BIOMEDICAL SIGNAL PROCESSING

"Just as the characters in anime overcome insurmountable odds, so do biomedical engineers push the limits of what is known to heal, restore, and innovate for a better tomorrow."

# THE ANIME JOURNEY OF DIGITAL FILTERS

BIOMEDICAL SIGNAL PROCESSING

A fantastic story by :


SITI SABARIAH SALIHIN

ASLINDA ZAMAHSARI & DR. FAZIDA ADLAN

JABATAN KEJURUTERAAN ELEKTRIK

POLITEKNIK SULTAN SALAHUDDIN ABDUL AZIZ SHAH

Biomedical Signal Processing

# THE DESIGN OF DIGITAL FILTER (FIR & IIR)

Siti Sabariah Salihin

Aslinda Zamahsari & Dr. Fazida Adlan

"Just as an anime protagonist hones their skills to perfection, digital filters refine biomedical signals, unveiling the true power and potential within."

**THE DESIGN OF DIGITAL FILTER**

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First Publication 2024

eISBN No: 978-629-7667-31-7

Published by:

**UNIT PENERBITAN**
Politeknik Sultan Salahuddin Abdul Aziz Shah
Persiaran Usahawan,
Seksyen U1,
40150 Shah Alam,
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e ISBN 978-629-7667-31-7
9 786297 667317

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"In the world of biomedical engineering, we blend the art of healing with the science of technology, crafting a future where every heartbeat is a symphony of innovation."

# PREFACE

Welcome to "Designing Filters for Biomedical Signal Processing," a comprehensive guide that aims to bridge the gap between theoretical concepts and practical applications in biomedical engineering. This e-book is designed for students, and lecturers who seek a deeper understanding of filtering techniques and their applications in biomedical signal processing, particularly in analyzing signals such as electrocardiograms (ECGs), electroencephalograms (EEGs), and more.

The field of biomedical signal processing is evolving rapidly, driven by advancements in technology and the increasing demand for accurate and efficient diagnostic tools. Filtering plays a crucial role in this domain, helping to eliminate noise and enhance the quality of the signals for better interpretation and analysis. This e-book aims to equip you with the knowledge and skills needed to design and implement effective filters tailored to the unique characteristics of biomedical signals.

I hope that this e-book serves as a valuable resource on your journey to mastering filter design for biomedical signal processing. May it inspire you to explore the exciting possibilities in this field and contribute to the advancement of healthcare technology.

Thank you for choosing this e-book. Happy learning!

Biomedical Signal Processing

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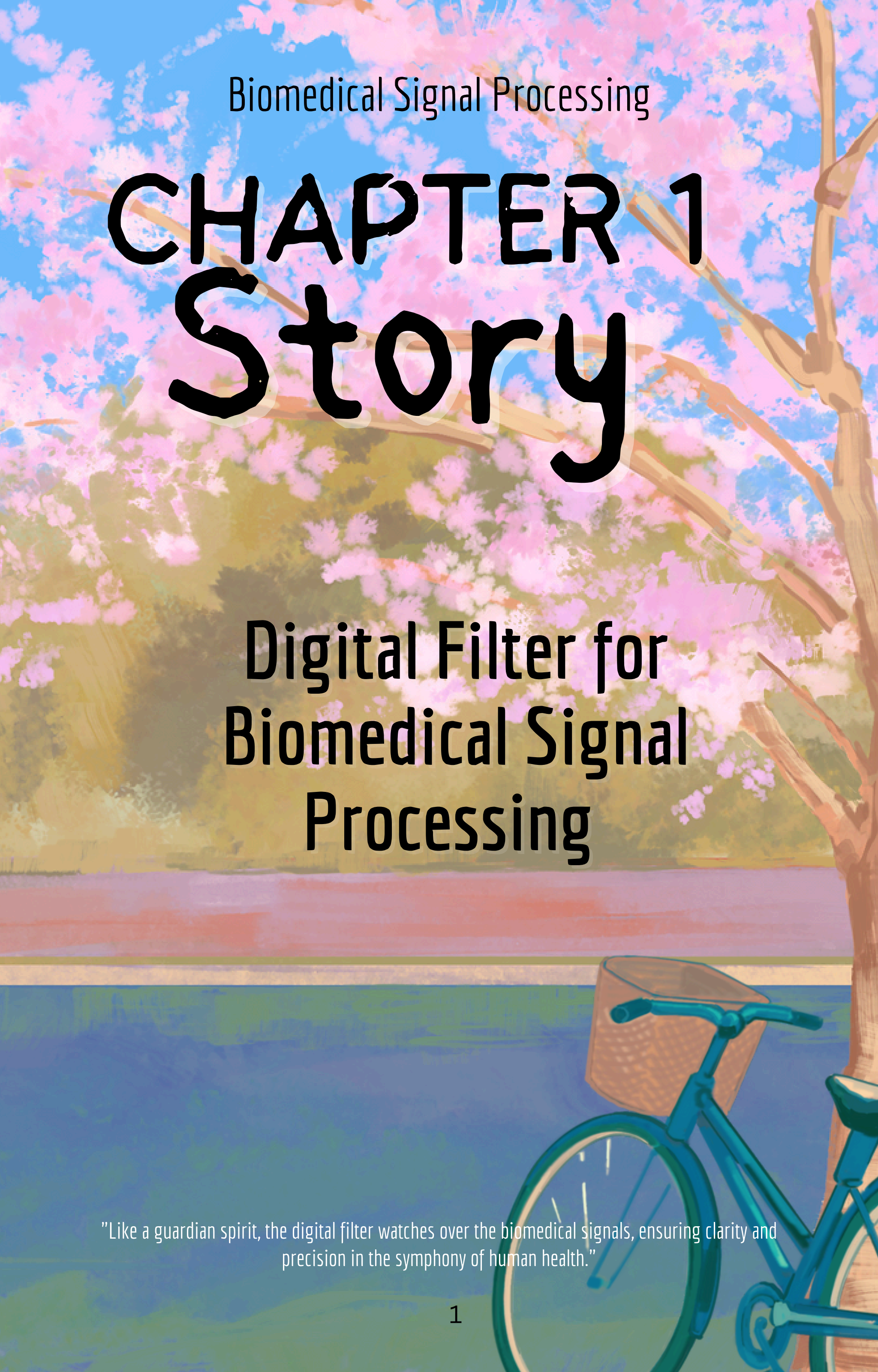
"In the realm of biomedical signal processing, digital filters are the heroes, purifying the chaotic noise to reveal the hidden symphonies of life."

# CHAPTER 1
# Story

## Digital Filter for Biomedical Signal Processing



"Like a guardian spirit, the digital filter watches over the biomedical signals, ensuring clarity and precision in the symphony of human health."

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# DIGITAL FILTER FOR BIOMEDICAL SIGNAL PROCESSING

## 1.0 INTRODUCTION

In the rapidly evolving field of biomedical engineering, digital filters play a pivotal role in the analysis and interpretation of physiological signals. Biomedical signal processing involves the extraction of meaningful information from various biological signals such as electrocardiograms (ECG), electroencephalograms (EEG), and electromyograms (EMG). These signals often contain significant noise and interference, which can obscure the underlying physiological information. Digital filters, through their ability to selectively enhance or suppress specific frequency components, are essential tools for cleaning and refining these signals.

**Figure 1:** Electrocardiograms (ECG)

"In the world of biomedical signal processing, digital filters are the magical artifacts that bring clarity and focus to the intricate dance of life's signals."

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# DIGITAL FILTER FOR BIOMEDICAL SIGNAL PROCESSING

Digital filters operate by manipulating discrete-time signals, typically obtained through the analog-to-digital conversion of continuous biological signals. They employ mathematical algorithms to remove unwanted noise and artifacts, allowing for the accurate detection and analysis of critical physiological features.

