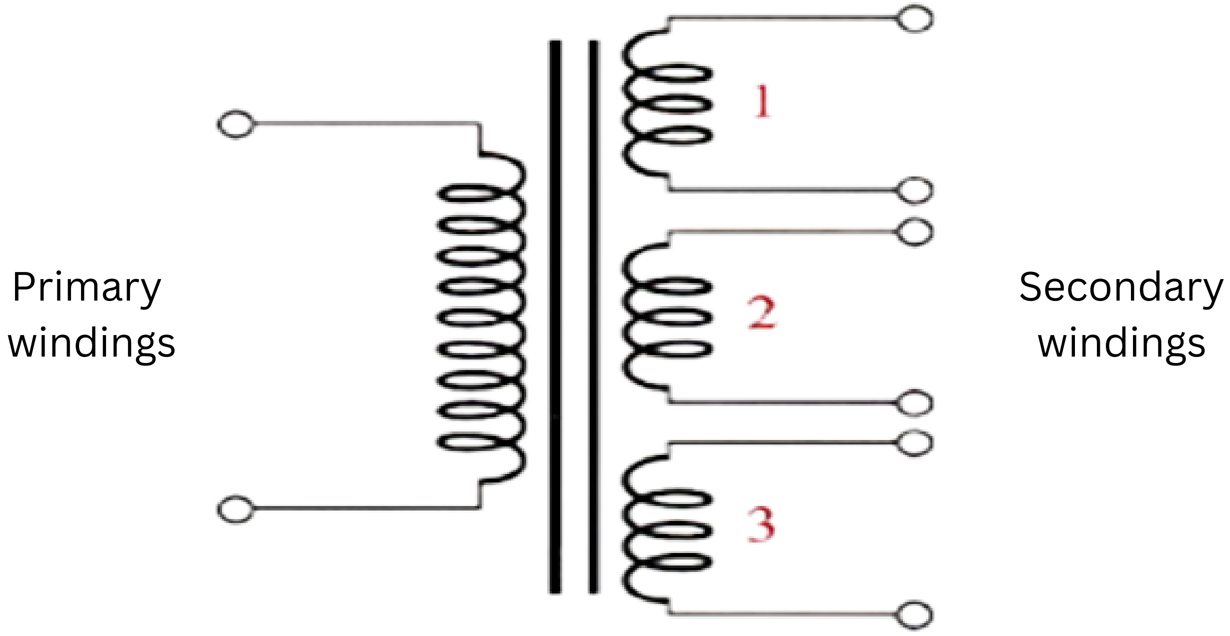
Primary windings in parallel for 110V ac operation



Primary windings in series for 220V ac operation

MULTIPLE-SECONDARY TRANSFORMER

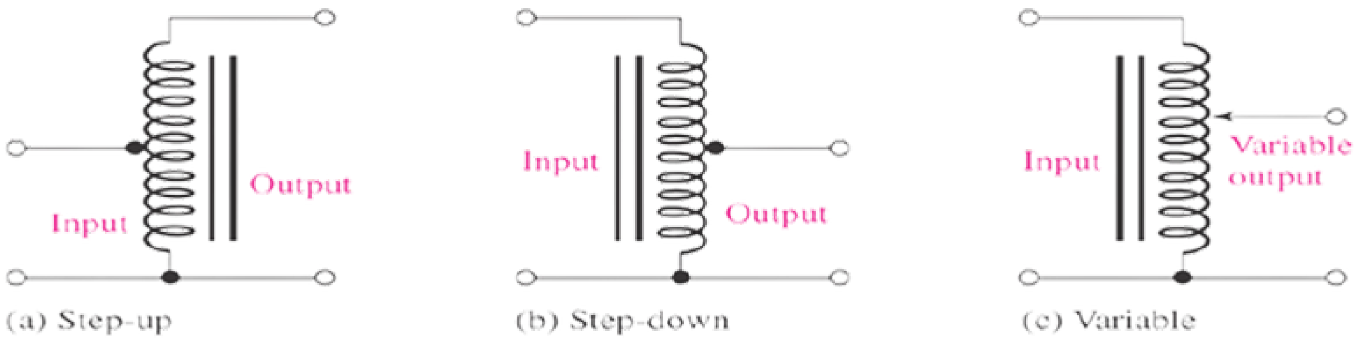
- There are transformers with multiple secondary windings.
- They are used to provide a variety of voltages from a single source.
- Common used in power distribution systems, electric motors, and electronic devices.



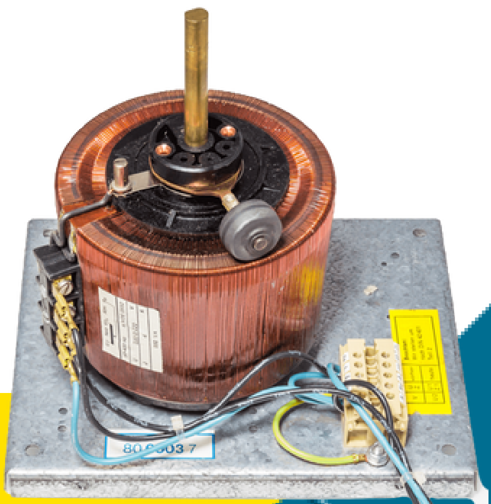
AUTO-TRANSFORMERS

- In an autotransformer, one winding serves as both the primary and the secondary.
- The winding is tapped at the proper points to achieve the desired turns ratio for stepping up or stepping down the voltage.
- Auto transformers differ from conventional transformers in that there is no electrical isolation between the primary and the secondary because both are on one winding

Schamatic Diagram



Physical View
Autotransfomer



AUTO-TRANSFORMERS

Advantages :

- A saving in cost since less copper is needed.
- Less volume, hence less weight.
- A higher efficiency, resulting from lower I2R losses.
- A continuously variable output voltage is achievable if a sliding contact is used.
- A smaller percentage voltage regulation.

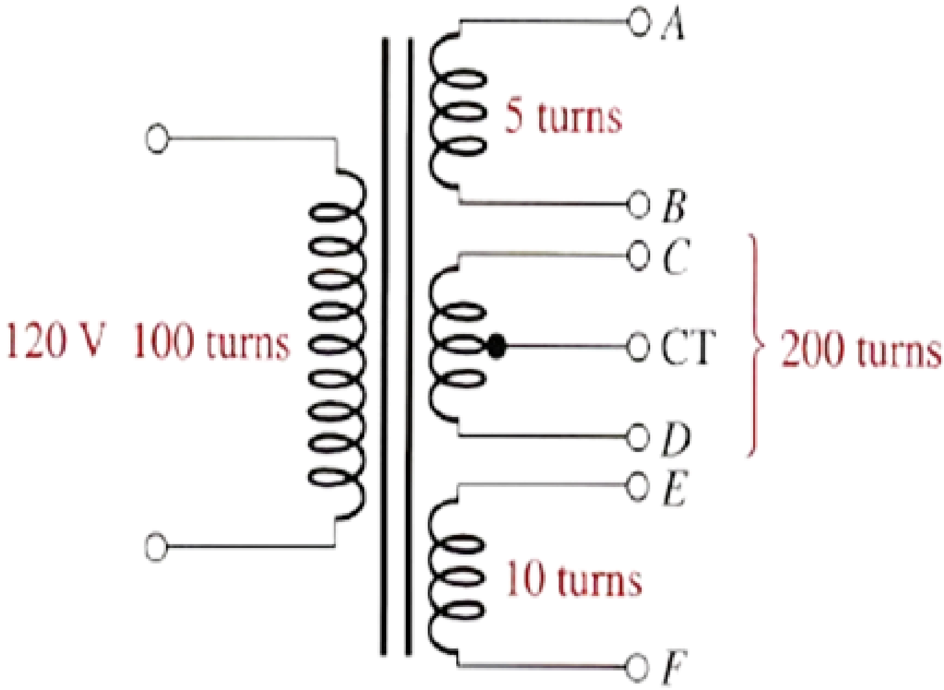
Disadvantages :

- The primary and secondary windings are not electrically separate, hence if an open-circuit occurs in the secondary winding the full primary voltage appears across the secondary.

EXAMPLE 1

QUESTION:

The transformer shown in figure below has the numbers of turns indicated. One of the secondaries is also center tapped. If 120 V ac are connected to the primary, determine each secondary voltage and the voltages with respect to the center tap (CT) on the middle secondary.



EXAMPLE 1

Answer :

$$V_{AB} = \frac{N_{sec}}{N_{pri}} \times V_{pri} = \frac{5}{100} \times 120\ V = 6\ V$$

$$V_{CD} = \frac{N_{sec}}{N_{pri}} \times V_{pri} = \frac{200}{100} \times 120\ V = 240\ V$$

$$V_{(CT)C} = V_{(CT)D} = \frac{240\ V}{2} = 120\ V$$

$$V_{EF} = \frac{N_{sec}}{N_{pri}} \times V_{pri} = \frac{10}{100} \times 120\ V = 12\ V$$

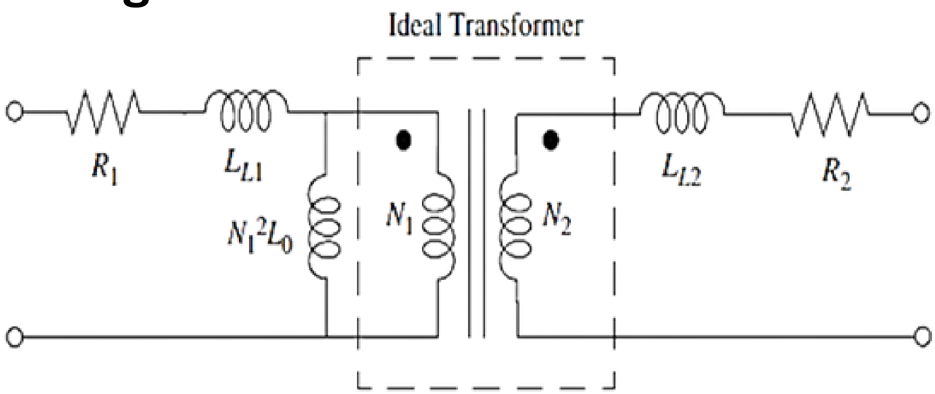
NON-IDEAL TRANSFORMER

Introduction

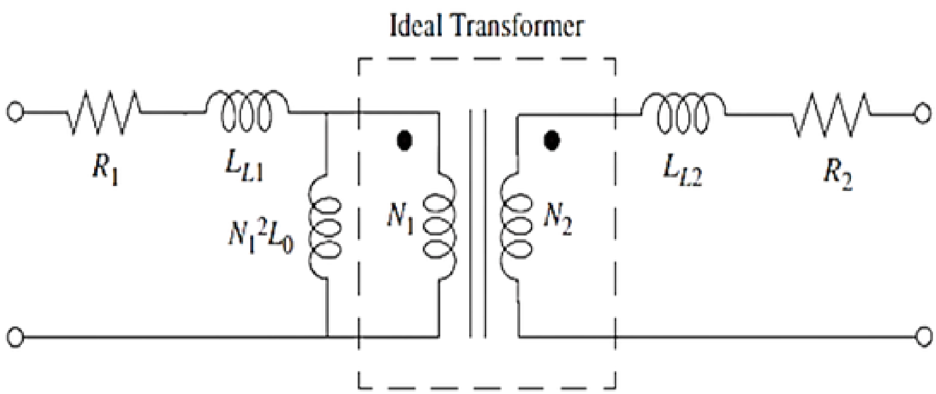
Real transformers are never ideal, and at times their non-idealities are important.

Such non-idealities include the resistance of the coils, the leakage inductance of the coils and the magnetizing inductance of the core.

These non-idealities can be added to the model shown in below figure



NON-IDEAL TRANSFORMER CHARACTERISTICS



- **R1, R2 are winding resistances of the Primary and secondary of the Transformer**
- **Leakage fluxes fl1, fl2 exist for Primary and Secondary windings respectively.**
- **Permeability of core is not infinite. Therefore mmf is needed to establish mutual flux fm in the core.**
- **Core Losses are included.**
- **The core saturates.**

NON-IDEAL TRANSFORMER CHARACTERISTICS

The losses which occur in a transformer on load can be divided into two groups:

a)Copper losses
b)Core losses

NON-IDEAL TRANSFORMER CHARACTERISTICS

COPPER LOSSES

Copper losses are variable and result in a heating of the conductors, due to the fact that they possess resistance.
If R1 and R2 are the primary and secondary winding resistances than the total copper loss is $I_1^2R_1 + I_2^2R_2$

$$\boxed{P_{cop} = (I_1)^2R_1 + (I_2)^2R_2}$$

CORE LOSSES

Core losses are constant for a given value of frequency and flux density and are of two types:

- **hysteresis loss**
- **eddy current loss.**

NON-IDEAL TRANSFORMER CHARACTERISTICS

CORE LOSSES

Hysteresis Loss

The heating of the core as a result of the internal molecular structure reversals which occur as the magnetic flux alternates.
The loss is proportional to the area of the hysteresis loop and thus low loss nickel iron alloys are used for the core since their hysteresis loop have small areas.

NON-IDEAL TRANSFORMER CHARACTERISTICS

CORE LOSSES

Eddy Current Loss

- The heating of the core due to e.m.f. ‘s being induced not only in the transformer windings but also in the core.
- These induced e.m.f.’s set up circulating currents called eddy currents.
- Owing to the low resistance of the core, eddy currents can be quite considerable and can cause a large power loss and excessive heating of the core.
- Eddy current losses can be reduced by increasing the resistivity of the core (i.e splitting it into layers or leaves) when very thin layers of insulating material; can be inserted between each pair of lamination's
- This increases the resistance of the eddy current path, and reduces the value of the eddy current

Total Losses in Transfomer

$$\boldsymbol{P_T = copper\ losses + core\ losses}$$

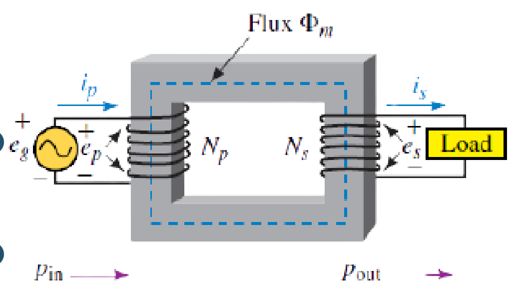
$$P_T = I_1^2 R_1 + I_2^2 R_2 + core\ losses$$

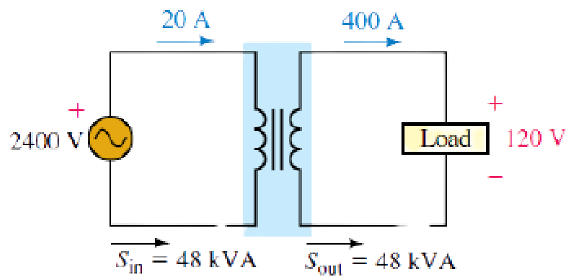
TRANSFORMER

POWER RATING OF A TRANSFORMER

- A transformer is usually described in ratings (volts) and sometimes in apparent power (kVA) in which the transformer can function without overheating.
- Referring to figure below, a transformer's rating can be obtained by either using EpIp or EsIs where Ip is the primary current and Is is the secondary current on full load.
- The rated current can be obtained from the transformers ratings;

$VA = E_pI_p = E_sI_s$ thus $I_p = \dfrac{VA}{E_p}$ and $I_s = \dfrac{VA}{E_s}$





POWER RATING OF A TRANSFORMER

- **The primary and secondary full-load rating transformer currents usually are not given but can be calculated from the rated VA or kVA as follows:**

Single-phase

$$Full\ load\ current = \frac{VA}{Voltage}$$

$$Full\ load\ current = \frac{kVA \times 1000}{Voltage}$$

Three-phase

$$Full\ load\ current = \frac{kVA \times 1000}{\sqrt{3} \times Voltage}$$

EFFICIENCY OF A TRANSFORMER

- **Efficiency is the ratio of output power to input power. It is usually stated in percentage.**

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power}} = \frac{\text{input power} - \text{losses}}{\text{input power}}$$

$$\eta = 1 - \frac{\text{losses}}{\text{input power}}$$

- **Large power transformers are exceptionally efficient, around a figure of 98% to 99%.**
- **Smaller transformers are around 95%.**

EFFICIENCY OF A TRANSFORMER

For Full Load

$$Output\ power = V_2 I_2 \cos \emptyset_2 = S \cos \theta$$
$$Total\ losses = copper\ losses + iron\ losses$$

For Half Load

$$Output\ power = (V_2 I_2 \cos \emptyset_2) * 0.5 = (S \cos \theta) * 0.5$$
$$Total\ losses = (0.5^2 *\ copper\ losses) + iron\ losses$$
$$input\ power = output\ power + losses$$

Maximum Efficiency

A transformer achieves maximum efficiency when the copper loss variable is equal to the constant iron or core loss.

$$\text{Copper loss} = \text{Iron loss}$$
$$(I_1)^2 R_1 + (I_2)^2 R_2 = V_0 I_0 \cos \theta_0$$

EXAMPLE 2

A certain type of transformer has a primary current of 5 A and a primary voltage of 4800 V. The secondary current is 90 A, and the secondary voltage is 240 V. Determine the efficiency of this transformer.