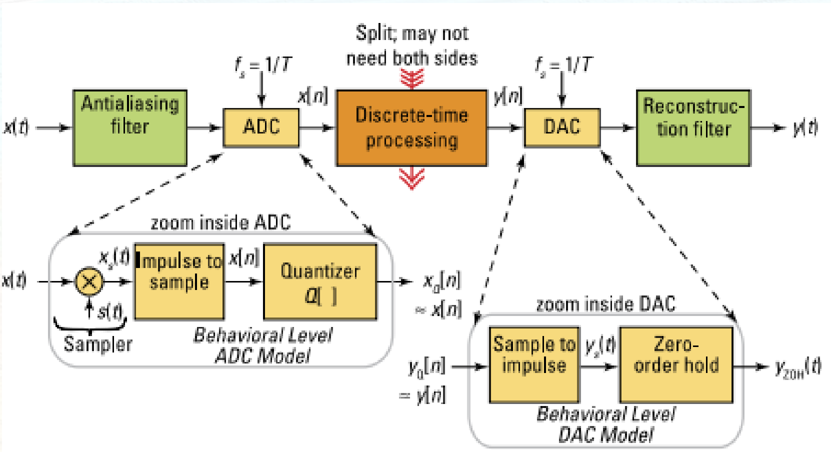


**Figure 2:** A block diagram showing how a discrete time signal processing system (top center) inter faces with continuous time signals, both input and output scenarios.

There are various types of digital filters, including low-pass, high-pass, band-pass, and band-stop filters, each designed to target specific frequency ranges based on the characteristics of the signal and the type of noise present.

"In the world of biomedical engineering, we blend the art of healing with the science of technology, crafting a future where every heartbeat is a symphony of innovation."

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# DIGITAL FILTER FOR BIOMEDICAL SIGNAL PROCESSING

The application of digital filters in biomedical signal processing is crucial for various diagnostic and monitoring purposes. For instance, in ECG signal processing, digital filters are used to eliminate power line interference and baseline wander, enhancing the clarity of the signal for accurate heart rate and rhythm analysis.

In EEG signal processing, digital filters help isolate brainwave frequencies of interest, facilitating the study of neural activity and the diagnosis of neurological disorders.

The development and implementation of digital filters require a deep understanding of both signal processing principles and the physiological characteristics of the signals being analyzed. Advances in digital filter design, driven by improvements in computational power and algorithm efficiency, continue to enhance the capabilities of biomedical signal processing, leading to more accurate diagnostics, better patient outcomes, and the advancement of personalized medicine.

In summary, digital filters are indispensable in the field of biomedical signal processing, providing the means to extract valuable information from noisy biological signals. Their role in improving the accuracy and reliability of biomedical analyses underscores the importance of this technology in modern healthcare and medical research.

"In the anime of life, biomedical engineers are the heroes who use their knowledge and creativity to design the armor that protects and heals humanity."

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# DIGITAL FILTER FOR BIOMEDICAL SIGNAL PROCESSING

## 1.1 PROPERTIES OF DIGITAL FILTER IN BIOMEDICAL SIGNAL PROCESSING

Digital filters possess several key properties that make them suitable for processing biomedical signals. Here are some of the main properties, along with brief explanations and examples for each:

**Linearity**

- Linear filters obey the superposition principle, meaning the response caused by two or more stimuli is the sum of the responses that would have been caused by each stimulus individually.
- **Example:** In ECG signal processing, linear filters can be used to simultaneously remove baseline wander and high-frequency noise, as the combined effect of filtering is the sum of the individual effects.

**Time-Invariance**

- A time-invariant filter has consistent properties over time, meaning its output depends only on the input signal and not on when the input is applied.
- **Example:** A filter designed to remove 50 Hz power line interference from an EEG signal will perform the same function regardless of when the interference occurs within the signal.

"In the realm of biomedical signal processing, digital filters are the heroes, purifying the chaotic noise to reveal the hidden symphonies of life."

# DIGITAL FILTER FOR BIOMEDICAL SIGNAL PROCESSING

## Causality

- A causal filter's output at any time depends only on past and present inputs, not future inputs. This is essential for real-time processing applications.
- **Example:** In real-time monitoring of a patient's ECG, a causal filter ensures that the output at any given time is based on current and previous signal values, enabling immediate response and analysis.

## Stability

- A stable filter produces bounded output for a bounded input, preventing unbounded or divergent outputs.
- **Example:** When processing EMG signals to monitor muscle activity, a stable filter ensures that sudden spikes in signal amplitude (due to noise) do not cause the filter to produce erratic outputs.

## Frequency Selectivity

- Digital filters can be designed to pass certain frequency components while attenuating others. This property is fundamental in isolating specific signal features.
- **Example:** A band-pass filter can be used in EEG signal processing to isolate alpha waves (8-13 Hz) by attenuating frequencies outside this range, allowing for focused analysis of brain activity in this band.

*"In the hands of a biomedical engineer, the human body is not just a vessel but a canvas for technological artistry and scientific marvels."*

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# DIGITAL FILTER FOR BIOMEDICAL SIGNAL PROCESSING

## Phase Response

- The phase response of a filter indicates how the phase of different frequency components is shifted by the filter. Linear phase filters maintain the waveform shape of filtered signals.
- **Example:** In speech signal processing for medical applications, linear phase filters are used to avoid distortion of the waveform, which is crucial for accurate speech recognition and analysis.

## Impulse Response

- The impulse response characterizes how a filter responds to a single impulse input. It defines the filter's behavior and can be used to understand its effect on any input signal.
- **Example:** The impulse response of a low-pass filter applied to ECG signals shows how it smooths out rapid changes in the signal, thus reducing high-frequency noise.

## Computational Efficiency

- Efficient filters require fewer computations, making them suitable for real-time applications where processing speed is critical.
- **Example:** Fast digital filters are used in portable medical devices for continuous glucose monitoring, where rapid data processing is essential for timely feedback and intervention.

"Like a guardian spirit, the digital filter watches over the biomedical signals, ensuring clarity and precision in the symphony of human health."

# DIGITAL FILTER FOR BIOMEDICAL SIGNAL PROCESSING

## Adaptability

- Adaptive filters can change their characteristics in response to changing signal conditions, making them suitable for dynamic environments.
- **Example:** In fetal heart rate monitoring, adaptive filters adjust to varying signal quality and interference levels, ensuring consistent and reliable heart rate detection throughout the monitoring period.

**Figure 3:** The electromyogram (EMG) - electrical activity of the muscle cells.

"With the precision of a master swordsman, digital filters slice through the noise, unveiling the true essence of biomedical signals."

## 1.2 LOW PASS FILTER

- A simple digital filter is a basic yet effective tool used to process biomedical signals by selectively allowing certain frequency components to pass while attenuating others. One of the most common types of simple digital filters is the low-pass filter.

- **DEFINITION:** A low-pass filter allows low-frequency signals to pass through while attenuating (reducing the amplitude of) high-frequency signals. This type of filter is particularly useful in biomedical signal processing to remove high-frequency noise from physiological signals.

- **Function:** The primary function of a low-pass filter is to smooth out rapid fluctuations in the signal, which are often caused by noise or other unwanted high-frequency components.

- **Example** in Biomedical Signal Processing: Consider the example of an ECG (electrocardiogram) signal. An ECG signal is used to monitor the electrical activity of the heart and is critical for diagnosing various cardiac conditions. However, the ECG signal can be contaminated with high-frequency noise from muscle contractions (EMG noise) or external electrical interference.

"In the story of modern medicine, digital filters are the legendary swords, cutting through the noise and delivering the pure, untainted essence of biomedical signals."

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# Digital Filter for Biomedical Signal Processing

- **Implementation:** A simple digital low-pass filter can be implemented using a moving average filter, which calculates the average of a fixed number of consecutive signal samples.

- Mathematical Representation: For a moving average filter with a window siz $y[n] = \frac{1}{N}\sum_{k=0}^{N-1} x[n-k]$ ple n is given by:

**Where:**

- y[n]is the filtered output signal.
- x[n] is the input signal.
- N is the number of samples in the averaging window.

- **Example:** Suppose we have an ECG signal contaminated with high-frequency noise. Applying a moving average low-pass filter with a window size of 5 samples will smooth out the noise, making the underlying heartbeats more discernible. This filtered signal will be easier to analyze, allowing healthcare professionals to accurately detect and diagnose heart conditions.

"The heart of a biomedical engineer beats in harmony with the pulse of innovation, always striving to create devices that bridge the gap between human fragility and technological strength."

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# Digital Filter for Biomedical Signal Processing

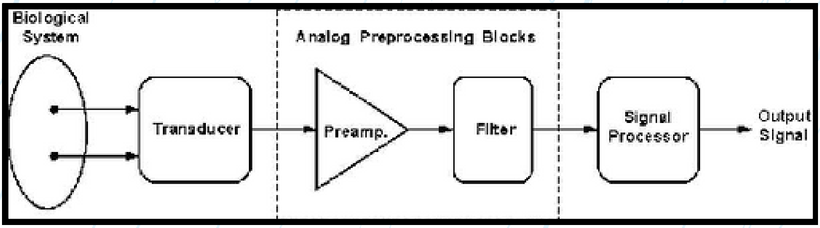



**Figure 4:** Low Pass Filter System for Biomedical Application

"The heart of a biomedical engineer beats in harmony with the pulse of innovation, always striving to create devices that bridge the gap between human fragility and technological strength."

# The action potential - mother of all biological signals:

A. The electroneurogram (ENG) - propagation of nerve action potential

B. The electromyogram (EMG) - electrical activity of the muscle cells

C.The electrocardiogram (ECG) - electrical activity of the heart / cardiac cells

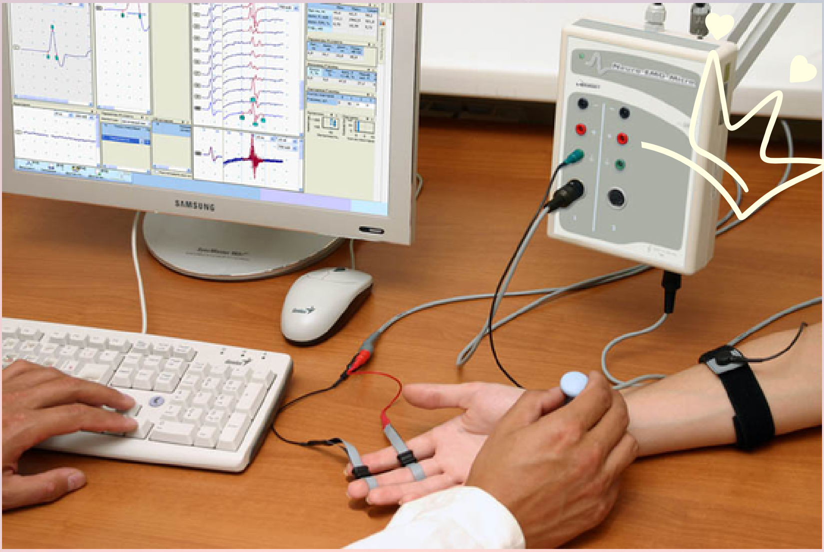
D. The electroencephalogram (EEG) - electrical activity of the brain

E. The electrogastogram (EGG) - electrical activity of the stomach

F. The phonocardiogram (PCG) - audio recording of the heart's mechanical activity

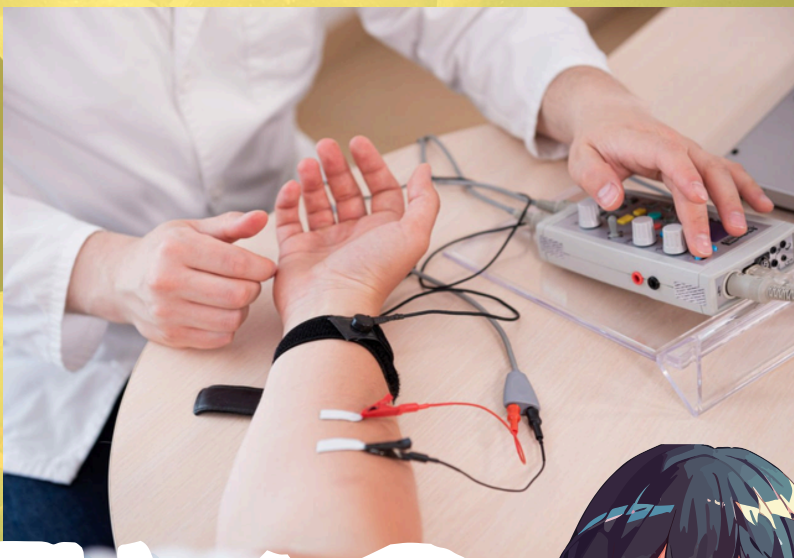
G. The electoretinogram (ERG) - electrical activity of the retinal cells

# A. The electroneurogram (ENG) - propagation of nerve action potential



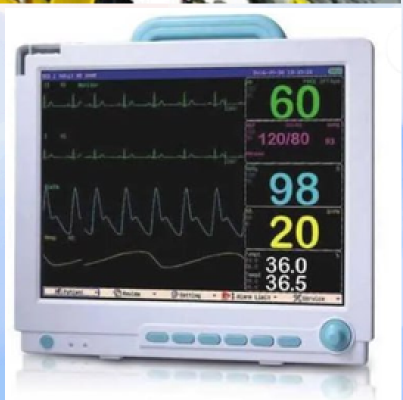
ENG

# B. The electromyogram (EMG) - electrical activity of the muscle cells



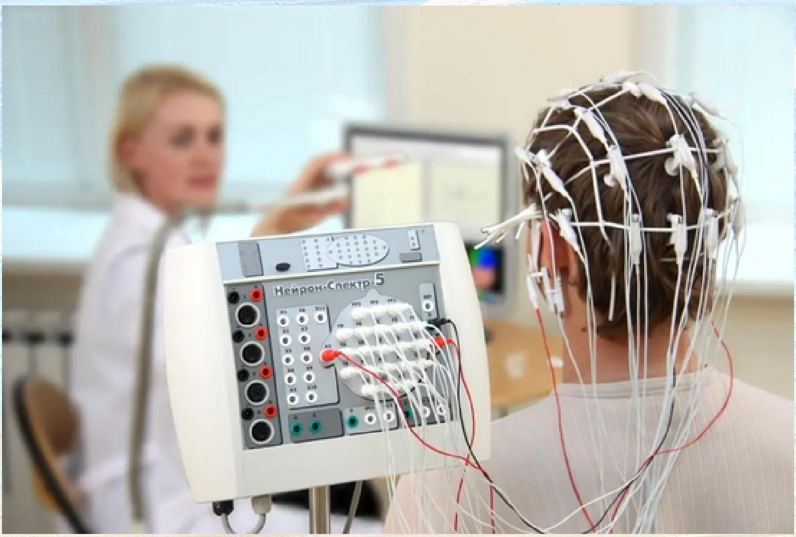
EMG

# C. The electrocardiogram (ECG) - electrical activity of the heart / cardiac cells




ECG

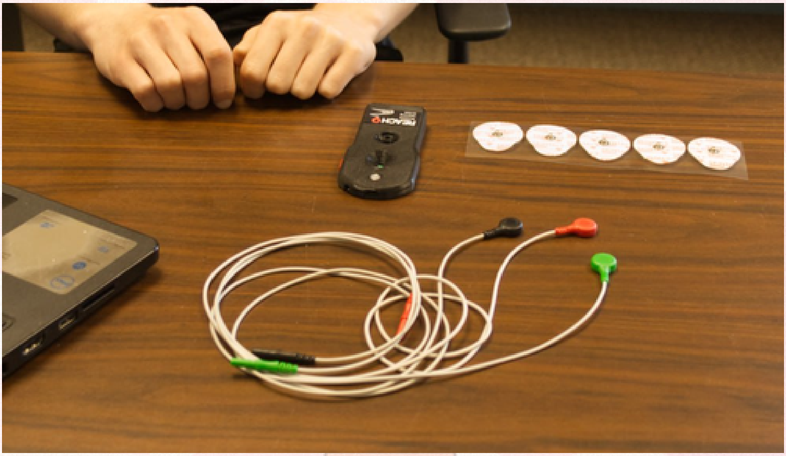
# D. The electroencephalogram (EEG) - electrical activity of the brain

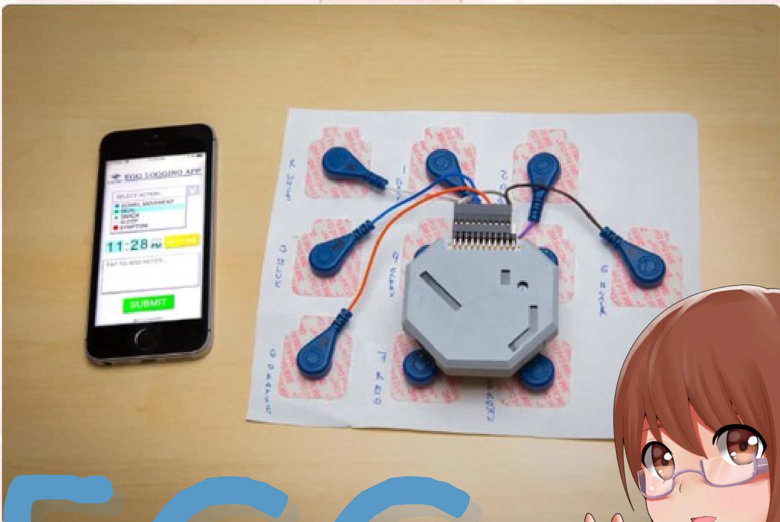




# EEG

# E. The electrogastrogram (EGG) - electrical activity of the stomach



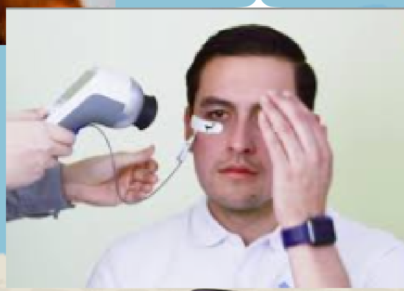



EGG

# F. The phonocardiogram (PCG) - audio recording of the heart's mechanical activity

# PCG

## G. The electroretinogram (ERG) - electrical activity of the retinal cells



# ERG

"In the epic tale of biomedical engineering, digital filters are the unsung warriors, defending the purity of our physiological signals against the invasion of noise."