Answer:

The input power;

$$P_{in} = I_{Pri}V_{Pri}$$
$$P_{in} = (5\ A)(4800\ V)$$
$$P_{in} = 24\ kVA$$

The output power;

$$P_{sec} = I_{sec}V_{sec}$$
$$P_{sec} = (90\ A)(240\ V)$$
$$P_{sec} = 21.6\ kVA$$

The efficiency is;

$$\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{21.6\ kVA}{24\ kVA} \times 100 = 90\%$$

EXAMPLE 3

A certain transformer has a rating of 10 kVA. If the secondary voltage is 250 V how much load current can the transformer handle?

Answer:

$$I_L = \frac{P_{sec}}{V_{sec}} = \frac{10k\,VA}{250\,V} = 40\ A$$

EXAMPLE 4

A 400kVA transformer has a primary winding resistance of 0.5Ω and a secondary winding resistance of 0.001Ω. The iron loss is 2.5kW and the primary and secondary voltages are 5kV and 320V respectively. If the power factor of the load is 0.85, determine the efficiency of the transformer

(a)on full load
(b)on half load

Answer:

a) Given,

Rating, S = 400kVA
$R1 = 0.5\Omega$
$R2 = 0.001\Omega$
Piron = 2.5kW
Ep = 5kV
Es = 320V
$pf = \cos\theta = 0.85$

Answer:

Find Ip & Is	Ip = 80A, Is = 1250A
Find total copper loss	4762.5W
Find total loss (iron+copper)	7.2625kW
Find output power on full load	340kW
Find input power	347.2625kW
Find Efficiency	97.91%

b) Since the copper loss varies as the square of the current, then

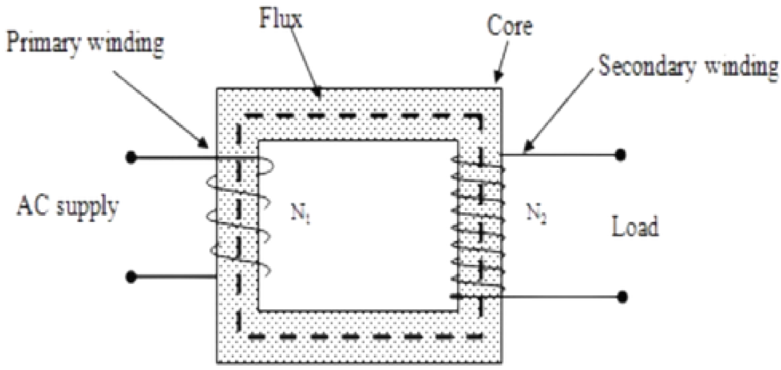
$$\text{total copper loss on half load} = 4762.5 \times \left(\frac{1}{2}\right)^2 = 1190.63\text{W}$$

Find total copper loss	1190.63W
Find total loss (iron+copper)	3.69kW
Find output power on full load	340kW
Find input power	343.69kW
Find Efficiency	99%

THE CONSTRUCTION AND THE OPERATION OF A TRANSFORMERS

PARTS OF BASIC TRANSFORMER

- A transformer consists of two windings connected by a magnetic core. One winding is connected to a power supply and the other to a load. A circuit diagram symbol for a transformer is shown in figure 1



- One coil is called primary winding and the other coil is called secondary winding as indicated. For standard operation, the source voltage is applied to the primary winding and a load is connected to the secondary winding.

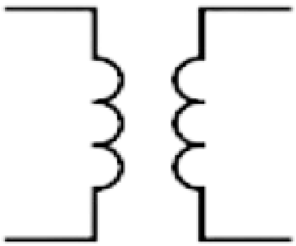
- The primary winding is the input winding, and the secondary winding is the output winding. It is common to refer to the side of the circuit that has the source voltage as the primary, and the side that has the induced voltage as the secondary.

- The winding of a transformer are formed around the core. The core provides both a physical structure for placement of the winding and magnetic path so that the magnetic flux is concentrated close to the coils.

THE CONSTRUCTION AND THE OPERATION OF A TRANSFORMERS

PARTS OF BASIC TRANSFORMER

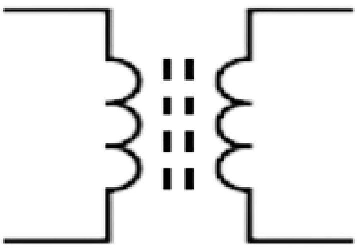
- Three general categories of core material are air, ferrite and iron. The schematic symbol for each type is:



Air Core

- Air-core and ferrite-core transformers generally are used for high frequency applications such as in radio circuit.
- The wire is typically covered by a varnish-type coating to prevent the windings from shorting together. The amount of magnetic coupling between the primary winding and secondary winding is set by the type of core material and by the relative positions of the windings.

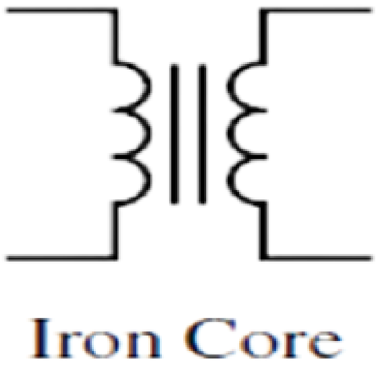
- Ferrite (a magnetic material made from powdered iron oxide) greatly increases coupling between coils (compared with air) while maintaining low losses.



Ferrite Core

THE CONSTRUCTION AND THE OPERATION OF A TRANSFORMERS

PARTS OF BASIC TRANSFORMER



- **Iron core** transformers are generally used for low frequency applications such as audio- and power-frequency applications.
- Iron (actually a special steel called transformer steel) is used for cores because it increases the coupling between coils by providing an easy path for magnetic flux.

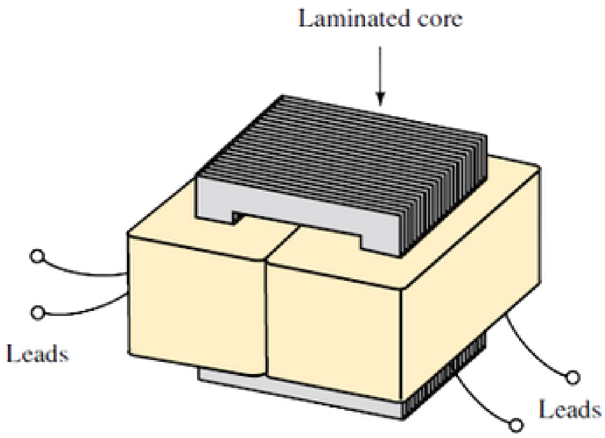
Figure : Example of iron-core transformers

THE CONSTRUCTION AND THE OPERATION OF A TRANSFORMERS

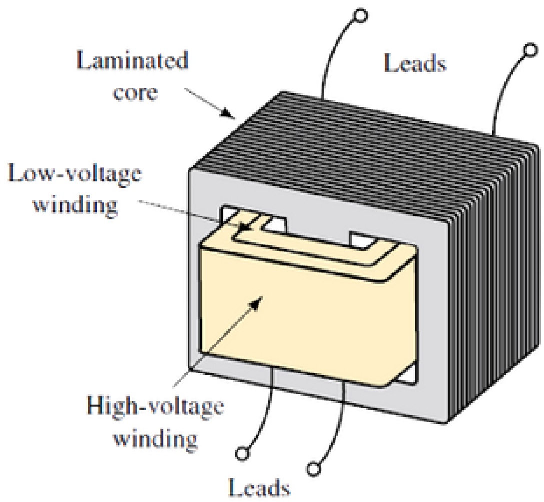
PARTS OF BASIC TRANSFORMER

Two basic types of iron-core construction are used:-

- **Core type**
- **Shell type.**



Figure(a): Core Type



Figure(b): Shell Type

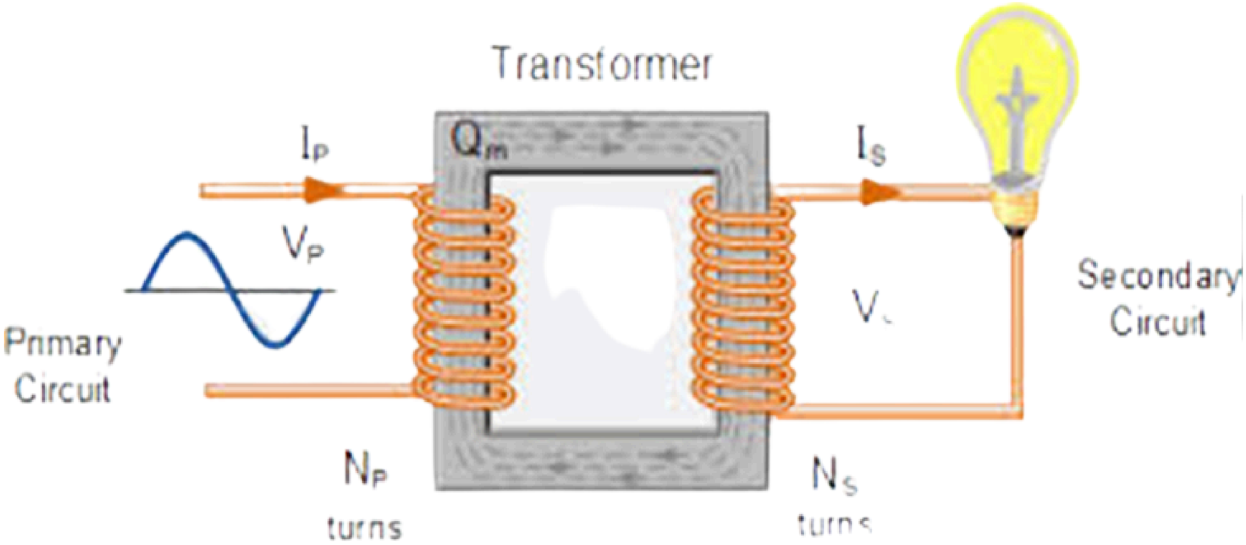
In both cases, cores are made from lamination's of sheet steel, insulated from each other by thin coatings of ceramic or other material to help minimize eddy current losses

In the core-type construction, shown figure(a), the windings are on separate legs of the laminated core.

In the shell-type construction, shown figure(b), both windings are on the same leg. The shell type can produce higher magnetic fluxes in the core, resulting in the need for fewer turns.

THE CONSTRUCTION AND THE OPERATION OF A TRANSFORMERS

PRIMARY WINDING AND SECONDARY WINDING



TURN RATIO

A transformer parameter that useful in understanding how a transformer operates is the turn ratio.
The turns ratio of a transformer is defined as the number of turns on its secondary divided by the number of turns on its primary.

$$n = \frac{N_2}{N_1} = \frac{secondary\ turns}{primary\ turns}$$

$$N_1 : N_2$$

EXAMPLE 5

A certain transformer used in radar system has a primary winding with 100 turns and a secondary winding with 400 turns. Determine the turns ratio.

Given, $N_1 = 100$
$N_2 = 400$

$$\eta = \frac{N_2}{N_1} = \frac{400}{100} = 4$$

So turns ratio can be expressed as 1 : 4

EXAMPLE 6

A certain transformer has a turns ratio of 10, if N1 = 500, determine N2.

$$n = \frac{N_2}{N_1}$$

$$10 = \frac{N_2}{100}$$

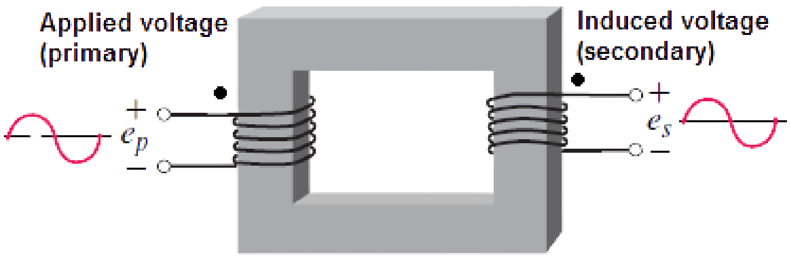
$$10 * 500 = N_2$$

$$N_2 = 5000\ turns$$

THE CONSTRUCTION AND THE OPERATION OF A TRANSFORMERS

DIRECTION OF WINDING

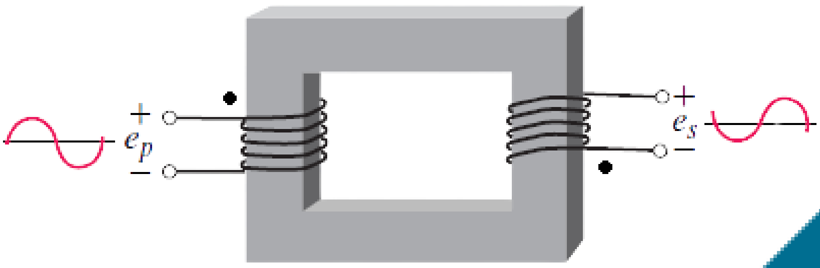
- The important transformer parameter is the direction in which the windings are placed around the core.
- As illustrate below, the direction of the windings determines the polarity of the voltage across the secondary winding (secondary voltage) with respect to the voltage across the primary winding (primary voltage).



(a) 0° phase shift

The primary and secondary voltages are in phase when the windings are in the same effective direction around the magnetic path.

The primary and secondary voltages are 180 degree out of phase when the windings are in the opposite direction



(b) 180° phase shift

THE CONSTRUCTION AND THE OPERATION OF A TRANSFORMERS

INCREASES AND DECREASES VOLTAGE USING TRANSFORMER

Step Up Transformer

- A transformer in which the secondary voltage (V2) is greater than the primary voltage (V1) is called a step-up transformer.
- The ratio of secondary voltage (V2) to primary voltage (V1) is equal to the ratio of the number of turns in the secondary winding (N2) to the number of turns in the primary winding (N1)

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

- Recall that N2/N1 defines the turn ratio, η. Therefore, the relationship , V2 can be expressed as

$$V_2 = \eta V_1$$

THE CONSTRUCTION AND THE OPERATION OF A TRANSFORMERS

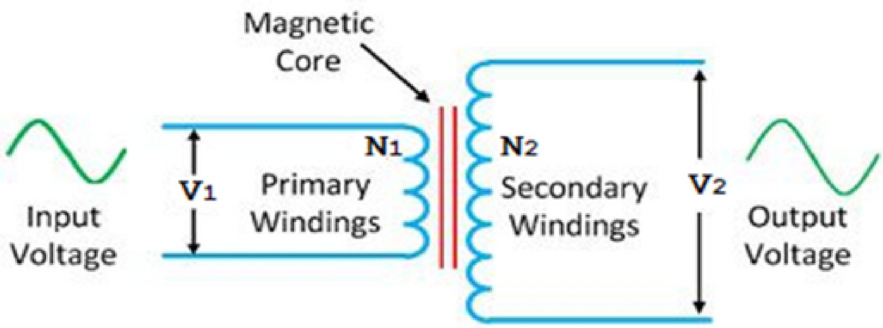
INCREASES AND DECREASES VOLTAGE USING TRANSFORMER

Step Up Transformer

- The secondary voltage is equal to the turns ratio times the primary voltage. This condition assumes that the coefficient of coupling is 1, and a good iron-core transformer approaches this value.