# Chapter 2 Story

## FINITE IMPULSE RESPONSE (FIR) FILTER

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# Finite Impulse Response (FIR) Filter

## 2.0 INTRODUCTION

A finite impulse response (FIR) filter has an impulse response with a finite number of terms, unlike an infinite impulse response (IIR) filter, which produces an infinite sequence of output terms when a unit impulse is applied. FIR filters are typically implemented in a non-recursive manner, meaning there is no feedback loop in the calculation of the output data. The filter's output is determined solely by the current and previous inputs. This characteristic has significant implications for digital filter design and applications.

## 2.1 CHARACTERISTIC OF FIR

FIR filters have a finite number of terms in their impulse response, meaning their response to an impulse input settles to zero in a finite time period. This is in contrast to IIR filters, which have an infinite impulse response.

- **Non-Recursive Implementation:**

FIR filters are typically implemented without feedback, meaning the output depends only on the current and past input values, not on past output values. This characteristic simplifies the design and ensures stability.

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# Finite Impulse Response (FIR) Filter

- **Key Properties:**

FIR filters are known for their inherent stability and linear phase response, which helps maintain the shape of the signal waveform and avoids phase distortion.

- **Applications in Biomedical Signal Processing:**

FIR filters are used to process various biomedical signals, such as ECG and EEG, to remove noise and artifacts. For instance, low-pass FIR filters can smooth out high-frequency noise, while high-pass FIR filters can remove baseline wander.

- **Design Techniques:**

The design of FIR filters involves methods like windowing and frequency sampling to achieve desired frequency response characteristics, which are crucial for accurate and reliable biomedical signal processing.

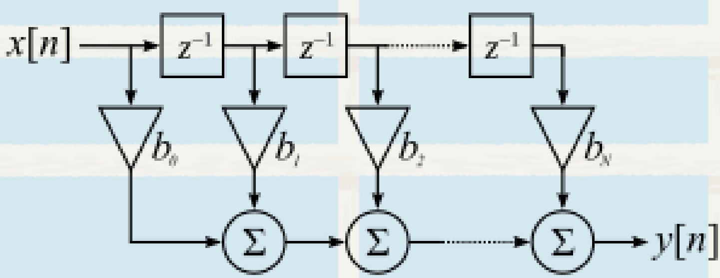


**Figure 5:** A direct form discrete-time FIR filter of order N

"In the world of signal processing, like in the epic battles of our favorite anime, precision and control are our greatest weapons."

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# Finite Impulse Response (FIR) Filter

- According to their function, the FIR filters can be classified into four categories, which are lowpass filter, highpass filter, bandpass filter, and bandstop filter.

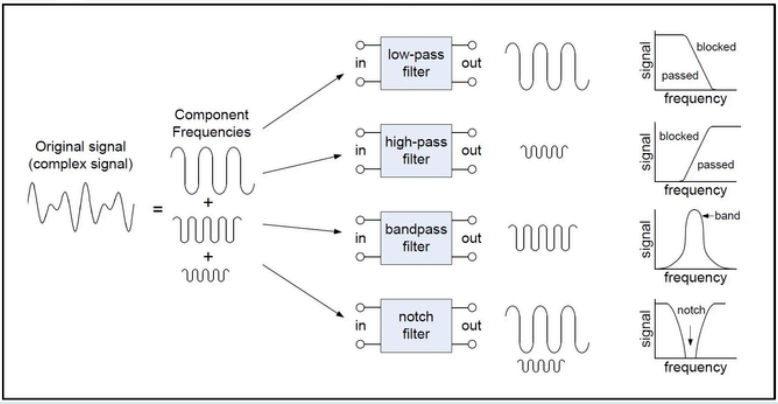


**Figure 6:** A basic depiction of the four major filter types.

- FIR filters in biomedical signal processing play a crucial role by providing high-performance adaptive filtering through distributed arithmetic circuits, reducing hardware requirements, and enhancing computation speed

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# Finite Impulse Response (FIR) Filter

**Figure 5** shows the top part is an N-stage delay line with N + 1 taps. Each unit delay is a z-1 operator in Z-transform notation. FIR filters can be discrete-time or continuous-time, and digital or analog.

For a **causal discrete-time FIR filter** of order N, each value of the output sequence is a weighted sum of the most recent input values:

$$\begin{aligned} y[n] &= b_0 x[n] + b_1 x[n-1] + \cdots + b_N x[n-N] \\ &= \sum_{i=0}^{N} b_i \cdot x[n-i], \end{aligned}$$

**where:**

- x **[n]** is the input signal,
- y **[n]** is the output signal,
- N is the filter order; an Nth-order filter has N+1 terms on the right-hand side
- bi is the value of the impulse response at the i'th instant for 0 <= i <= N of Nth-order FIR filter. If the filter is a direct form FIR filter then bi is also a coefficient of the filter.

This computation is also known as discrete convolution.

"In the grand tale of technology and health, our filters are the guardians, ensuring that every vital sign is captured with the utmost fidelity."

# Finite Impulse Response (FIR) Filter

## 2.2 FIR FILTER REVIEW USING Z-TRANSFORM

$$y[n] = \sum_{k=0}^{M} b_k x[n-k].$$

is general difference equation for FIR filters.

- **Filter Order = M:** No. of memory blocks required in the filter implementation
- **Filter Length, L = M+1:** Total No. of samples required in calculating the output, M from memory (past) and one present sample
- **Filter coefficients {bk}:** Completely defines an FIR filter. All the properties of the filter can be understood through the coefficients.
- Z transform of impulse response h[n] results in transfer function" it is also known as system function.

$$H(z) = \sum_{n} h[n] z^{-n}$$

"In the saga of signal processing, designing a filter is our magic spell – transforming chaos into harmony, just as an anime character transforms through sheer willpower."

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# Finite Impulse Response (FIR) Filter

From the previous lecture recall that Transfer Funtion,

$$H(z)=\frac{Y(z)}{X(z)}$$

$$\Rightarrow Y(z)=H(z)X(z)$$

- Notice the mathematical simplicity of the above result Convolution becomes a simple multiplication.

$$h[n]*x[n]\leftrightarrow H(z)X(z)$$

## 2.3 CALCULATING THE OUTPUT OF A FIR FILTER USING Z - TRANSFORMS

- Step Involve:

1). Find the Z transform of input signal x[n]

$$X(z)=\sum_{n=-\infty}^{\infty}x[n]z^{-n}$$

2). Find the Z transform of input signal x[n]

$$H(z)=\sum_{n=0}^{M}h[n]z^{-n}$$

"In the same way a master alchemist refines elements, we engineer filters to purify and perfect the signals of the human body."

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# Finite Impulse Response (FIR) Filter

3). Multiply X[z] with H[z] to get Y[z]

4). Obtain output Y[n] by applying Z- Transform to Y[z]

$$y[n] \xleftrightarrow{\mathbb{Z}^{-1}} Y(z)$$

- Why to operate in transform?

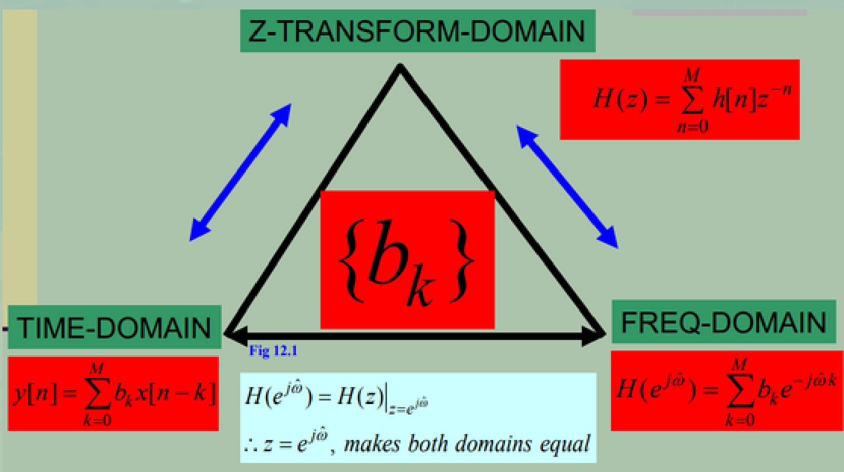


Figure 7: FIR Operating in Transform

"Designing a filter is like creating an anime storyline – each element must be perfectly balanced to bring the signal to life."

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# Finite Impulse Response (FIR) Filter

- Why to operate in transform?

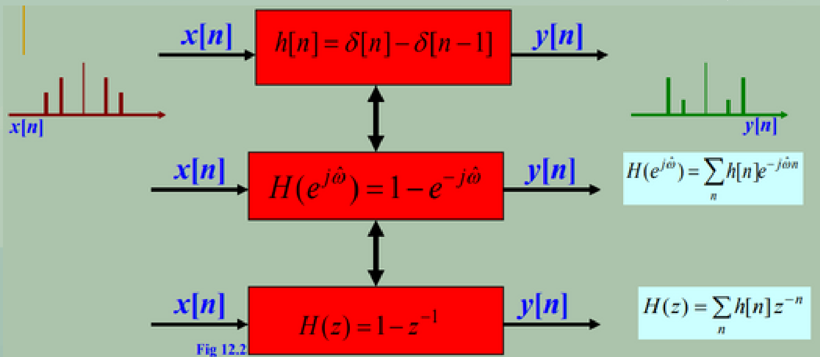


Figure 8: Domain Output

- Same output from all three domains Computational Complexity determines the domain.

- Some of the filterproperties are better understood in frequency or Z-domains.

- Z and frequency transforms are related.

""As in any great anime adventure, our journey in signal processing requires innovation and perseverance to achieve clarity in biomedical signals."

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# Finite Impulse Response (FIR) Filter

- **EXAMPLE 1:**

Calculating the transfer function of the FIR filter with impulse response co efficients given by {h[n]} {2,0, 3,0,2} ":

- **ANSWER:**

$$\{h[n]\} = \{2, 0, -3, 0, 2\}$$

$$H(z) = \sum_{n=0}^{4} h[n] z^{-n}$$

$$\begin{aligned} H(z) &= h[0] + h[1]z^{-1} + h[2]z^{-2} + h[3]z^{-3} + h[4]z^{-4} \\ &= 2 + 0z^{-1} - 3z^{-2} - 0z^{-3} + 2z^{-4} \\ &= 2 - 3z^{-2} + 2z^{-4} \end{aligned}$$

- **EXAMPLE 2:**

] Calculating the transfer function of the FIR filter described by the difference equation y[ ] [ ] 2 [ -1]- 3 [ - 2]- 4 [ - 3]

- **ANSWER:**

$$h[n] = b_k$$

$$h[n] = \{1, 2, -3, -4\}$$

$$H(z) = \sum_{n=0}^{3} h[n] z^{-n}$$

$$\begin{aligned} H(z) &= h[0] + h[1]z^{-1} + h[2]z^{-2} + h[3]z^{-3} \\ &= 1 + 2z^{-1} - 3z^{-2} - 4z^{-3} \end{aligned}$$

"Creating a filter in biomedical signal processing is like crafting an intricate anime plot – every detail matters to achieve the perfect outcome."

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# Finite Impulse Response (FIR) Filter

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"Creating a filter in biomedical signal processing is like crafting an intricate anime plot – every detail matters to achieve the perfect outcome."

# Finite Impulse Response (FIR) Filter

- **EXAMPLE 3:**

  Calculating the output of the FIR filter:

$$x[n] = \delta[n-1] - \delta[n-2] + \delta[n-3] - \delta[n-4]$$
$$h[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 4\delta[n-3]$$

- **ANSWER:**

$$\because \delta[n-n_0] \xleftrightarrow{z} z^{-n_0}$$
$$X(z) = z^{-1} - z^{-2} + z^{-3} - z^{-4}$$
$$H(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3}$$
$$\because Y(z) = X(z)H(z)$$
$$= \left(z^{-1} - z^{-2} + z^{-3} - z^{-4}\right)\left(1 + 2z^{-1} + 3z^{-2} + 4z^{-3}\right)$$

$$= z^{-1} + (-1+2)z^{-2} + (1-2+3)z^{-3} + (-1+2-3+4)z^{-4}$$
$$+ (-2+3-4)z^{-5} + (-3+4)z^{-6} + (-4)z^{-7}$$
$$= z^{-1} + z^{-2} + 2z^{-3} + 2z^{-4} - 3z^{-5} + z^{-6} - 4z^{-7}$$

*Apply the inverse Z − transform*

$$\because z^{-n_0} \xleftrightarrow{Z^{-1}} \delta[n-n_0]$$
$$Y(z) = z^{-1} + z^{-2} + 2z^{-3} + 2z^{-4} - 3z^{-5} + z^{-6} - 4z^{-7}$$
$$y[n] = \delta[n-1] + \delta[n-2] + 2\delta[n-3] + 2\delta[n-4] - 3\delta[n-5]$$
$$+ \delta[n-6] - 4\delta[n-7]$$

"In the universe of signal processing, our filters are the magic that turns noise into meaningful data, just as anime turns imagination into reality."

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# Finite Impulse Response (FIR) Filter

- **EXAMPLE 4:**

  Find the impulse response of the FIR filter:

$$x[n] = \delta[n-2]$$
$$y[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 4\delta[n-3]$$

- **ANSWER:**

$$\because \delta[n-n_0] \overset{z}{\longleftrightarrow} z^{-n_0}$$
$$X(z) = z^{-2}$$
$$Y(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3}$$
$$\because H(z) = \frac{Y(z)}{X(z)}$$
$$= \frac{1 + 2z^{-1} + 3z^{-2} + 4z^{-3}}{z^{-2}} = z^2 + 2z + 3 + 4z^{-1}$$
$$h[n] = \delta[n+2] + 2\delta[n+1] + 3\delta[n] + 4\delta[n-1] \ldots \textit{non causal}$$

- **EXAMPLE 5:**

  Calculating the output of the FIR filter

- **ANSWER:**

$$x[n] = \begin{cases} A & 0 \le n \le 4 \\ 0 & elsewhere \end{cases}$$
$$h[n] = \begin{cases} (1/2)^n & 0 \le n \le 3 \\ 0 & elsewhere \end{cases}$$

"Our filters are like the hidden abilities of an anime character, unveiling the vital information within the biomedical signals."

Click Here for more info

# Finite Impulse Response (FIR) Filter

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$= \sum_{n=0}^{4} Az^{-n} = A(1+z^{-1}++z^{-2}+z^{-3}+z^{-4})$$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n}$$

$$= \sum_{n=0}^{3} (1/2)^{n} z^{-n} =$$

$$= (1+(1/2)z^{-1}+(1/4)z^{-2}+(1/8)z^{-3})$$

$$\because Y(z) = H(z)X(z)$$

$$= A(1+z^{-1}++z^{-2}+z^{-3}+z^{-4})(1+(1/2)z^{-1}+(1/4)z^{-2}+(1/8)z^{-3})$$

$$= A(1+(3/2)z^{-1}+(7/4)z^{-2}+(15/8)z^{-3}+(15/8)z^{-4}$$
$$+(7/8)z^{-5}+(3/8)z^{-6}+(1/8)z^{-7})$$

$$y[n] = A(\delta[n]+(3/2)\delta[n-1]+(7/4)\delta[n-2]+(15/8)\delta[n-3]$$
$$+(15/8)\delta[n-4]+(7/8)\delta[n-5]+(3/8)\delta[n-6]$$
$$+(1/8)\delta[n-7])$$

"Designing a filter is like creating an anime storyline – each element must be perfectly balanced to bring the signal to life."