- The turns ratio for a step-up transformer is always greater than 1 because the number of turns in the secondary winding (N2) is always greater than the number of turns in the primary winding (N1)

- Considered a step-up transformer shown in the figure. The V1 and V2 are the voltages, and N1 and N2 are the number of turns on the primary and secondary winding of the transformer.



$$\frac{V_{pri}}{V_{sec}} = \frac{N_{pri}}{N_{sec}}$$

THE CONSTRUCTION AND THE OPERATION OF A TRANSFORMERS

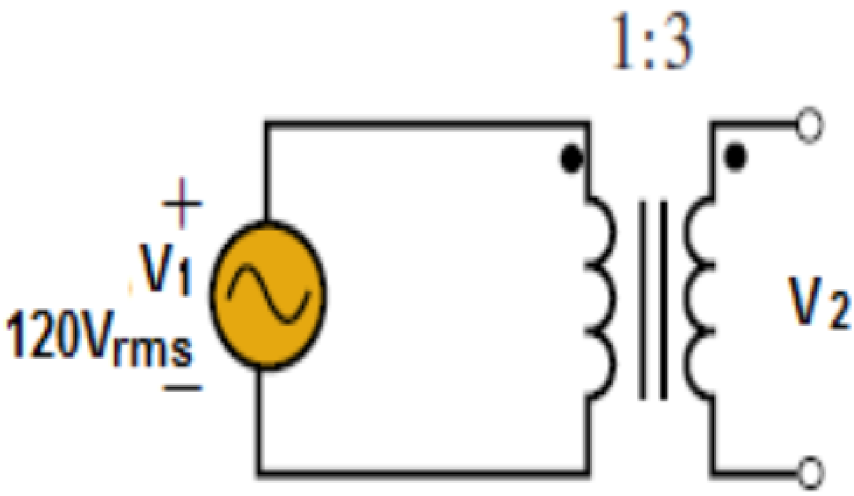
INCREASES AND DECREASES VOLTAGE USING TRANSFORMER

Application of Step Up Transformer

- Used in transmission lines for transforming the high voltage produced by the alternator. The power loss of the transmission line is directly proportional to the square of the current flows through it, $P=I^2 R$.
- The output current of the step-up transformer is less, and hence it is used for reducing the power loss.
- Used for starting the electrical motor, in the microwave oven, X-rays machines, etc.

EXAMPLE 7

The transformer in figure below has a turns ratio of 3. determine the voltage across the secondary?



SOLUTION

$$V_2 = \eta V_1 \longrightarrow \eta = \frac{N_2}{N_1}$$

$$= 3\ (120V)$$

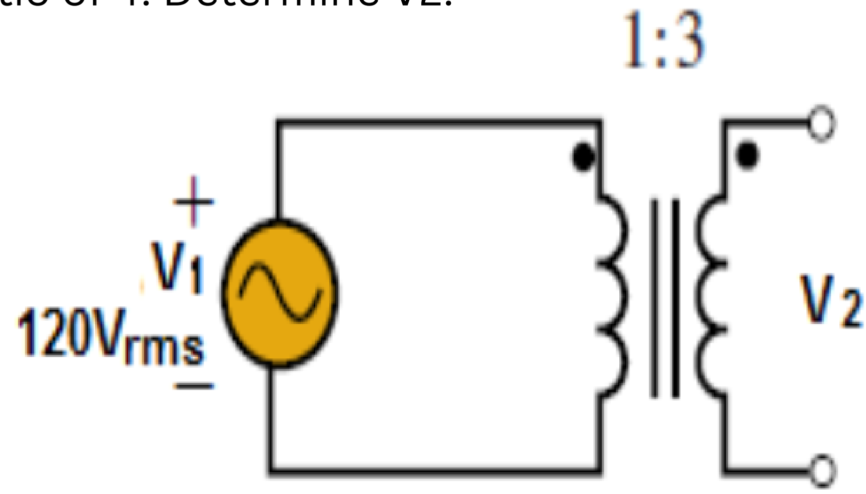
$$= \mathbf{360V}$$

$$= \frac{3}{1}$$

$$= 3$$

EXAMPLE 8

The transformer in previous figure is changed to one with a turns ratio of 4. Determine V2.



Solution:

$$\eta = \frac{N_2}{N_1} = \frac{4}{1} = 4$$

$$\therefore \eta = \frac{V_2}{V_1}$$

$$4 = \frac{V_2}{120}$$

$$V_2 = 480V$$

THE CONSTRUCTION AND THE OPERATION OF A TRANSFORMERS

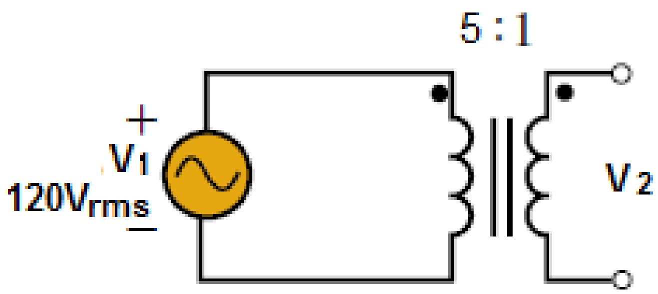
INCREASES AND DECREASES VOLTAGE USING TRANSFORMER

Step Down Transformer

- A transformer in which the secondary voltage is less than the primary voltage is called step-down transformer.
- Turn ratio, η, of a step-down transformer is always less than 1 because the number of turns in the secondary winding (N2) is always less than the number of turns in the primary winding (N1).

EXAMPLE 9

The transformer in figure below is part of laboratory power supply and has a turns ratio of 0.2. Determine the secondary voltage.



Solution:

$$V2 = \eta \, V1$$
$$= (0.2)\,(120V)$$
$$= 24V$$

THE CONSTRUCTION AND THE OPERATION OF A TRANSFORMERS

INCREASES AND DECREASES VOLTAGE USING TRANSFORMER

Application of Step Down Transformer

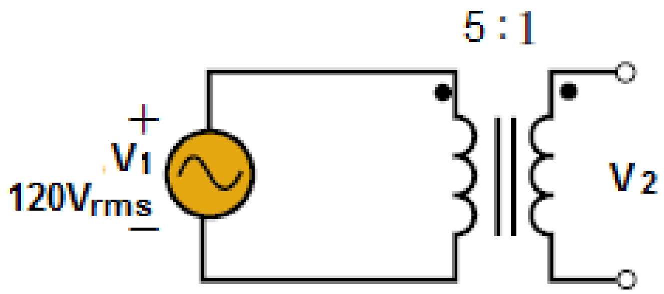
- **Used for electrical isolation**
- **In a power distribution network**
- **For controlling the home appliances,**
- **In a doorbell, etc.**

EXAMPLE 10

The transformer in previous figure is changed to one with a turns ratio of 0.48. Determine the secondary voltage.

Solution:

$$V2 = \eta\ V1$$
$$= (0.48)\ (120V)$$
$$= 57.6V$$



TRANSFORMERS

SUMMARY

Transformer	Turn Ratio (η = Ns/Np)
Step-up	>1
Step-down	< 1
Coupling	1

EXAMPLE 11

A transformer has 500 primary turns and 3000 secondary turns. If the primary voltage is 240V, determine the secondary voltage, assuming an ideal transformer.

$$\frac{V_{pri}}{V_{sec}} = \frac{N_{pri}}{N_{sec}}$$

$hence$

$$\boldsymbol{V_{sec} = \frac{N_{sec}}{N_{pri}} \times V_{pri}}$$

Thus

$$V_{sec} = \frac{3000}{500} \times 240 = 1.44kV$$

EXAMPLE 12

An ideal transformer with a turns ratio of 2:7 is fed from a 240V supply. Determine its output voltage.

$$\frac{V_{pri}}{V_{sec}} = \frac{N_{pri}}{N_{sec}}$$

hence

$$\boldsymbol{V_{sec} = \frac{N_{sec}}{N_{pri}} \times V_{pri}}$$

Thus

$$V_{sec} = \frac{7}{2} \times 240 = 840V$$

DC ISOLATION

- Transformers are useful in providing electrical isolation between primary circuit and the secondary circuit because there is no electrical connection between the two windings. In a transformer, energy is transferred entirely by magnetic coupling.

  - If there is nonchanging direct current through the primary circuit of a transformer, nothing happens in the secondary circuit.

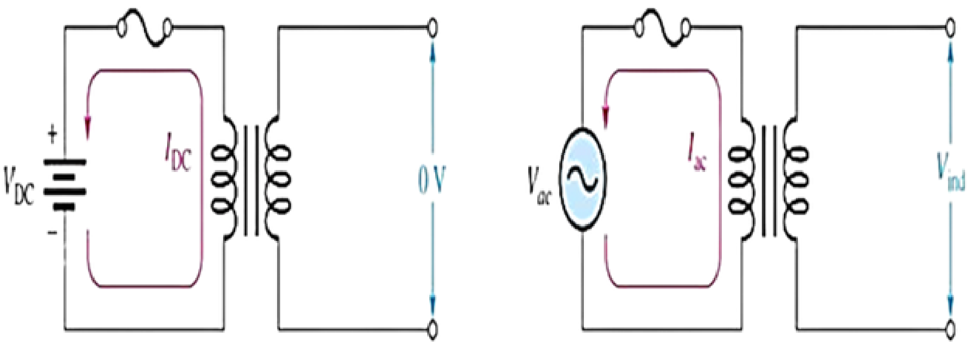


Figure : DC Isolation

  - A changing current in the primary winding is necessary in order to create a changing magnetic field.

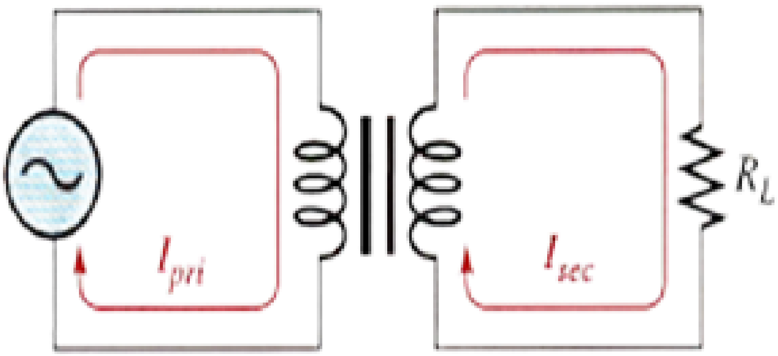
- This will cause voltage to be induced in the secondary circuit.

  - Therefore, the transformer isolates the secondary circuit from any dc voltage in the primary circuit.

THE EFFECT OF A RESISTIVE LOAD ACROSS THE SECONDARY WINDING

THE CURRENT DELIVERED BY THE SECONDARY WHEN A STEP-UP AND STEP-DOWN TRANSFORMER IS LOADED

- When a load resistor is connected to the secondary winding, there is current through the resulting secondary circuit because of the voltage induced in the secondary coil.
- It can be shown that the ratio of the primary current, Ipri to the secondary current, Isec is equal to the turns ratio, as expressed in the following equation:



$$\frac{I_{pri}}{I_{sec}} = \frac{N_{sec}}{N_{pri}}$$

THE EFFECT OF A RESISTIVE LOAD ACROSS THE SECONDARY WINDING

THE CURRENT DELIVERED BY THE SECONDARY WHEN A STEP-UP AND STEP-DOWN TRANSFORMER IS LOADED

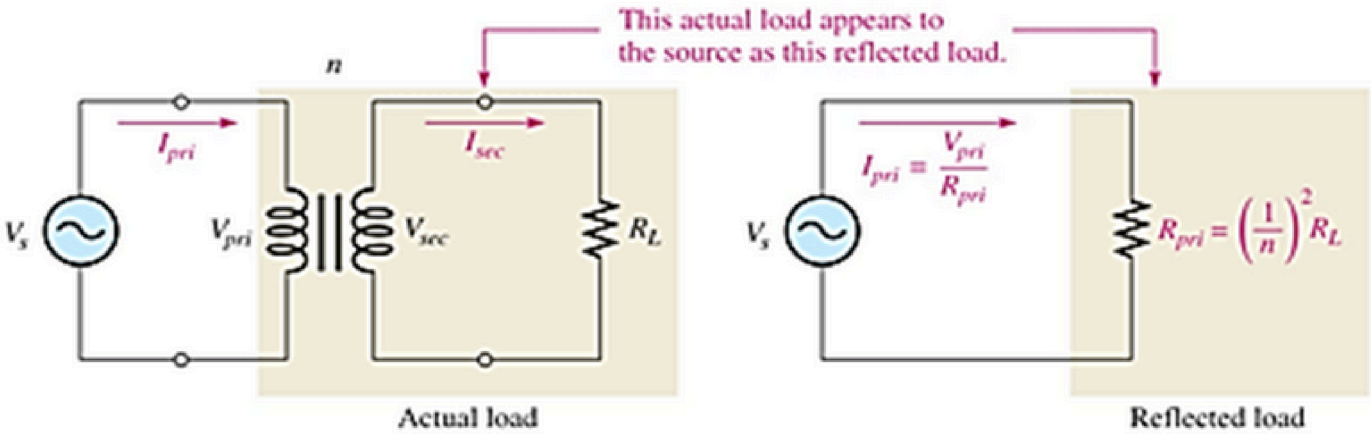
- Thus, for a step-up transformer, in which Nsec / Npri is greater than 1, the secondary is less than the primary current.
- For a step-down transformer, Nsec / Npri is less than 1, and Isec, is greater than Ipri.
- A transformer changes both the voltage and current on the primary side to different values on the secondary side.
- This makes a load resistance appear to have a different value on the primary side.
- The current delivered by the secondary when a step-up and step-down transformer is loaded and given as per below formula:

$$R_{pri} = \frac{V_{pri}}{I_{pri}} \quad and \qquad R_L = \frac{V_{sec}}{I_{sec}}$$

$$\frac{R_{pri}}{R_L} = \left(\frac{V_{pri}}{V_{sec}}\right)\left(\frac{I_{sec}}{I_{pri}}\right) = \left(\frac{1}{n}\right)\left(\frac{1}{n}\right) = \frac{1}{n^2}$$

THE EFFECT OF A RESISTIVE LOAD ACROSS THE SECONDARY WINDING

THE CURRENT DELIVERED BY THE SECONDARY WHEN A STEP-UP AND STEP-DOWN TRANSFORMER IS LOADED



$$R_{pri} = \frac{V_{pri}}{I_{pri}} \quad and \quad R_L = \frac{V_{sec}}{I_{sec}}$$

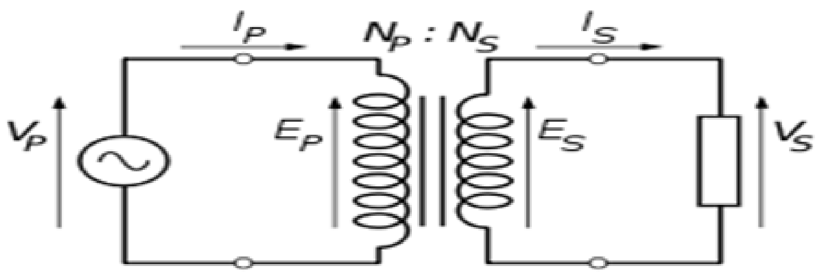
$$\frac{R_{pri}}{R_L} = \left(\frac{V_{pri}}{V_{sec}}\right)\left(\frac{I_{sec}}{I_{pri}}\right) = \left(\frac{1}{n}\right)\left(\frac{1}{n}\right) = \frac{1}{n^2}$$

TRANSFORMERS

POWER IN A TRANSFORMER

When a load is connected to the secondary winding of a transformer, the power transferred to the load can never be greater than the power in the primary winding.

For an ideal transformer, the power delivered to the primary equals the power delivered by the secondary to the load.



When losses are considered, some of the power is dissipated in the transformer rather than the load; therefore, the load power is always less than the power delivered to the primary.

POWER IN A TRANSFORMER

Power is dependent on voltage and current, and there can be no increase in power in a transformer.

Therefore, if the voltage is stepped up, the current is stepped down and vice versa.

In an ideal transformer, the secondary power is equal to the primary power regardless of the turns ratio, as the following equations show.;

The power delivered to primary is;

$$P_{pri} = I_{Pri}V_{Pri}$$

And the power delivered to the load is;

$$P_{sec} = I_{sec}V_{sec}$$

From equation,

$$I_{sec} = \frac{N_{pri}}{N_{sec}} \times I_{pri} \qquad and \qquad V_{sec} = \frac{N_{sec}}{N_{pri}} \times V_{pri}$$

By substitution,

$$P_{sec} = \left(\frac{N_{pri}}{N_{sec}}\right)\left(\frac{N_{sec}}{N_{pri}}\right) I_{pri}V_{pri}$$

Cancelling terms yield;

$$P_{sec} = I_{pri}V_{pri} = P_{pri}$$

POWER IN A TRANSFORMER

EXAMPLE 13

An ideal transformer has a turns ratio of 8:1 and the primary current is 3A when it is supplied at 240V. Calculate the secondary voltage and current.