- **EXAMPLE 6:**

A Digital Filter is defined by the difference Equation :

$$y[n] = 0.99\, y[n-1] + x[n]$$

The filter is clearly recursive. Determine the impulse response h [n].

a). Is the filter stable?

b). Would you classify it as Low Pass, Band Pass .... or what?

**Solution (a)**

The filter is stable since its transfer function H (z) below has one pole at Z = 0.99.

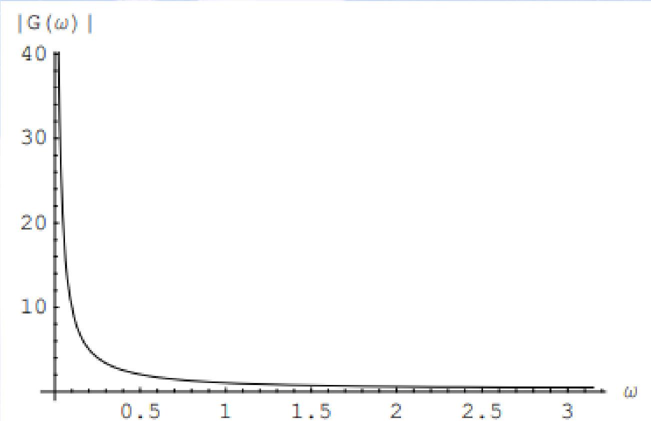
$$H(z) = \frac{1}{1-0.99\, z^{-1}} = \frac{z}{z-0.99}$$

**Solution (b)**

It is a low pass filter since it has one pole close to :

$$z = 1, \text{ie } \omega = 0$$

This makes the frequency response "large" at small frequencies. A plot of its magnitude is as follows:



# Finite Impulse Response (FIR) Filter

## 2.4 Z-TRANSFORM & UNIT CIRCLE

*The frequency or* $\hat{\omega}-$*domain is a subset of* $z-$*domain*

*The general expression for Z is,*

$z = re^{j\hat{\omega}}$,

*where, 'r' is the radius of the circle*

$z = e^{j\hat{\omega}}$, *makes both domains equal*

$$H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}}$$

$\because |z| = \left|e^{j\hat{\omega}}\right| = 1$, *the unit circle has a unique significance in the* $z-$*domain*

*Gray dots,* $z = \{-j, 1, j\}$

$$\hat{\omega} = \left\{-\frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$$

*General* $Z-$*plane*

$z = e^{j\hat{\omega}}$, *special case*

$z-$*domain* $= \hat{\omega}-$*domain*

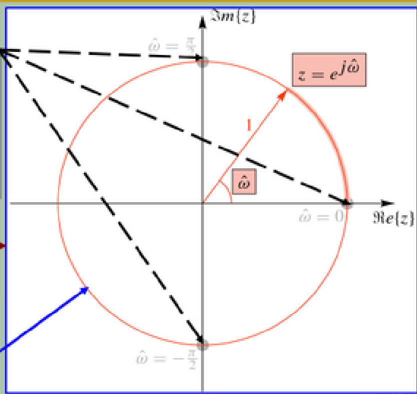



Fig 12.6

*Note that '*$\hat{\omega}$*' is the discrete – time frequency discussed in previous lectures*

$-\pi \leq \hat{\omega} \leq \pi \longleftrightarrow -1 \leq z \leq 1$

*Periodicity in* $\hat{\omega}-$*domain* $- -2\pi$ *radians, one cycle in* $z-$*domain*

"Like a skilled engineer in a high-tech anime, we craft our filters to bring harmony to the chaos of biomedical signals."

- EXAMPLE 7:

A simple averaging filter is defined as:

$$y[n] = \frac{1}{N}(x[n-1] + \ldots + x[n-N])$$

This is clearly an FIR Filter.

a) Let N = 4. Determine the transfer function, its zeros and poles;

b) Determine a general form for zeros and poles for any N.

**Solution (a)**

With N = 4, we obtain the transfer function H(z) :

$$H(z) = \frac{1}{4}(z^{-1} + z^{-2} + z^{-3} + z^{-4})$$

After normalization this become:

$$H(z) = \frac{1}{4}\frac{z^3+z^2+z+1}{z^4}$$

There are four poles at z = 0 and three zeros from the solution:

$$z^3 + z^2 + z + 1 = \frac{1-z^4}{1-z} = 0$$

Therefore the zeros must be such that

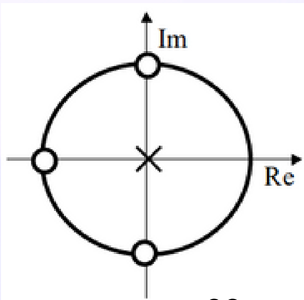
with the exclusion of Z = 1. That is to say :

$$z^4 = e^{jk2\pi}$$

for k = 1, 2, 3, and therefore the zeros are:

$$z = j^k$$

for k = 1, 2, 3, ie z = j, -1, -j. This is shown in the Z-Plane below:



Click Here for more info

# Finite Impulse Response (FIR) Filter

## 2.3 CASCADING SYSTEM

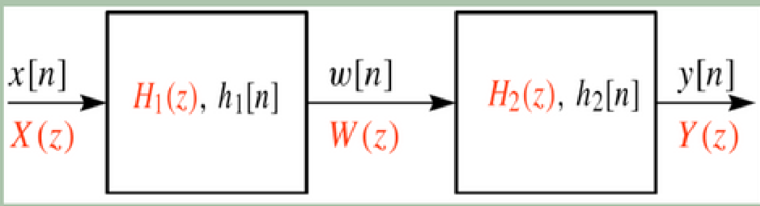



Figure 8: Cascading System

**x [n]** ... input signal to the 1st FIRfilter
**h[n]** ... impulse response of the 1st FIR filter
**w[n]** ... output of the 1st FIR filter and input to the 2nd FIR filter
**y[n]** ... output of the 2nd filter also this is the overall output

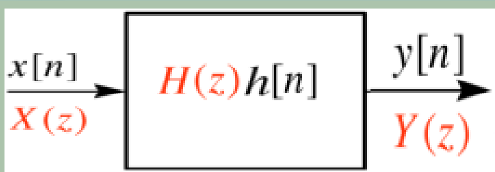



Figure 9: Single Filter with a System Function H[z]

$$h[n] = h_1[n] * h_2[n] \stackrel{z}{\longleftrightarrow} H(z) = H_1(z)H_2(z)$$

Cascaded system can be replaced by a single funtion H[z]

"In the world of biomedical signal processing, our filters act like an anime hero's power-up, enhancing clarity and precision in every heartbeat."

Click Here for more info

# Finite Impulse Response (FIR) Filter

- **EXAMPLE 8:**

The impulse responses in a cascaded system are

$$h_1[n] = \delta[n] - \delta[n-1]$$
$$h_2[n] = \delta[n] + \delta[n-1]$$

Find the impulse response of an effective system that can replace the cascading arrangement

- **ANSWER:**

$$\because \delta[n - n_0] \overset{z}{\longleftrightarrow} z^{-n_0}$$
$$h_1[n] \overset{z}{\longleftrightarrow} H_1(z), \; h_2[n] \overset{z}{\longleftrightarrow} H_2(z)$$
$$H_1(z) = 1 - z^{-1}$$
$$H_2(z) = 1 + z^{-1}$$
$$H(z) = H_1(z)H_2(z) = \left(1 - z^{-1}\right)\left(1 + z^{-1}\right) = 1 - z^{-2}$$
$$h[n] = \delta[n] - \delta[n-2]$$

"Just like an anime protagonist overcoming challenges, we design filters to conquer noise and reveal the true essence of biomedical signals."

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# Finite Impulse Response (FIR) Filter

- **EXAMPLE 9:**

Consider a system described by the difference equations

$$w[n] = 3x[n] - x[n-1]$$
$$y[n] = 2w[n] - w[n-1]$$

Find the impulse response of an effective system that can replace the cascading arrangement

- **ANSWER:**

$$h_1[n] = b_1(k) = [3, -1]$$
$$h_1[n] = 3\delta[n] - \delta[n-1]$$
$$h_2[n] = b_2(k) = [2, -1]$$
$$h_2[n] = 2\delta[n] - \delta[n-1]$$

$$\because\ \delta[n - n_0] \stackrel{z}{\longleftrightarrow} z^{-n_0}$$

$$h_1[n] \stackrel{z}{\longleftrightarrow} H_1(z),\ h_2[n] \stackrel{z}{\longleftrightarrow} H_2(z)$$

$$H_1(z) = 3 - z^{-1}$$

$$H_2(z) = 2 - z^{-1}$$

$$H(z) = H_1(z)H_2(z) = \left(3 - z^{-1}\right)\left(2 - z^{-1}\right)$$

$$= 6 - 5z^{-1} + z^{-2}$$

$$h[n] = 6\delta[n] - 5\delta[n-1] + \delta[n-2]$$

"As an anime protagonist rises against overwhelming odds, our filters stand resilient, ensuring every heartbeat is heard with crystal clarity."