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \quad hence \ \frac{240}{V_2} = \frac{8}{1}$$

Thus the secondary voltage;

$$V_2 = \frac{1}{8} \times 240 = 30V$$

$$\frac{N_1}{N_2} = \frac{I_2}{I_1}$$

hence

$$I_2 = \frac{N_1}{N_2} \times I_1 = \frac{8}{1} \times 3 = 24A$$

POWER IN A TRANSFORMER

EXAMPLE 14

An ideal transformer, connected to a 240V mains, supplies a 12V, 150W lamp. Calculate the transformer turns ratio and the current taken from the supply

$$turns\ ratio = \frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{240}{12} = 20$$

$$Output\ power = I_2 V_2$$

hence

$$I_2 = \frac{Output\ power}{V_2} = \frac{150}{12} = 12.5A$$

$$\frac{V_1}{V_2} = \frac{I_2}{I_1}$$

hence

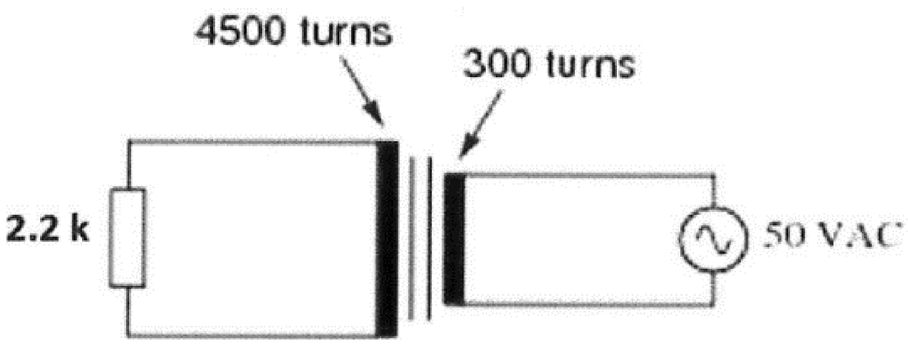
$$I_1 = \frac{V_2}{V_1} \times I_2 = \frac{12}{240} \times 12.5 = 0.625A$$

TRANSFORMERS

QUESTIONS

QUESTION 1:

Referring to figure below, calculate Vp, Vs, Ip and Is.



Solution:

$$\frac{V_P}{V_S} = \frac{N_P}{N_S}$$
$$V_P = \frac{N_P}{N_S} x V_S$$
$$= \frac{4500}{300} x50$$
$$= 750\, VAC$$

$$V_P = I_P R_P$$
$$I_P = \frac{V_P}{R_P}$$
$$= \frac{750}{2.2k}$$
$$= 0.34\ amp$$

$$\frac{V_P}{V_S} = \frac{I_S}{I_P}$$
$$I_S = \frac{V_P}{V_P} x I_1$$
$$= \frac{750}{50} x0.34$$
$$= 5.1\ amp$$

Answer:

$$\therefore V_S = 50Vac$$
$$I_P = 0.34A$$
$$V_S = 750\text{Vac}$$
$$I_S = 5.1A$$

QUESTIONS

QUESTION 2:

Referring to Figure B4(c), calculate:

i. The Voltage Induced In The Primary
ii. The Secondary Current
iii. The Primary Current

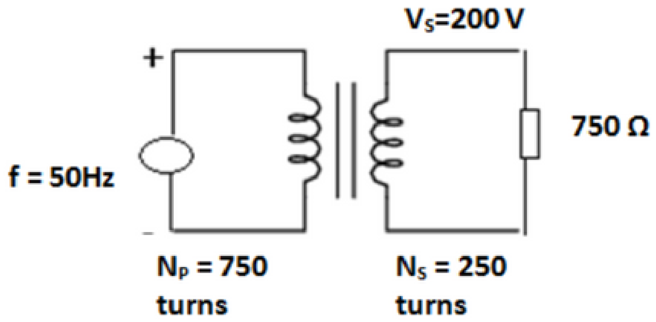


Figure B4(c) / *Rajah B4(c)*

Solution:

$$\frac{V_P}{V_S} = \frac{N_P}{N_S}$$
$$V_P = \frac{N_P}{N_S} x V_S$$
$$= \frac{750}{250} x200$$
$$= 600\ V$$

$$V_s = I_s R_s$$
$$I_s = \frac{V_s}{R_s}$$
$$= \frac{200}{750}$$
$$= 0.27\ amp$$

$$\frac{V_P}{V_S} = \frac{I_S}{I_P}$$
$$I_p = \frac{V_s}{V_P} x I_s$$
$$= \frac{200}{600} x0.27$$
$$= 0.09\ amp$$

Answer:

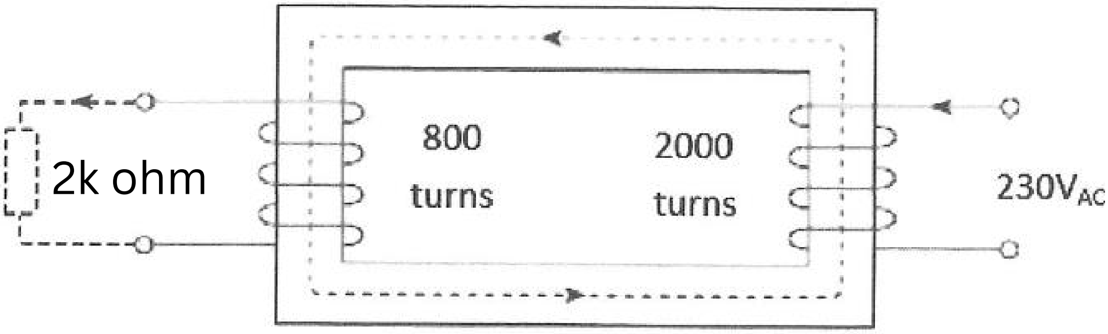
$$\therefore V_s = 200V$$
$$I_s = 0.27A$$
$$V_P = 600V$$
$$I_p = 0.09A$$

QUESTIONS

QUESTION 3:

Referring to Figure;

Calculate the secondary voltage (Vs), primary current (Ip) and secondary current (Is).



Solution:

$$\frac{V_P}{V_S} = \frac{N_P}{N_S}$$
$$V_P = \frac{N_P}{N_S} x V_S$$
$$= \frac{800}{2000} x230$$
$$= 92\ V$$

$$V_p = I_p R_p$$
$$I_p = \frac{V_p}{R_p}$$
$$= \frac{92}{2000}$$
$$= 0.046\ amp$$

$$\frac{V_P}{V_S} = \frac{I_S}{I_P}$$
$$I_s = \frac{V_p}{V_s} x I_p$$
$$= \frac{92}{2k} x0.046$$
$$= 0.002\ amp$$

Answer:

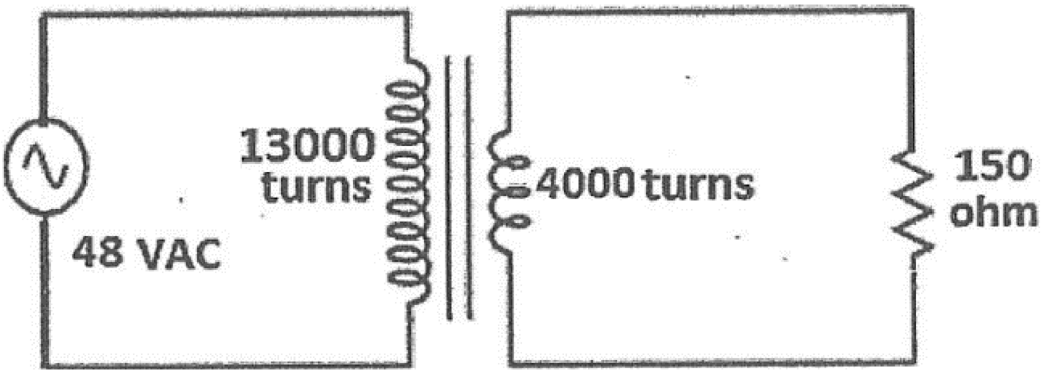
$$\therefore V_p = 92V$$
$$I_p = 0.046A$$
$$I_S = 0.002A$$

QUESTIONS

QUESTION 4:

By referring to Figure, calculate :

i. Primary voltage
ii. Secondary voltage
iii. Secondary current
iv. Primary current



SOLUTION

QUESTION 4:

$$\frac{V_P}{V_S} = \frac{N_P}{N_S}$$
$$V_s = \frac{N_s}{N_p} x V_p$$
$$= \frac{4000}{13000} x48$$
$$= 14.8\ V$$

$$V_s = I_s R_s$$
$$I_s = \frac{V_s}{R_s}$$
$$= \frac{14.8}{150}$$
$$= 0.099 \approx 0.1\ amp$$

$$\frac{V_P}{V_S} = \frac{I_S}{I_P}$$
$$I_p = \frac{V_s}{V_p} x I_s$$
$$= \frac{14.8}{48} x0.1$$
$$= 0.03\ amp$$

Answer:

$$\therefore V_p = 48V$$
$$V_s = 14.8V$$
$$I_s = 0.1A$$
$$I_p = 0.03A$$

QUESTIONS

QUESTION 5:

An ideal single phase transformer has 2000 windings on it primary coil and 700 windings in its secondary coil. The main coil is connected to a 240 volt AC, 50Hz. Calculate :

1. Secondary voltage.
2. Current at a primary coil if the current in a secondary coil is 5A.

Solution:

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

$$V_s = \frac{N_s}{N_p} x V_p$$

$$= \frac{700}{2000} x240$$

$$= 84\ V$$

$$\frac{V_P}{V_S} = \frac{I_S}{I_P}$$

$$I_S = \frac{V_P}{V_S} x I_P$$

$$= \frac{240}{84} x5$$

$$= 14.3\ amp$$

Answer:

$$\therefore V_S = 84V$$

$$I_s = 14.3A$$

TRANSFORMERS

TUTORIAL

- List THREE (3) / FOUR (4) types of transformer.
- Outline FIVE (5) / SIX (6) characteristics of an ideal transformer.
- Explain the principle of a transformer.
- Draw and label clearly windings, voltages and current for a set-up transformer.
- Draw and label clearly winding, voltages and current for a step-down transformer.
- Explain the characteristics of the step-up transformer.
- What does it mean by step-up and step-down transformers?
- List THREE (3) types of losses which happen to a transformer.
- The amount of mutual inductance depends upon several factors. Identify FIVE (5) factors that affect mutual inductance.
- Define transformer ratio.
- What is the relationship between the currents in the primary and secondary windings of transformer?

TRANSFORMERS

TUTORIAL

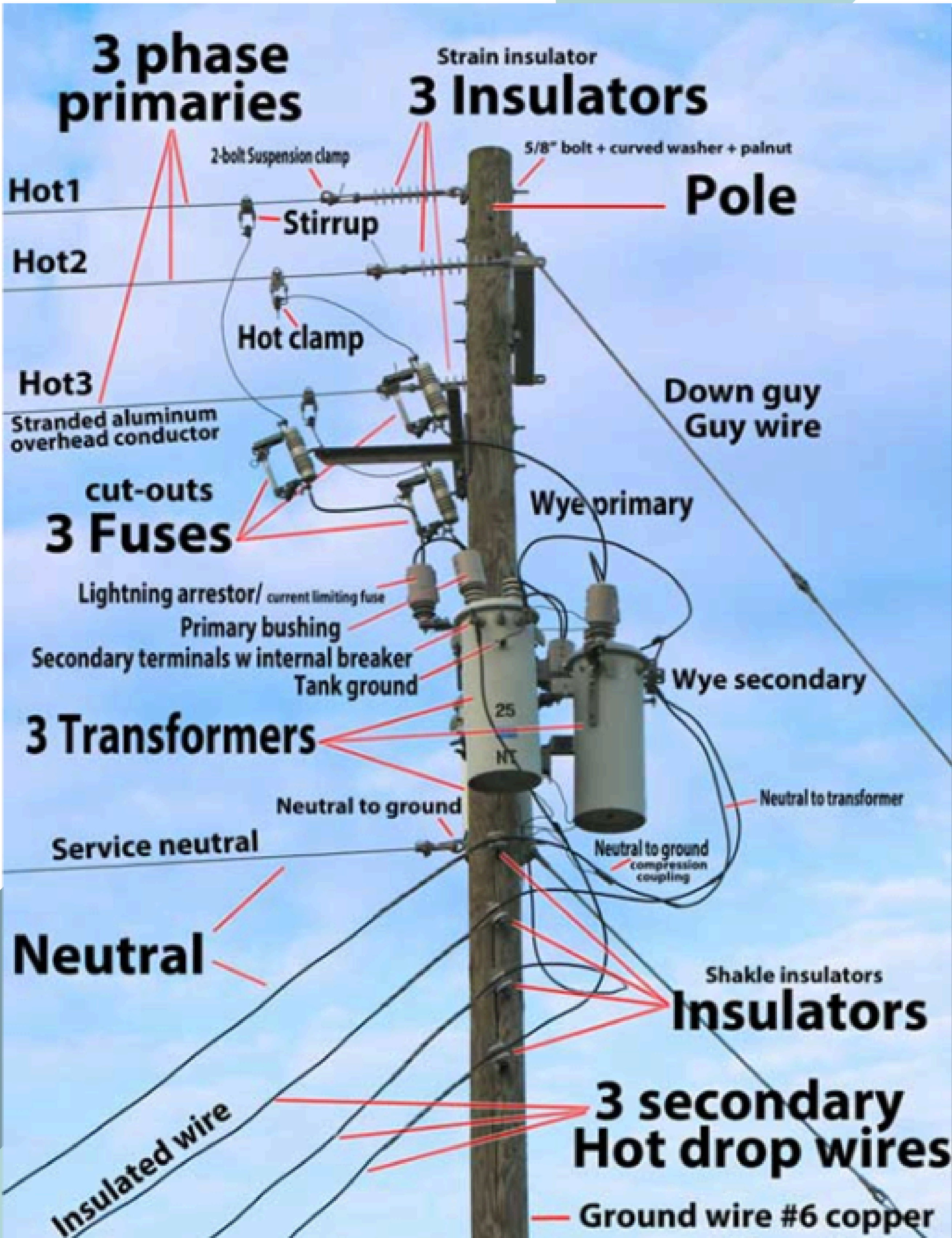
- A 5kVA single-phase transformer has a turn ratio of 10:1 and is fed from a 2.5kV supply. Neglecting the losses, calculate the full load secondary current, the minimum load resistance which can be connected across the secondary winding to give full load kVA and the primary current at full load kVA.
- An ideal transformer is connected to a power supply 240V, supplies a lamp of 12V, 150W. Calculate the transformer turns ratio and the current taken from the supply.
- A step-down transformer has a turn ratio of 20:1, a primary voltage of 4kV and a load of 10kW. Neglecting losses, calculate the value of the secondary current.
- A transformer with 3000 primary turns and 1250 secondary turns is supplied from a 415V AC supply. Calculate the secondary voltage.
- A 12Ω resistor is connected across the secondary winding of an ideal transformer whose secondary voltage is V_s = 120V. Determine the primary voltage V_p, if the supply current I_p = 4A.

TUTORIAL

- The primary winding of a particular transformer has 200 turns and is connected across a 230V, 50Hz, single phase supply. If there are 1200 turns in the secondary winding, the voltage across it will be?

CHAPTER 5

THREE PHASE SYSTEMS



Course Content Outline

Understand three phase system.
a.Explain the basic principles of a three phase system.
b.Explain the three phase e.m.f generation.
c.Explain balanced load in a three phase system.

Apply the understanding of three phase system configurations.

- **List the advantages and application of three phase system compared to single phase system.**
- **Construct the waveform and vector of three phase e.m.f.**
- **Construct phasor diagram for: a. delta system b. star system**
- **Calculate the phase voltage, phase current, line voltage and line current for delta and star systems.**
- **Compare the delta and star quantities.**
- **Calculate total power for three phase system using formula $P=\sqrt{(3V_L)}(I_L\cos\theta)$.**
- **Solve problems related to three phase system.**

Introduction

Generation, transmission and distribution of electricity via the National Grid system is accomplished by three phase alternating currents.

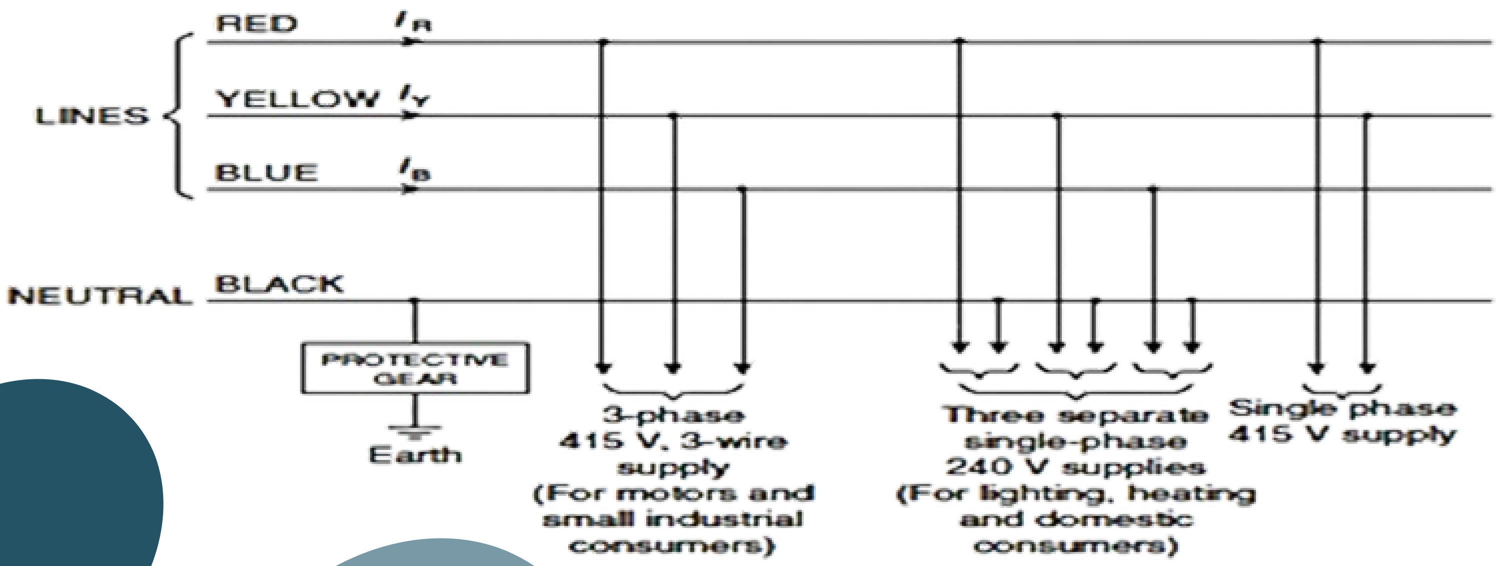


Figure A : National grid system

The majority of single-phase supplies are obtained by connection to a three-phase supply

BASIC PRINCIPLES OF A THREE PHASE SYSTEM

- 3 phase supply is generated when 3 coils are placed 120° apart and the whole rotated in a uniform magnetic field.
- The result is three independence supplies of equal voltages which are each displaced by 120°.

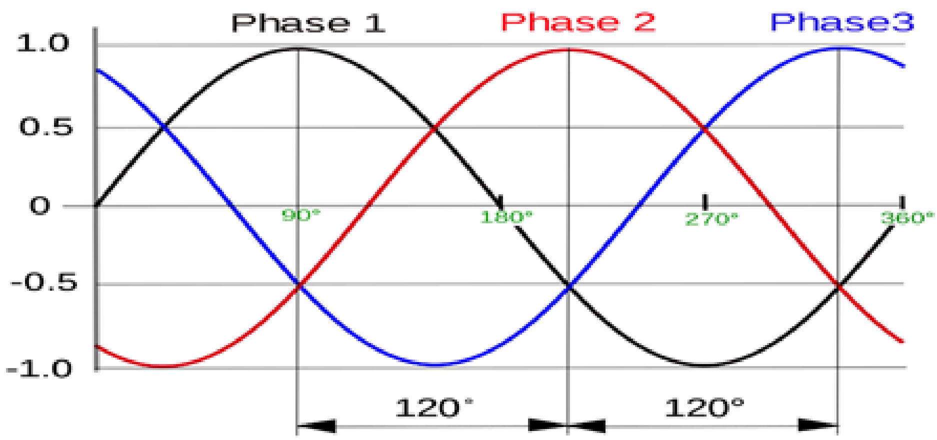


Figure 1: Voltage in each phase of the generator connected to three identical loads

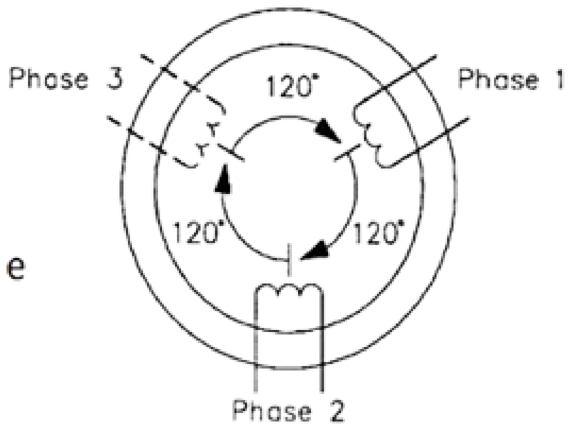


Figure 2: Three-phase generator consisting of three single-phase sources equal in magnitude and 120 degree apart in phase

BASIC PRINCIPLES OF A THREE PHASE SYSTEM

- The convention adopted to identify each of the phase voltages is:

  R-red, Y-yellow and B-blue.

- The phase sequence is given by the sequence in which the conductors pass the point initially taken by the red conductor.

- The national standard phase sequence is R,Y,B.

- 2 ways to interconnect the three phases:
  (a) Star connection
  (b) Delta connection

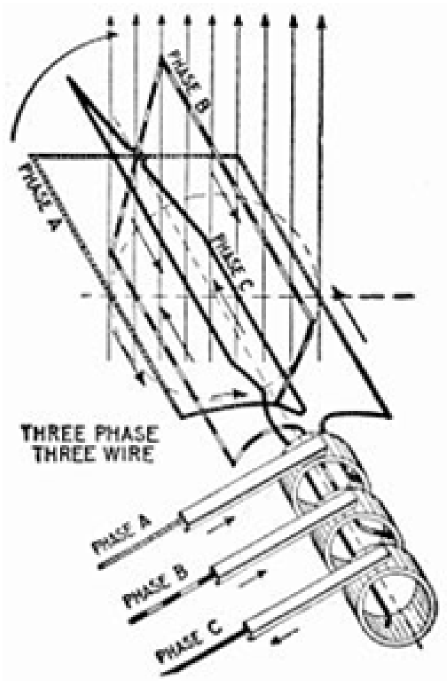
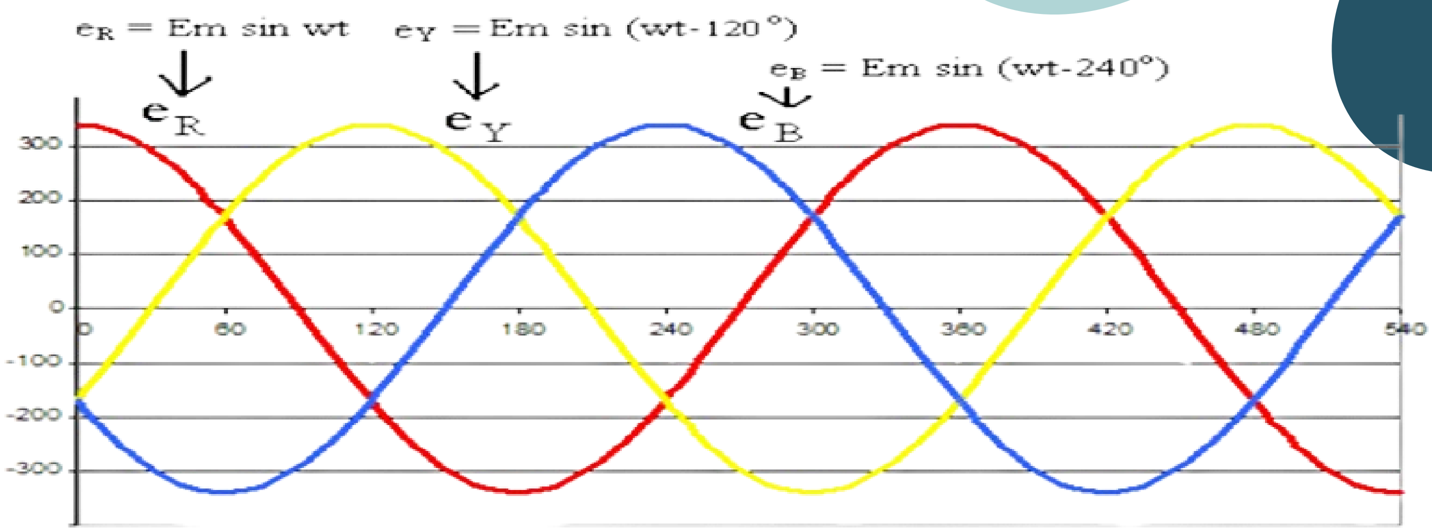


Figure 3: Three phase three wire

BASIC PRINCIPLES OF A THREE PHASE SYSTEM

THREE PHASE WAVEFORM



- The convention adopted to identify each of the phase voltages is: R-red, Y-yellow, and B-blue
- The phase-sequence is given by the sequence in which the conductors pass the point initially taken by the red conductor.
- The national standard phase sequence is R,Y, B.

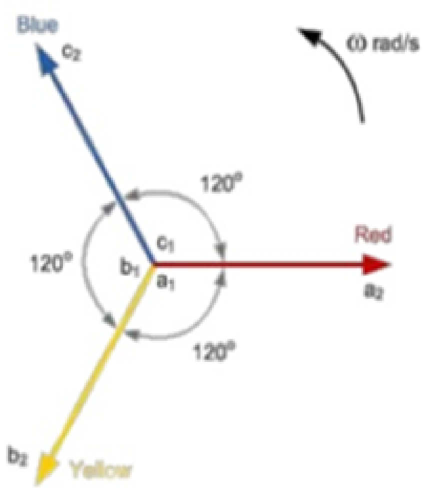
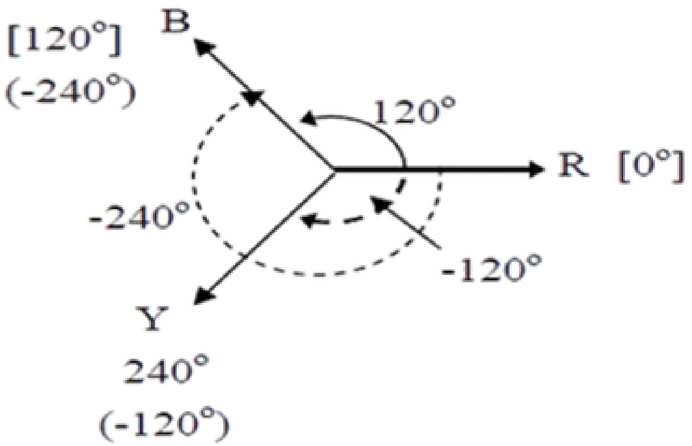
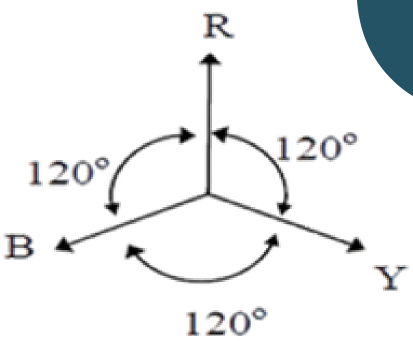


Figure 4: Three-phase generator consisting of three single-phase sources equal in magnitude and 120 degree apart in phase

BASIC PRINCIPLES OF A THREE PHASE SYSTEM

THREE PHASE VECTOR (PHASE) DIAGRAM





Since the three generated voltages are sinewaves of the same frequency, mutually out-of-phase by 120°, then they may be represented both on a waveform diagram using the same angular or time axis, and as phasors.

$$V_R = V_m \sin(\omega t)$$

$$V_Y = V_m \sin(\omega t - 120°)$$

$$V_B = V_m \sin(\omega t - 240°)$$

UNDERSTAND THREE PHASE SYSTEM CONFIGURATIONS

LIST THE ADVANTAGES AND APPLICATION OF THREE PHASE SYSTEM COMPARED TO SINGLE PHASE SYSTEM

APPLICATIONS:

Two voltage are available (star & delta connection):

(a) Star connection: domestic user

(b) Delta connection: industrials user

ADVANTAGES:

Table 1: Comparison an advantages between three phase and single phase system

CRITERIA	3 PHASE	SINGLE PHASE
COST- Cheap	The size of conductor in 3 phase is 75% only the size of conductors in a single phase with the same rating of power.	For a given amount of power transmitted through a system, the single phase system requires conductors with a **bigger cross sectional area.**
	Three phase system requires conductors with **a smaller cross sectional area.** This means a saving of copper (or aluminum) and thus the original installation **costs are less.**	This means use more copper so the **cost will increase.**
POWER RANGE & SIZE	Also **available in a range of sizes** (1 horse-power to many thousands of horse-power) and they usually have only one moving part. Generally smaller	A single phase motor **power is limited.** Bigger size,

ADVANTAGES:

Table 1: Comparison an advantages between three phase and single phase system

CRITERIA	THREE PHASE	SINGLE PHASE
POWER	There is never a time when all the voltage go to 0, the power stays constant throughout the whole cycle. This give the **vibration free drive.**	Voltage drops to 0 every half-cycle. Therefore the amount of power not constant over time. This will **cause a vibration** in a large motor application.
MAINTENANCE	Three phase motors are very robust, relatively **cheap,** provide steadier output and require little maintenance compared with single phase motors.	**Expensive**, unsteady output and need high maintenance.
SELF-STARTING	Three phase motors can have a **high starting torque** and the rotating magnetic field in very smooth.	Do **not have good starting torque** and they have a complex starting switch in some cases.

APPLICATIONS:

- A three-phase machine of a given physical size gives more output than a single-phase machine of the same size and most electrical power generation is carried out by means of three-phase synchronous generators.

- There is a considerable amount of saving in conductor material to be gained by using three-phase rather than single-phase for the purposes of power transmission by overhead lines or underground cables.

- The three-phase induction motor is the cheapest and most robust of machines and accounts for the vast majority of the world's industrial machines.

- The phase currents tend to cancel out one another, summing to zero in the case of a linear balanced load. This makes it possible to eliminate the neutral conductor on some lines; all the phase conductors carry the same current and so can be the same size, for a balanced load.

- Power transfer into a linear balanced load is constant, which helps to reduce generator and motor vibrations.

- Three-phase systems can produce a magnetic field that rotates in a specified direction, which simplifies the design of electric motors. Three is the lowest phase order to exhibit all of these properties.

UNDERSTAND THREE PHASE SYSTEM CONFIGURATIONS

EXPLAIN THE THREE PHASE E.M.F GENERATION:

1) Three-phase generators have three coils fixed at 120° to each other with the same amplitude and frequency.

2) The phases are normally called red (R), yellow (Y) and blue (B).

3) The loops are being rotated anti-clockwise and each loop is producing exactly the same emf with the same amplitude and frequency but the loop Y Lags loop R by 120 and the loop B lags loop Y by 120.

4) This is the same for the associated loop Y 1, B 1, and R 1. At any moment the e.m.f generated in the three loops are as follows.

5) $e_R = E_m \sin(\theta)$
$e_Y = E_m \sin(\theta - 120)$
$e_B = E_m \sin(\theta - 240)$

6) 2 ways to generate 3 phase:
- Rotate the coil in the constant magnetic field.
- Rotate the magnetic field around the static coils.