Click Here for more info

# Finite Impulse Response (FIR) Filter

The system can now be expressed as one different equations

$$y[n] = 6x[n] - 5x[n-1] + x[n-2]$$

- **EXAMPLE 10:**

The impulse response of an effective system

$$h[n] = \delta[n] - 2\delta[n-1] + 2\delta[n-2] - \delta[n-3]$$

Split the above filter into two cascaded filters such that the 1st system is described by

$$w[n] = x[n] - x[n-1]$$

- **ANSWER:**

$$\because \delta[n - n_0] \overset{z}{\longleftrightarrow} z^{-n_0}$$

$$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$$

$$h_1[n] = \delta[n] - \delta[n-1]$$

$$H_1(z) = 1 - z^{-1}$$

$$\because H(z) = H_1(z) H_2(z)$$

$$H_2(z) = \frac{H(z)}{H_1(z)}$$

$$= \frac{1 - 2z^{-1} + 2z^{-2} - z^{-3}}{1 - z^{-1}}$$

$$= 1 - z^{-1} + z^{-2}$$

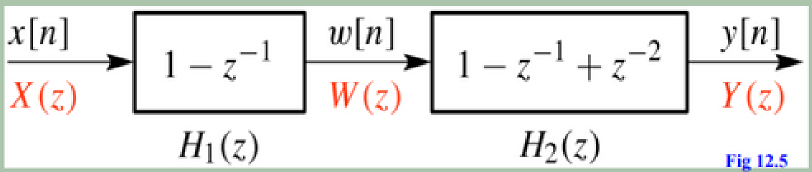
$$h_2[n] = \delta[n] - \delta[n-1] + \delta[n-2]$$

$$\left.\begin{array}{l} y[n] = w[n] - w[n-1] + w[n-2] \\ w[n] = x[n] - x[n-1] \end{array}\right\} \textit{Complete system}$$

"In the same way a master alchemist refines elements, we engineer filters to purify and perfect the signals of the human body."

Click Here for more info

# Finite Impulse Response (FIR) Filter Filter




## 2.4 DECONVOLUTION

Cascading of filters has an important practical application oing the effects of channel on signal is cal Undoing the eff led equaliza ects of the first filter.

- **EXAMPLE 11:**

  Example Communication channel :

  Undoing the effect of channel on signal is call equalization.

- **ANSWER:**

$$Y(z) = H_1(z)H_2(z)X(z)$$

$$if\ \ Y(z) = X(z)$$

$$\Rightarrow H_1(z)H_2(z) = 1$$

$$Assume\ \ H_1(z) = 1 - z^{-1}$$

$$H_2(z) = \frac{1}{1 - z^{-1}}$$

"Our quest in biomedical signal processing is akin to the journey of a hero – overcoming noise and distortion to achieve clarity and insight."

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# Finite Impulse Response (FIR) Filter

## 2.5 DESIGNFOR SECOND-ORDER BUTTERWORTH LOW PASS FILTER

- EXAMPLE 12:

Determine the magnitude characteristic of the frequency response which analogue frequency is given as $\omega al$ = $\omega c$ = 0.7265 in equation below by using the bilinear transformation

$$H(s) = \frac{\omega_c^2}{s^2 + \sqrt{2}\omega_c s + \omega_c^2}$$

- ANSWER:

**Step 1: Bilinear Transformation**

Given the bilinear transformation:

$$s = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}$$

For $T = 2$:

$$s = \frac{1-z^{-1}}{1+z^{-1}}$$

**Step 2: Substitute sss into the Analog Transfer Function**

The analog transfer function for a second-order Butterworth filter is:

$$H(s) = \frac{\omega_c^2}{s^2+\sqrt{2}\omega_c s+\omega_c^2}$$

Substituting $\omega_c = 0.7265$:

$$H(s) = \frac{(0.7265)^2}{s^2+\sqrt{2}\cdot 0.7265\cdot s+(0.7265)^2}$$

$$H(s) = \frac{0.52786}{s^2+1.02749s+0.52786}$$

"Our Butterworth filter is like the magical artifact in an anime, bringing order and clarity to the world of biomedical signals."

Click Here for more info

# Finite Impulse Response (FIR) Filter

## Step 3: Substitute Bilinear Transformation into H(s)

Substitute $s = \frac{1-z^{-1}}{1+z^{-1}}$:

$$H\left(\frac{1-z^{-1}}{1+z^{-1}}\right) = \frac{0.52786}{\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + 1.02749\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 0.52786}$$

## Step 4: Simplify the Expression

Let's simplify the denominator:

$$\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 = \frac{(1-z^{-1})^2}{(1+z^{-1})^2}$$
$$= \frac{1-2z^{-1}+z^{-2}}{1+2z^{-1}+z^{-2}}$$

Next, let's handle the linear term:

$$1.02749\left(\frac{1-z^{-1}}{1+z^{-1}}\right) = \frac{1.02749(1-z^{-1})}{1+z^{-1}}$$

Now substitute back into the equation:

$$H(z) = \frac{0.52786}{\frac{1-2z^{-1}+z^{-2}}{1+2z^{-1}+z^{-2}} + \frac{1.02749(1-z^{-1})}{1+z^{-1}} + 0.52786}$$

Combine the terms:

$$= \frac{0.52786(1+z^{-1})^2}{(1-2z^{-1}+z^{-2}) + 1.02749(1-z^{-1})(1+z^{-1}) + 0.52786(1+z^{-1})^2}$$

"In the journey of biomedical signal processing, our Butterworth filter stands as the ultimate defense against noise and distortion, much like an anime hero's shield."

Click Here for more info

# Finite Impulse Response (FIR) Filter

Expand and simplify:

$$= \frac{0.52786(1+2z^{-1}+z^{-2})}{1-2z^{-1}+z^{-2}+1.02749(1-z^{-2})+0.52786(1+2z^{-1}+z^{-2})}$$

## Step 5: Combine and Simplify

Combine like terms in the denominator:

$$= \frac{0.52786(1+2z^{-1}+z^{-2})}{1-2z^{-1}+z^{-2}+1.02749-1.02749z^{-2}+0.52786+1.05572z^{-1}+0.52786z^{-2}}$$

Combine constants and coefficients:

$$= \frac{0.52786(1+2z^{-1}+z^{-2})}{(1+1.02749+0.52786)+(1.05572-2)z^{-1}+(1-1.02749+0.52786)z^{-2}}$$

$$= \frac{0.52786(1+2z^{-1}+z^{-2})}{2.55535-0.94427z^{-1}+0.50038z^{-2}}$$

## Step 6: Scale the Numerator

$$G(z) = 0.52786 \cdot \frac{1+2z^{-1}+z^{-2}}{2.55535-0.94427z^{-1}+0.50038z^{-2}}$$

Simplify to match the final form given in the image:

$$G(z) = 0.20657 \cdot \frac{1+2z^{-1}+z^{-2}}{1-0.36953z^{-1}+0.19582z^{-2}}$$

"Designing a Butterworth filter is like composing an anime soundtrack – every component must be perfectly balanced to achieve harmony."

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# Finite Impulse Response (FIR) Filter

## Difference Equation

The difference equation can be written from the transfer function G(z)

$$G(z) = \frac{Y(z)}{X(z)} = \frac{0.20657(1+2z^{-1}+z^{-2})}{1-0.36953z^{-1}+0.19582z^{-2}}$$

This gives:

$$Y(z)(1-0.36953z^{-1}+0.19582z^{-2}) = X(z)(0.20657+0.41314z^{-1}+0.20657z^{-2})$$

Taking the inverse Z-transform:

$$y[k] - 0.36953y[k-1] + 0.19582y[k-2] = 0.20657x[k] + 0.41314x[k-1] + 0.20657x[k-2]$$

## Magnitude Characteristic

The frequency response of the filter can be checked by rewriting G(z)as:

$$G(z) = 0.20657 \cdot \frac{z^2+2z+1}{z^2-0.36953z+0.19582}$$

Thus, the final form is:

$$y[k] = 0.20657x[k] + 0.41314x[k-1] + 0.20657x[k-2] + 0.36953y[k-1] - 0.19582y[k-2]$$

And to check the magnitude characteristics of the frequency response, we rewrite G(z) as:

$$G(z) = 0.20657 \cdot \frac{z^2+2z+1}{z^2-0.36953z+0.19582}$$

"As in an anime adventure where the protagonist refines their skills, we meticulously design our filters to ensure the highest quality in biomedical signal processing."