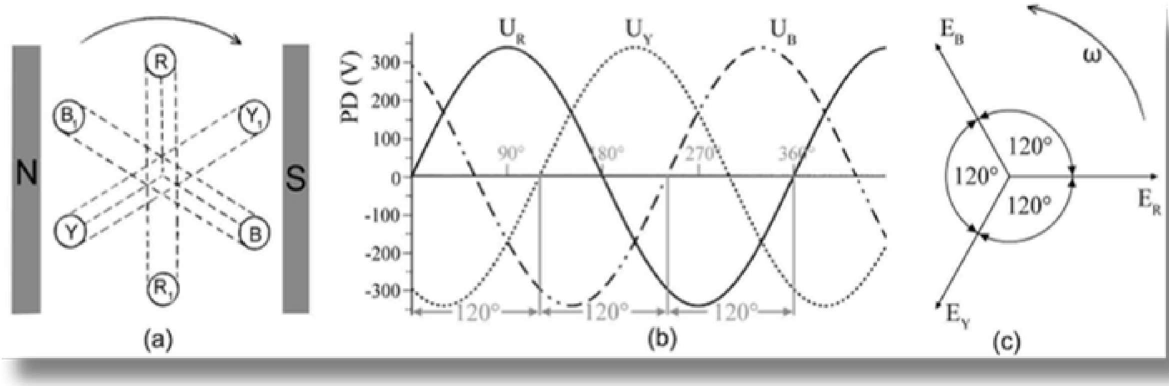
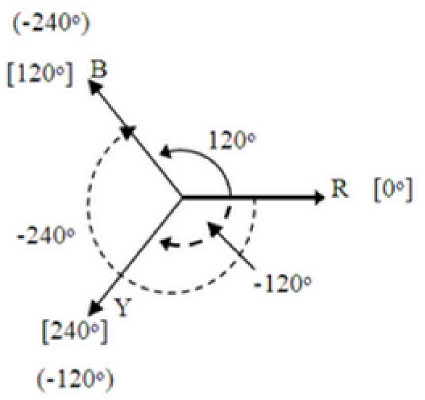


Figure 5 : A three-phase AC supply with 240V rms generated in each coil: (a) Generation; (b) Wave diagram; (c) Phasor diagram

UNDERSTAND THREE PHASE SYSTEM CONFIGURATIONS

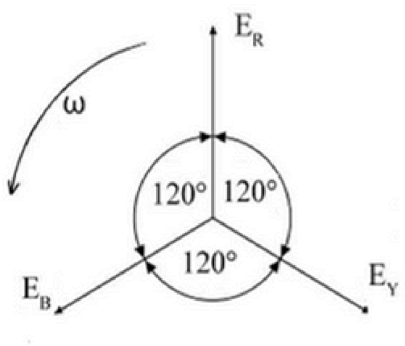
EXPLAIN THE THREE PHASE E.M.F GENERATION:

Three phase e.m.f vector diagram (geometry)



- Select 1 phase as reference.
- Show the concept of leading and lagging.

Three phase e.m.f vector diagram (conversional)



- Show that each phase have difference 120 degree

IDENTIFY CONNECTION AND VECTOR/PHASE DIAGRAM FOR:

(a) DELTA SYSTEM

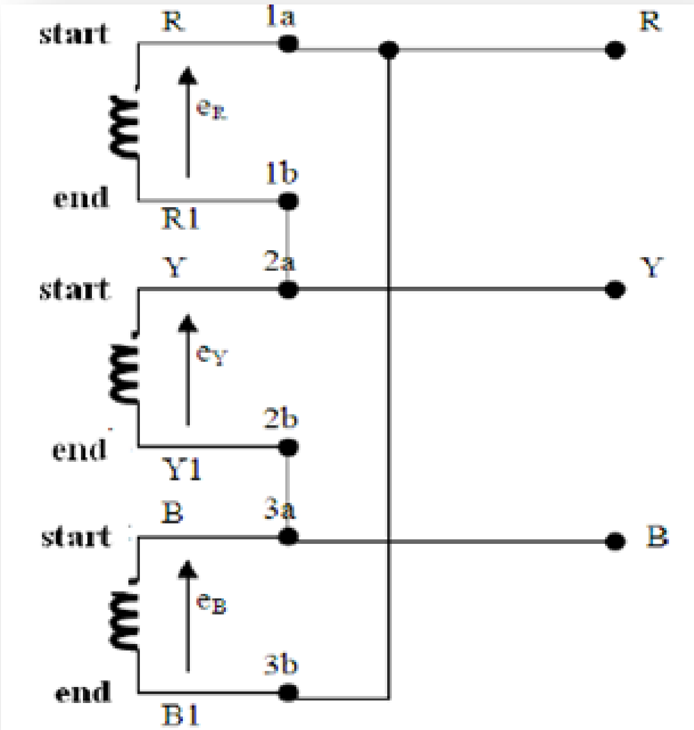


Figure 6: Physical connection

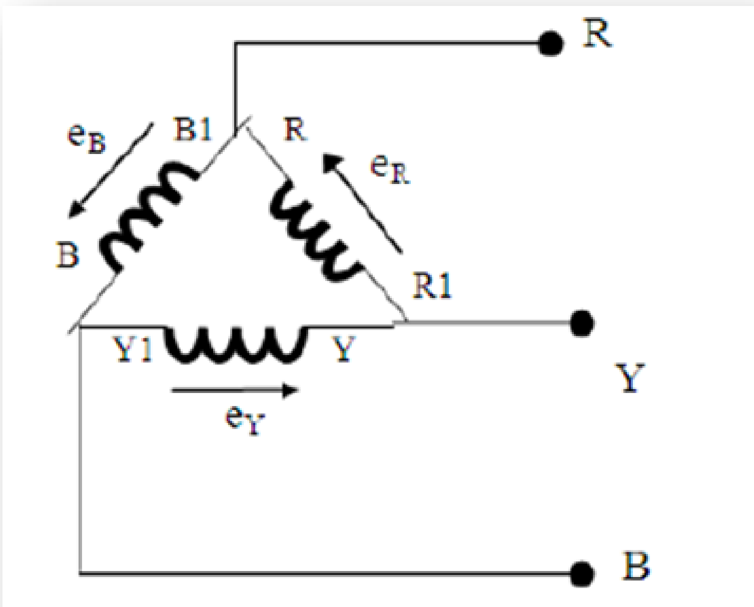


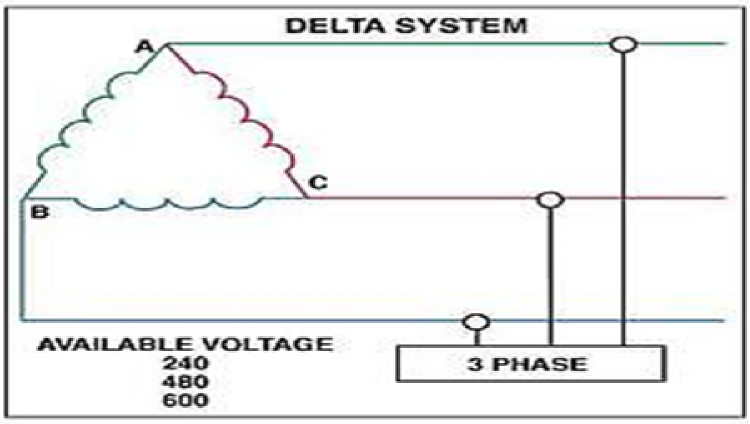
Figure 7: Conventional connection

IDENTIFY CONNECTION AND VECTOR/PHASE DIAGRAM FOR:

(a) DELTA SYSTEM

- The delta connection is formed by connecting one end of winding to starting end of other and connections re continued to form a closed loop.
- The supply terminals are taken out from the three junction points.
- The delta connection is a common three-phase, three-wire system in which the voltage between each pair of line wires is the actual transformer voltage.
- This system is especially popular for use on ungrounded systems. A delta system can be grounded, but this introduces some complications.

- A delta (or mesh) connected load as shown in figure where the end of one load is connected to the start of the next load.



IDENTIFY CONNECTION AND VECTOR/PHASE DIAGRAM FOR:

(a) DELTA SYSTEM

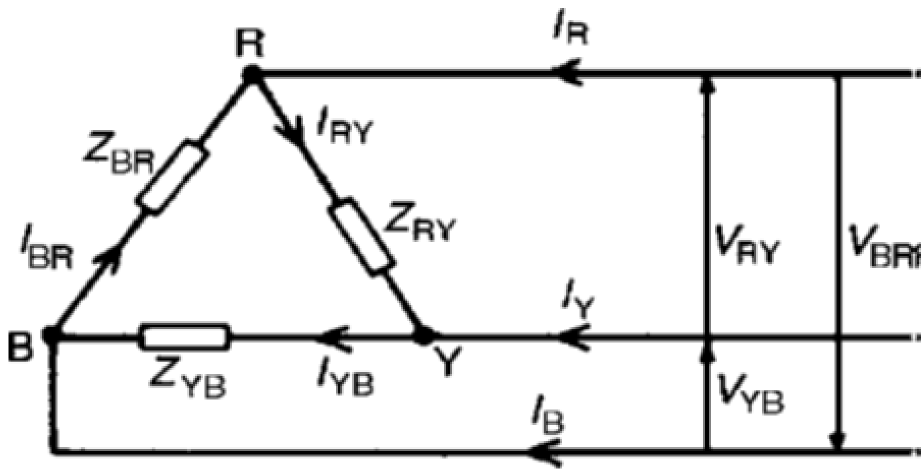


Figure 8 : Three phase delta connection

From figure , it can be seen that the line voltages V_{RY}, V_{YB} and V_{BR} are the respective phase voltages, i.e. for a delta connection:

$$V_L = V_P$$

Using Kirchhoff's current law in Figure;

$$I_R = I_{RY} - I_{BR} = I_{RY} + (-I_{BR})$$

IDENTIFY CONNECTION AND VECTOR/PHASE DIAGRAM FOR:

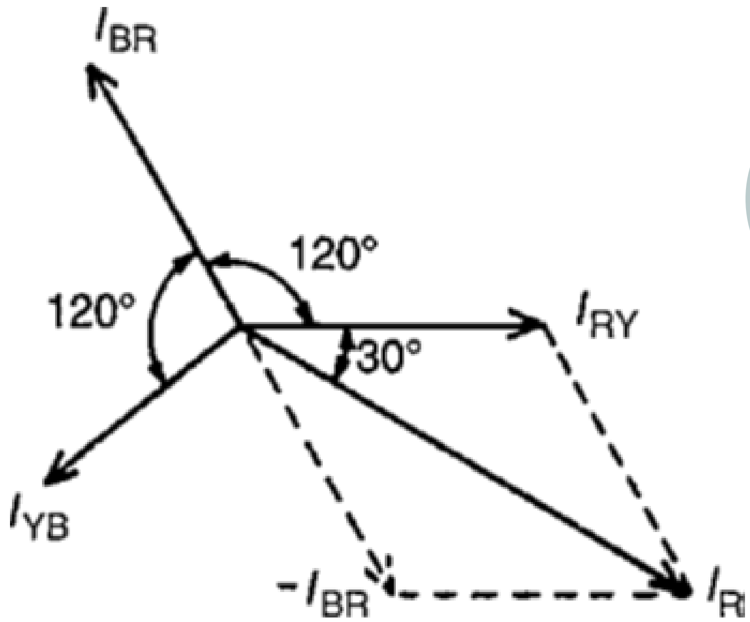
(a) DELTA SYSTEM (PHASOR)

From the phasor diagram shown in Figure, by trigonometry or by measurement,

$$I_R = \sqrt{3} I_{RY}$$

i.e. for a delta connection:

$$I_L = \sqrt{3} I_P$$



IDENTIFY CONNECTION AND VECTOR/PHASE DIAGRAM FOR:

(a) DELTA SYSTEM (POWER)

For a delta connection,

$$V_P = V_L \quad and \quad I_P = \frac{I_L}{\sqrt{3}}$$

Hence

$$P = 3\frac{I_L}{\sqrt{3}}V_L \cos\phi = \sqrt{3}V_L I_L \cos\phi$$

Hence for either a **star or a delta balanced connection** the total power P is given by:

$$P = 3\frac{I_L}{\sqrt{3}}V_L \cos\phi \text{ watts} \quad \text{or} \quad \mathbf{P = \sqrt{3}V_L I_L \cos\phi \text{ watts}}$$

Total volt-amperes

$$\mathbf{S} = \sqrt{3}V_L I_L \, volt - amperes$$

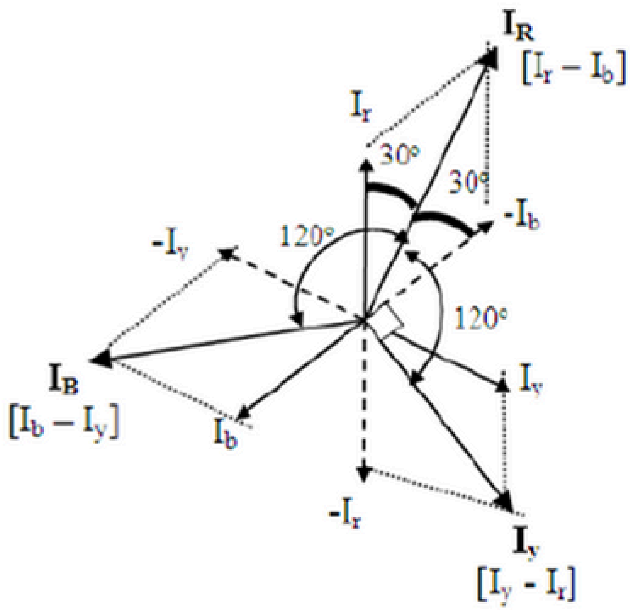
Power factor

$$\cos\theta = \frac{total\ power, P}{total\ volt - amperes, S}$$

IDENTIFY CONNECTION AND VECTOR/PHASE DIAGRAM FOR:

(a) DELTA SYSTEM

VECTOR DIAGRAM



Persamaan vector:

$\vec{I_R} = \vec{I_r} - \vec{I_b}$

$\vec{I_Y} = \vec{I_y} - \vec{I_r}$

$\vec{I_B} = \vec{I_b} - \vec{I_y}$

$\vec{I_R} = \vec{I_r} - \vec{I_b}$ Line current R is contribute to current phase R and B

$\vec{I_Y} = \vec{I_y} - \vec{I_r}$ Line current Y is contributeto current phase Y and R

$\vec{I_B} = \vec{I_b} - \vec{I_y}$ Line current B is contribute to current phase B and Y

IDENTIFY CONNECTION AND VECTOR/PHASE DIAGRAM FOR:

(b) STAR SYSTEM

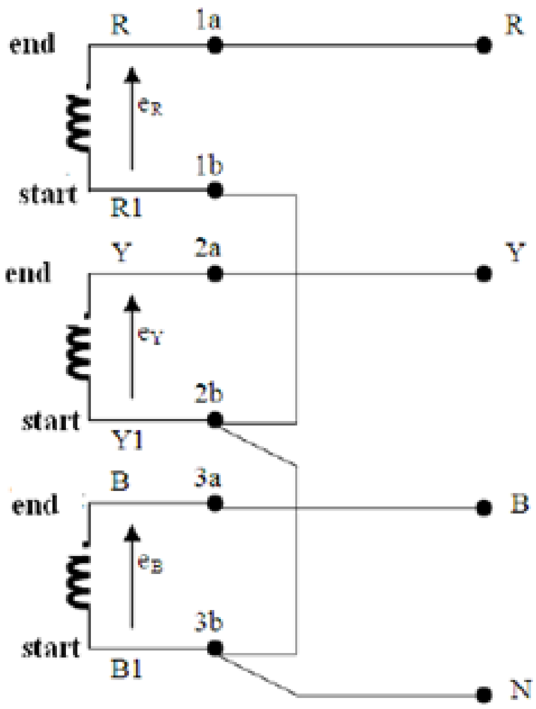


Figure: Physical connection

The star connection is formed by connecting starting or terminating ends of all the three windings together.

The common point is called Neutral point.

The voltages, VR, VY and VB are called phase voltages or line to neutral voltages.
Phase voltages are generally denoted by Vp.
The voltages, VRY, VYB and VBR are called line voltages.

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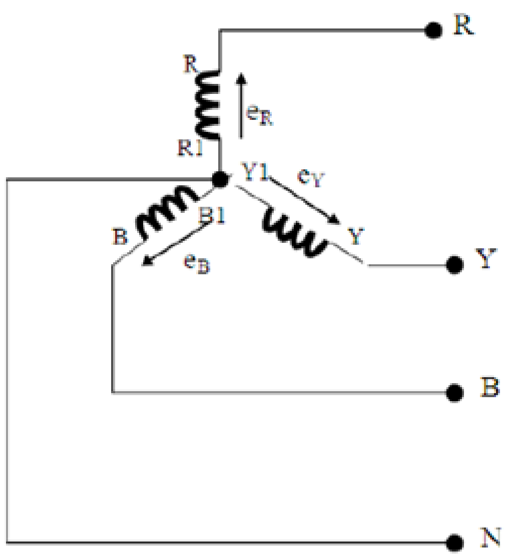
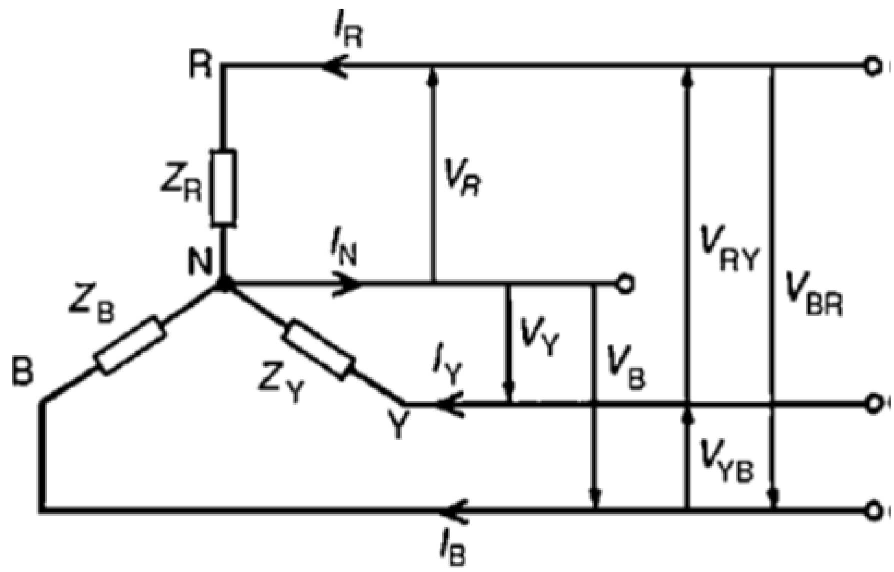


Figure: Convensional connection

IDENTIFY CONNECTION AND VECTOR/PHASE DIAGRAM FOR:

(b) STAR SYSTEM

From Figure it can be seen that the phase currents (generally denoted by Ip) are equal to their respective line currents IR, IY and IB, i.e. for a star connection:



IDENTIFY CONNECTION AND VECTOR/PHASE DIAGRAM FOR:

(b) <u>STAR SYSTEM</u>

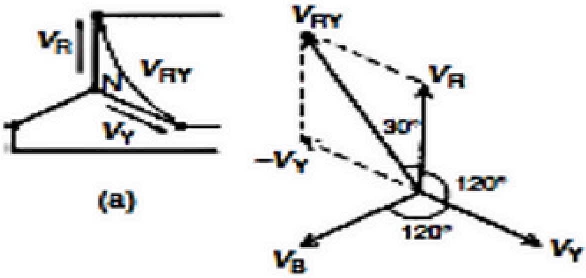
For a balanced system;

$$I_R = I_Y = I_B$$
$$V_R = V_Y = V_B$$
$$V_{RY} = V_{YB} = V_{BR}$$
$$Z_R = Z_Y = Z_B$$

and the current in the neutral conductor, IN = 0 when a star-connected system is balanced, then the neutral conductor is unnecessary and is often omitted.

The line voltage, V_{RY}, is given by $V_{RY} = V_R - V_Y$ (V_Y is negative since it is in the opposite direction to V_{RY}).

In the phasor diagram of Figure, phasor V_Y is reversed (shown by the broken line) and then added phasorially to VR (i.e. $V_{RY} = V_R + (-V_Y)$).



IDENTIFY CONNECTION AND VECTOR/PHASE DIAGRAM FOR:

(b) STAR SYSTEM (POWER)

By trigonometry, or by measurement,

$$V_{RY} = \sqrt{3}V_R$$

For a balanced star connection:

$$V_L = \sqrt{3} \times V_{PH}$$

The power dissipated in a three-phase load is given by the sum of the power dissipated in each phase.

If a load is balanced then the total power P is given by:

P=3×power consumed by one phase.

The power consumed in one phase;

$$P_{ph} = I_P^2 R_P \quad or \quad P_{ph} = V_P I_P \cos\phi$$

where Φ is the phase angle between Vp and Ip.

For a star connection,

$$V_P = \frac{V_L}{\sqrt{3}} \quad and \quad I_P = I_L$$

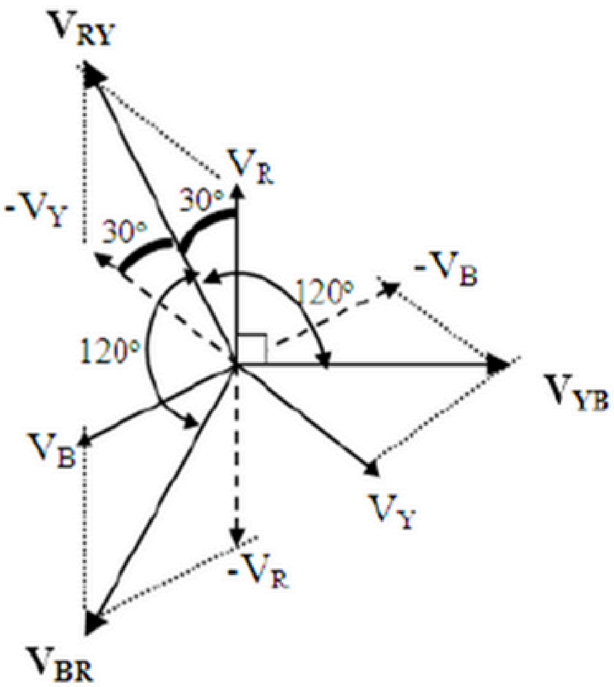
Hence

$$P = 3\frac{V_L}{\sqrt{3}} I_L \cos\phi = \sqrt{3} \times V_L \times I_L \times \cos\phi$$

IDENTIFY CONNECTION AND VECTOR/PHASE DIAGRAM FOR:

(b) <u>STAR SYSTEM</u>

VECTOR DIAGRAM



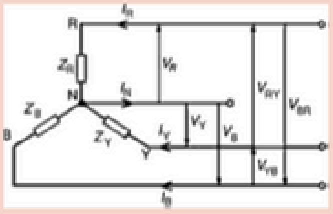
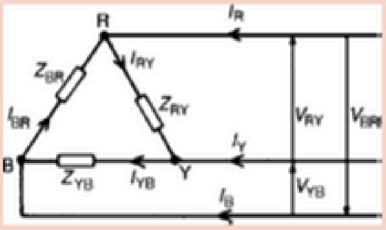
<u>Vector equation:</u>

$$\overrightarrow{V_{RY}} = \overrightarrow{V_R} + (\overrightarrow{-V_Y})$$
$$= \overrightarrow{V_R} - \overrightarrow{V_Y}$$

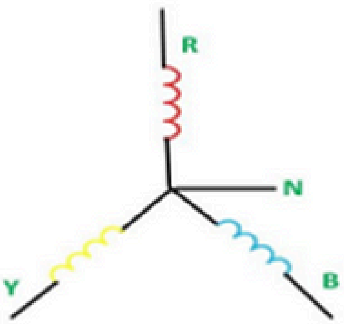
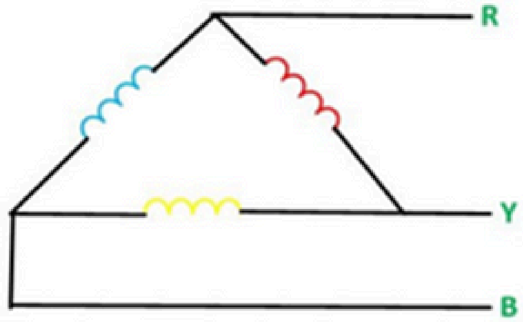
$$\overrightarrow{V_{YB}} = \overrightarrow{V_Y} - \overrightarrow{V_B}$$

$$\overrightarrow{V_{BR}} = \overrightarrow{V_B} - \overrightarrow{V_R}$$

COMPARISON OF STAR AND DELTA CONNECTION

	STAR CONNECTION	DELTA CONNECTION
Connection		
Line Current	$I_L = I_P$	$I_L = \sqrt{3}I_P$
Phase Current	$I_P = I_L$	$I_P = \frac{I_L}{\sqrt{3}}$
Line Voltage	$V_L = \sqrt{3}V_P$	$V_L = V_P$
Phase Voltage	$V_P = \frac{V_L}{\sqrt{3}}$	$V_P = V_L$
Impedance	$Z = \frac{V_p}{I_p}$	
Total power, P	$P = \sqrt{3}V_L I_L \cos\phi \ watts$	
Total volt-amperes	$S = \sqrt{3}V_L I_L \ volt - amperes$	

COMPARISON OF STAR AND DELTA CONNECTION

BASIS	STAR CONNECTION	DELTA CONNECTION
Diagram		
Basic Definition	The terminals of the three branches are connected to a common point. The network formed is known as Star Connection	The three branches of the network are connected in such a way that it forms a closed loop known as Delta Connection
Connection of terminals	The starting and the finishing point that is the similar ends of the three coils are connected together	The end of each coil is connected to the starting point of the other coil that means the opposite terminals of the coils are connected together.
Neutral point	Neutral or the star point exists in the star connection.	Neutral point does not exist in the delta connection.

COMPARISON OF STAR AND DELTA CONNECTION

BASIS	STAR CONNECTION	DELTA CONNECTION
Relation between line and phase current	Line current is equal to the Phase current.	Line current is equal to root three times of the Phase Current.
Relation between line and phase voltage	Line voltage is equal to root three times of the Phase Voltage	Line voltage is equal to the Phase voltage.
Speed	The Speed of the star connected motors is slow as they receive $1/\sqrt{3}$ of the voltage.	The Speed of the delta connected motors is high because each phase gets the total of the line voltage.
Phase voltage	Phase voltage is low as $1/\sqrt{3}$ times of the line voltage.	Phase voltage is equal to the line voltage.

COMPARISON OF STAR AND DELTA CONNECTION

BASIS	STAR CONNECTION	DELTA CONNECTION
Number of turns	Requires less number of turns	Requires large number of turns.
Insulation level	Insulation required is low.	High insulation is required.
Network Type	Mainly used in the Power Transmission networks.	Used in the Power Distribution networks.
Received voltage	In Star Connection each winding receive 230 volts	In delta connection each winding receives 414 volts.
Type of system	Both Three phase four wire and three phase three wire system can be derived in star connection.	Three phase four wire system can be derived from the Delta connection.

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Type of system	Both Three phase four wire and three phase three wire system can be derived in star connection.	Three phase four wire system can be derived from the Delta connection.

EXAMPLE 1:

Three loads, each of resistance 30Ω, are connected in star to a 415V, 3-phase supply. Determine :

(a) the system phase voltage
(b) the phase current and
(c) the line current

Solution

A 415 V, 3-phase supply means that 415 V is the line voltage, V_L

For a star connection,

$$V_L = \sqrt{3}V_p$$

Hence phase voltage

$$V_P = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6V \ \ or\ 240V$$

Phase current

$$I_P = \frac{V_p}{R_p} = \frac{240}{30} = 8A$$

For a star connection $I_P = I_L$ hence the line current $I_L = 8A$

EXAMPLE 2:

A star-connected load consists of three identical coils each of resistance 30Ω and inductance 127.3 mH. If the line current is 5.08A, calculate the line voltage if the supply frequency is 50 Hz.

Solution

Inductive reactance

$$X_L = 2\pi f L = 2\pi(50)(127.3m) = 40\Omega$$

Impedance of each phase

$$Z_P = \sqrt{R^2 + X_L^2} = \sqrt{30^2 + 40^2} = 50\Omega$$

For a star connection

$$I_L = I_P = \frac{V_p}{Z_p} \ hence \ V_P = I_P Z_P = (5.08)(50) = 254V$$

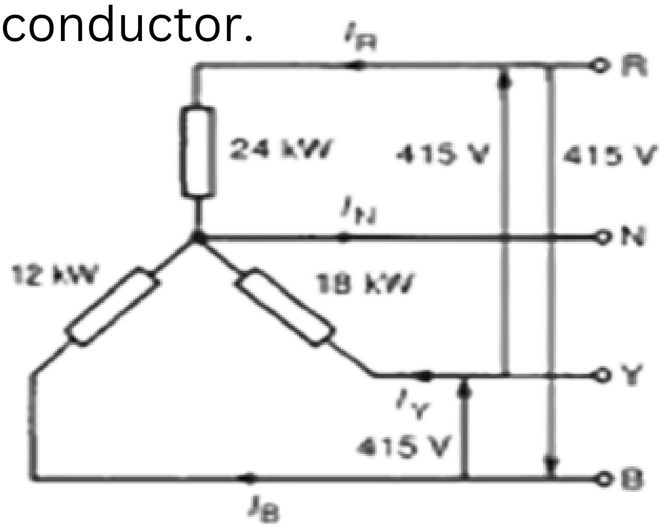
Line voltage

$$V_L = \sqrt{3}V_p = \sqrt{3}(254) = 440V$$

EXAMPLE 3:

A 415V, 3-phase, 4 wire, star-connected system supplies three resistive loads as shown in below figure. Determine :

(a) the current in each line and
(b) the current in the neutral conductor.



Solution

For a star connected system $V_L = \sqrt{3}V_p$ hence,

$$V_P = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 240V$$

Since for resistive load;

$$current, I = \frac{power, P}{voltage, V}$$

Then

$$I_R = \frac{P_R}{V_R} = \frac{24000}{240} = 100A$$

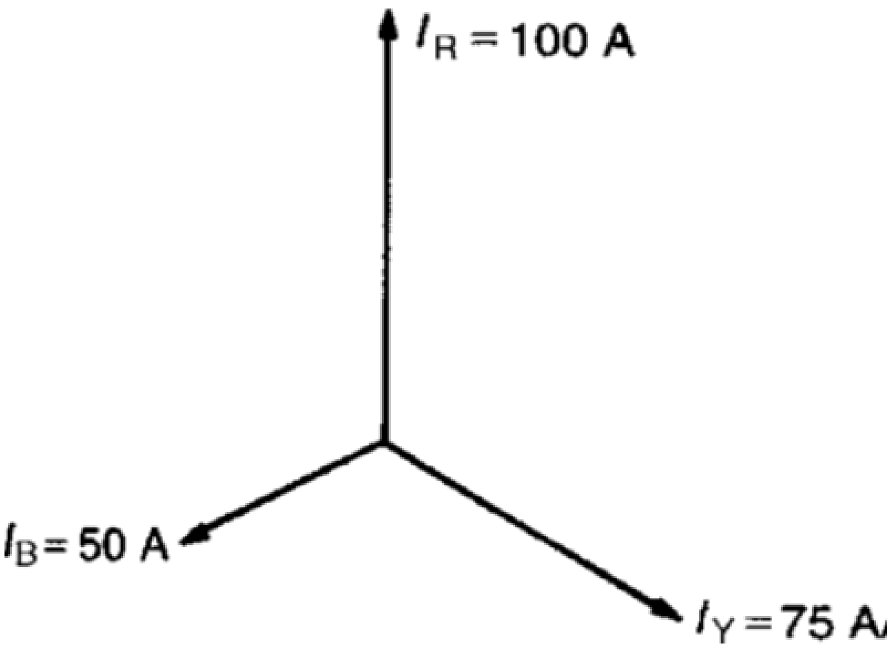
$$I_Y = \frac{P_Y}{V_Y} = \frac{18000}{240} = 75A$$

$$I_B = \frac{P_B}{V_B} = \frac{12000}{240} = 50A$$

EXAMPLE 4:

The three line currents are shown in the phasor diagram. Since each load is resistive the currents are in phase with the phase voltages and are hence mutually displaced by 120 ◦ . The current in the neutral conductor is given by IN =IR +IY +IB phasorially.

Answer:



EXAMPLE 4:

Answer:

By calculation,

By calculation; I_R consider at 90^o, I_B at 210^o and I_Y at 330^o

Then;

$$total\ horizontal\ component = I_R \cos 90 + I_B \cos 210 + I_Y \cos 330$$

$$total\ horizontal\ component = 100 \cos 90 + 50 \cos 210 + 75 \cos 330$$

$$total\ horizontal\ component = 0 + (-43.3) + 64.95$$

$$total\ horizontal\ component = 21.65$$

$$total\ vertical\ component = I_R \sin 90 + I_B \sin 210 + I_Y \sin 330$$

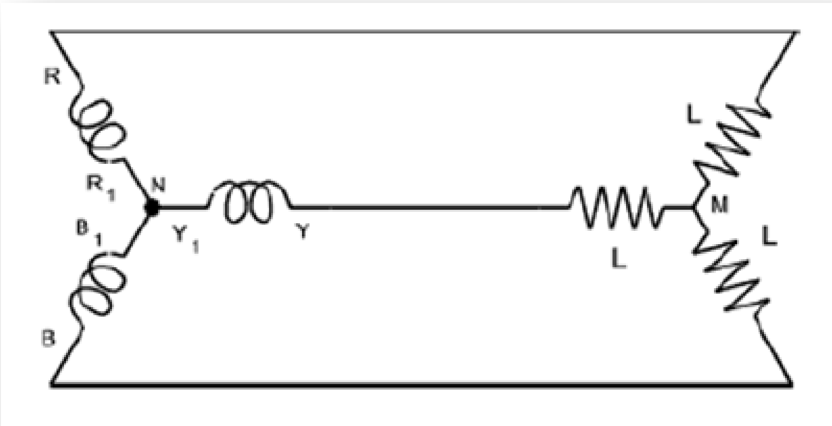
$$total\ vertical\ component = 100 \sin 90 + 50 \sin 210 + 75 \sin 330$$

$$total\ vertical\ component = 100 + (-25) + (-37.5)$$
$$total\ vertical\ component = 37.5$$

$$\mathrm{I_N} = \sqrt{(\text{total horizontal component})^2 + (\text{total vertical component})^2}$$

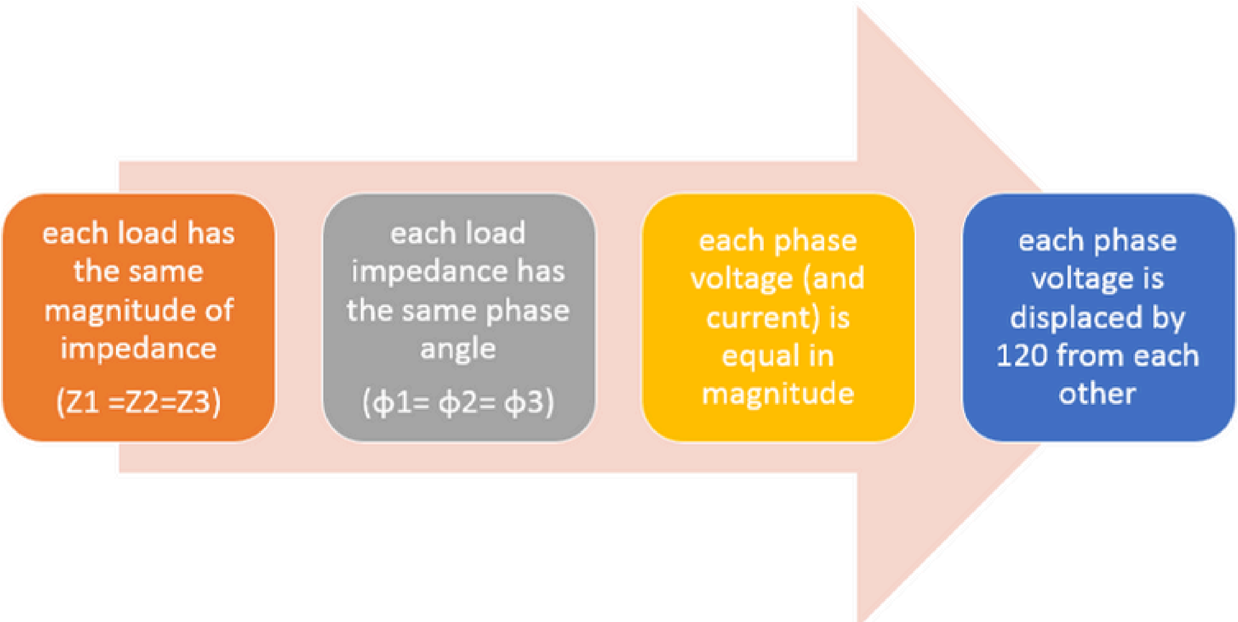
$$\mathrm{I_N} = \sqrt{(21.65)^2 + (37.5)^2} = 43.3\mathrm{A}$$

DEFINE BALANCED LOAD IN A THREE PHASE



1) Hence with balanced loads there is no load in the neutral line at any instant and the neutral line can be removed resulting in the three wire star system as shown below.

2) It can be proved that at every instant, for balanced loads the algebraic sum of the currents flowing in the three conductors is zero



Calculate Total Power For 3 Phase System Using Formula P = √3 VL IL cos θ

Single phase power :

Each phase : $V_{PH} . I_{PH} . \text{kos}\ \varphi$

3 phase power :

Phase element : $3 . V_{PH} . I_{PH} . \text{kos}\ \varphi$

Line element : $\sqrt{3} . V_L . I_L . \text{kos}\ \varphi$

Real power (P) : - Also call true power
- Measure in watt (W)
- $\sqrt{3}\, V_L I_L \cos\theta = 3\, V_p I_p \cos\theta$

Reactive power (Q) : - measure in volt amperes reactive (VAR).
- $\sqrt{3}\, V_L I_L \sin\theta = 3\, V_p I_p \sin\theta$

Apparent power (S) : - measure in volt amperes (VA).
- $\sqrt{(P^2 + Q^2)} = 3\, V_p I_p$

STAR AND DELTA CONNECTION SUMMARY

CRITERIA	STAR CONNECTION	DELTA CONNECTION
Symbol	⅄ @ Y	△
Voltan	$V_L = \sqrt{3}\ V_{PH}$	$V_L = V_{PH}$
Current	$I_L = I_{PH}$	$I_L = \sqrt{3}\ I_{PH}$
Balance condition	$I_N = I_R + I_Y + I_B = 0$	$V_{close\ circuit} = V_{RY} + V_{YB} + V_{BR} = 0$
1 phase power in each coil	$V_{PH} . I_{PH}$. kos φ	$V_{PH} . I_{PH}$. kos φ
3 phase power: (i) Phase element: (ii) Line element:	3 .$V_{PH} . I_{PH}$. kos φ $\sqrt{3}$.$V_L . I_L$. kos φ	3 .$V_{PH} . I_{PH}$. kos φ $\sqrt{3}$.$V_L . I_L$. kos φ

SOLVE PROBLEM RELATED TO THREE PHASE SYSTEM

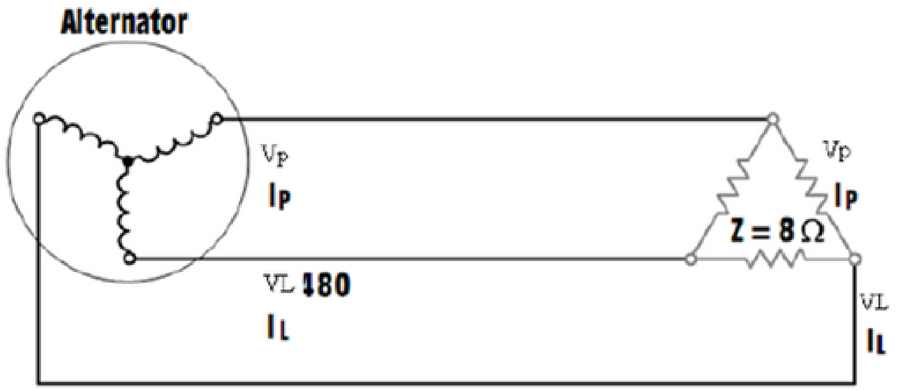
Example 5:

A wye-connected three-phase alternator supplies power to a delta-connected resistive load, Figure below. The alternator has a line voltage of 480V. Each resistor of the delta load has 8 Ω of resistance. Find the following values:

a) VL(Load) – line voltage of the load

b) VP(Load) – phase voltage of the load

c) IP(Load) – phase current of the load



SOLVE PROBLEM RELATED TO THREE PHASE SYSTEM

Solution Example 5:

- The load is connected directly to the alternator. Therefore, the line voltage supplied by the alternator is the line voltage of the load.

$$V_L(\text{Load}) = 480\ V$$

- The three resistors of the load are connected in a delta connection. In a delta connection, the phase voltage is the same as the line voltage.

$$Vp(\text{Load}) = V_L(\text{Load})$$
$$Vp(\text{Load}) = 480$$

- Each of the three resistors in the load is one phase of the load. Now that the phase voltage is known (480 V), the amount of phase current can be computed using Ohm's Law.

$$Ip(load) = \frac{Vp(load)}{Z}$$
$$Ip(load) = \frac{480}{8}$$
$$Ip(load) = 60\,A$$

- The three load resistors are connected as a delta with 60 A of current flow in each phase. The line current supplying a delta connection must be √3 times greater than the phase current.

$$I_{L(Load)} = I_{P(Load)} \times 1.732$$
$$I_{L(Load)} = 60 \times 1.732$$
$$I_{L(Load)} = 103.92\ A$$

** note:
$\sqrt{3} = 1.732$

- The alternator must supply the line current to the load or loads to which it is connected. In this example, only one load is connected to the alternator. Therefore, the line current of the load will be the same as the line current of the alternator.

$$I_{L(Alt)} = 103.92\ A$$

- The phase windings of the alternator are connected in a wye connection. In a wye connection, the phase current and line current are equal. The phase current of the alternator will, therefore, be the same as the alternator line current.

$$I_{P(Alt)} = 103.92\ A$$

SOLVE PROBLEM RELATED TO THREE PHASE SYSTEM

continue Solution Example 5:

- The phase voltage of a wye connection is less than the line voltage by a factor of the square root of 3. The phase voltage of the alternator will be:

$$Vp(Alt) = \frac{V_{L(Alt)}}{\sqrt{3}}$$
$$Vp(Alt) = \frac{480}{1.732}$$
$$Vp(Alt) = 277.13V$$

- In this circuit, the load is pure resistive. The voltage and current are in phase with each other, which produces a unity power factor of 1. The true power in this circuit will be computed using the formula:

$$P = \sqrt{3} \times V_{L(Alt)} \times I_{L(Alt)} \times PF$$
$$P = 1.732 \times 480 \times 103.92 \times 1$$
$$P = 86394.93W$$
$$P = 86.39KW$$

EXERCISE

QUESTION 1

Three identical capacitors are connected in delta to a 415V, 50 Hz, 3-phase supply. If the line current is 15A, determine the capacitance of each of the capacitors.

QUESTION 2

Three coils each having resistance 3Ω and inductive reactance 4Ω are connected;
(i) in star
(ii) in delta
to a 415V, 3-phase supply.

Calculate for each connection
(a) the line and phase voltages
(b) the phase and line currents

QUESTION 3

Three 12Ω resistors are connected in star to a 415V, 3-phase supply. Determine the total power dissipated by the resistors.

QUESTION 4

The input power to a 3-phase a.c. motor is measured as 5kW. If the voltage and current to the motor are 400V and 8.6A respectively, determine the power factor of the system.

QUESTION 5

Three balanced loads with each of resistance is 20Ω and the inductance 40mH are connected in delta to 415V, 50Hz, three phase supply. Calculate the total power dissipated, apparent power and reactive power in the circuit.

EXERCISE QUESTION 1

ANSWER:

For a delta connection;

$$I_L = \sqrt{3} I_P$$

Hence a phase current;

$$I_P = \frac{I_L}{\sqrt{3}}$$

$$I_P = \frac{15}{\sqrt{3}}$$

$$I_P = 8.66A$$

Capacitive reactance per phase (since for a delta connection $V_L = V_P$

$$X_C = \frac{V_P}{I_P} = \frac{V_L}{I_P} = \frac{415}{8.66} = 47.92\Omega$$

$$X_C = \frac{1}{2\pi f C}$$

$then$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(50)(47.92)} = 66.43\mu F$$

EXERCISE QUESTION 2

ANSWER:

For a star connection $I_L = I_P$ and $V_L = \sqrt{3}V_p$:

A 415 V, 3-phase supply means that the line voltage $V_L = 415V$

Phase voltage

$$V_P = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 240V$$

Impedance per phase

$$Z_P = \sqrt{R^2 + X_L^2} = \sqrt{3^2 + 4^2} = 5\Omega$$

Phase current

$$I_P = \frac{V_p}{Z_p} = \frac{240}{5} = 48A$$

Line current

$$I_L = I_P = 48A$$

For a delta connection $V_L = V_p$ and $I_L = \sqrt{3}I_p$:

Line voltage $V_L = 415V$

Phase voltage $V_P = V_L = 415V$

Phase current

$$I_P = \frac{V_p}{Z_p} = \frac{415}{5} = 83A$$

Line current

$$I_L = \sqrt{3}I_p = \sqrt{3}(83) = 144A$$

EXERCISE QIESTION 3

ANSWER:

Line voltage, V_L=415V

Phase voltage

$$V_P = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 240V$$

$$I_P = \frac{V_p}{Z_p} = \frac{240}{12} = 20A$$

For a star connection

$$I_L = I_P = 20A$$

For purely resistive load, the power factor $= \cos\phi = 1$

Hence

$$P = \sqrt{3}V_L I_L \cos\phi = \sqrt{3}(415)(20)(1) = 14.4kW$$

EXERCISE QUESTION 4

ANSWER:

Power $P = 5000W$,

Line voltage $V_L = 400\ V$,

Line current, $I_L = 8.6A$

Power,

$$P = \sqrt{3}V_L I_L \cos\phi$$

Hence

$$power\ factor = \frac{P}{\sqrt{3}V_L I_L}$$

$$power\ factor = \frac{5000}{\sqrt{3}(400)(8.6)} = 0.839$$

EXERCISE QUESTION 5

ANSWER:

Inductive reactance

$$X_L = 2\pi f L = 2\pi(50)(40m) = 12.57\ \Omega$$

Impedance of each phase

$$Z_P = \sqrt{R^2 + X_L^2} = \sqrt{20^2 + 12.57^2} = 23.62\ \Omega$$

Or

$$Z = R + jX_L = 20 + j12.57 = 23.62\angle 32.15°\ \Omega$$

Phase current

$$I_P = \frac{V_p}{Z_p} = \frac{415}{23.62} = 17.57A$$

Line current

$$I_L = \sqrt{3} I_p = \sqrt{3}(17.57) = 31.43A$$

Total power dissipated

$$\mathrm{P} = \sqrt{3}\mathrm{V_L I_L} \cos\theta$$
$$P = \sqrt{3} \times 415 \times 31.43 \times \cos 32.15°$$
$$P = 19.128\ kW$$

Apparent power

$$\mathrm{S} = \sqrt{3}\mathrm{V_L I_L}$$
$$S = \sqrt{3} \times 415 \times 31.43$$
$$S = 22.59\ kVA$$

Reactive power

$$\mathrm{Q} = \sqrt{3}\mathrm{V_L I_L} \sin\theta$$
$$Q = \sqrt{3} \times 415 \times 31.43 \times \sin 32.15°$$
$$Q = 12.02\ \ kVAR$$

TUTORIAL

1. List THREE (3) / FOUR (4) advantages of a three phase system.
2. With the aid of a diagram, detail the circuit diagram for three phase system in STAR connection.
3. A three-phase load are supplied with line voltage 415V and frequency 50Hz. Each phase consists of 15Ω resistor and connected in series with inductor, 0.05H in DELTA connection. Calculate the phase and line current.
4. Three balanced loads with each of resistance is 10Ω and the inductance 42mH are connected in delta to a 415V, 50Hz, three-phase supply. Calculate the total power dissipated, apparent power and reactive power in the circuit.
5. With the aid of circuit diagram, differentiate between the star and delta connection in a three-phase system.
6. List the phasor equations of the three phase voltages, EA, EB and EC generated by a three-phase generator and sketch its phasor diagram.
7. A three load resistance of 60Ω is connected in delta to a 500V, 3 phase supply. Determine phase voltage and line current for the system.
8. A three coil balanced positive sequence Y-connected source with EAN = 100 <10° V is connected to a Δ- connected balance load with (8+j4) Ω per phase. Calculate the phase current and line current.
9. Each phase in Delta connection consists of 10Ω resistor and connected in series with the inductor, 0.019H. This three phase load is supplied with line voltage, 415V and frequency, 50Hz. Calculate the phase and line current.
10. A three coil balanced load has 10Ω resistor and 100mH inductor, connected in star connection with a three phase supply system with 415V, 50Hz. Calculate the phase current (IPH), the line current (IL) and the power in three phase.

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