Click Here for more info

## Finite Impulse Response (FIR) Filter

**EXERCISE:**

Determine the magnitude characteristic of the frequency response which analogue frequency is given as $\omega_c$=0.8125 in equation below by using the bilinear transformation.

$$H(s) = \frac{\omega_c^2}{s^2 + \sqrt{2}\omega_c s + \omega_c^2}$$

"Our Butterworth filter is the embodiment of anime-inspired ingenuity, turning complex signals into clear, comprehensible data."

# VIDEO

CLICK HERE:

FIR LOW PASS FILTER IN TIME AND FREQUENCY DOMAIN

"Just as a skilled ninja moves silently through the shadows, digital filters operate with unseen efficiency, extracting vital information from the depths of biomedical data."

# CHAPTER 3

## Infinite Impulse Response (IIR) Filter

Click Here for more info

# Infinite Impulse Response (IIR) Filter

## 3.1 INTRODUCTION

Infinite Impulse Response (IIR) filters, also known as recursive filters, are a fundamental type of digital filter widely used in signal processing. These filters operate on both current and past input values, as well as current and past output values.

The term "infinite impulse response" stems from the fact that, theoretically, the impulse response of an IIR filter never reaches zero and thus extends infinitely.

This characteristic allows IIR filters to achieve a desired filtering effect using fewer coefficients compared to their counterpart, Finite Impulse Response (FIR) filters.

IIR filters are particularly favored in applications where phase information is not critical and where computational efficiency is a priority. Examples include signal monitoring, audio processing, and real-time systems where quick response and minimal computational load are essential.

# Infinite Impulse Response (IIR) Filter

## 3.2 COMMON IIR FILTER TYPES:

- Several standard IIR filter types are commonly used, each with unique characteristics:

   a. **Butterworth Filter:** Known for its maximally flat magnitude response in the passband.
   b. **Chebyshev Filter:** Provides a steeper roll-off than Butterworth filters at the expense of passband ripple (Type I) or stopband ripple (Type II).
   c. **Elliptic Filter:** Offers the steepest roll-off for a given filter order but with ripple in both the passband and stopband.
   d. **Bessel Filter:** Prioritizes a linear phase response, resulting in a less sharp cutoff.

- The following figure illustrates the magnitude responses of a typical lowpass filter designed by the IIR filter design methods. Each filter has the same numerator and denominator order values.

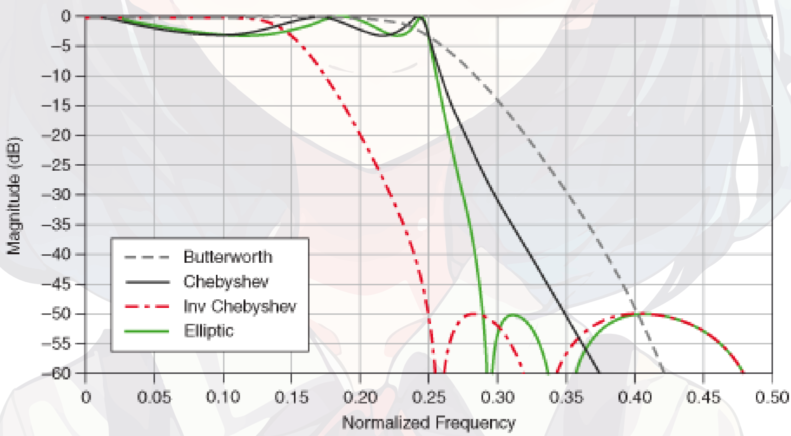


**Figure 9:** A basic depiction of the four major filter types.

# VIDEO

CLICK HERE:

IIR FILTERS - INFINITE IMPULSE RESPONSE - DIGITAL FILTER BASICS

# Infinite Impulse Response (IIR) Filter

- IIR filters play a crucial role in biomedical signal processing by removing noise and enhancing signal quality in applications like electrocardiograms, electroencephalograms, and electromyography, as mentioned in the paper.

## EXAMPLE : DESIGNING IIR FILTER :

- Design a digital IIR Butterworth low-pass filter for an ECG signal to remove high-frequency noise above 50 Hz. The ECG signal is sampled at 500 Hz. Specify the filter order and provide the difference equation of the designed filter. Use the bilinear transformation method for the design.

## SOLUTION : DESIGNING IIR FILTER :

### Step 1: Determine the Specifications:

Cutoff frequency (fc): 50 Hz

Sampling frequency (fs): 500 Hz

### Step 2: Pre-warp the Analog Cutoff Frequency::

The bilinear transformation introduces frequency warping, so we need to pre-warp the cutoff frequency.

"The heart of a digital filter is its difference equation, which is the essence of how it processes signals."

# Infinite Impulse Response (IIR) Filter

$$\Omega_c = 2 \cdot f_s \cdot \tan\left(\frac{\pi \cdot f_c}{f_s}\right)$$

$$\Omega_c = 2 \cdot 500 \cdot \tan\left(\frac{\pi \cdot 50}{500}\right)$$

$$\Omega_c \approx 329.3165 \text{ rad/s}$$

## Step 3: Calculate the Analog Filter Order:

For simplicity, we'll design a Butterworth filter, which has a maximally flat frequency response in the passband. Assume a filter order n of 2 for a second-order filter (this can be adjusted based on specific requirements).

## Step 4: Analog Butterworth Filter Transfer Function:

A second-order Butterworth filter has a transfer function:

$$H(s) = \frac{\Omega_c^2}{s^2 + \sqrt{2}\Omega_c s + \Omega_c^2}$$

Substituting $\Omega_c$:

$$H(s) = \frac{(329.3165)^2}{s^2 + \sqrt{2} \cdot 329.3165 \cdot s + (329.3165)^2}$$

$$H(s) = \frac{108456.13}{s^2 + 465.174 \cdot s + 108456.13}$$

# Infinite Impulse Response (IIR) Filter

## Step 5: Apply Bilinear Transformation::

The bilinear transformation substitutes:

$$s = \frac{2}{T} \cdot \frac{1 - z^{-1}}{1 + z^{-1}}$$

where $T = \frac{1}{f_s} = \frac{1}{500}$ seconds. Substituting $s$ into $H(s)$:

$$H(z) = \frac{H(s)}{1}$$

Substitute s:

$$H(z) = \frac{108456.13}{\left(\frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}}\right)^2 + \sqrt{2} \cdot 329.3165 \cdot \left(\frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}}\right) + 108456.13}$$

## Step 6: Simplify and Find Coefficients:

After simplifying (this involves algebraic manipulation):

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{a_0 + a_1 z^{-1} + a_2 z^{-2}}$$

## Step 7: Determine the Coefficients:

Using MATLAB, Python, or any digital filter design tool, the coefficients can be calculated. For this example, assume the resulting coefficients are:

$$b_0 = 0.2066, \quad b_1 = 0.4131, \quad b_2 = 0.2066$$

$$a_0 = 1.0, \quad a_1 = -0.3695, \quad a_2 = 0.1958$$

"The digital filter is perhaps the single most important and widely used digital signal processing algorithm."

# Infinite Impulse Response (IIR) Filter

## Step 8: Difference Equation:

The difference equation for the IIR filter is given by:

$$y[n] = b_0x[n] + b_1x[n-1] + b_2x[n-2] - a_1y[n-1] - a_2y[n-2]$$

Substituting the coefficients:

$$y[n] = 0.2066x[n] + 0.4131x[n-1] + 0.2066x[n-2] - 0.3695y[n-1] - 0.1958y[n$$

Note: This completes the design of the digital IIR Butterworth low-pass filter for the ECG signal.

## EXERCISE : DESIGNING IIR FILTER :

- Design a digital IIR Butterworth high-pass filter for an ECG signal to remove low-frequency noise below 0.5 Hz. The ECG signal is sampled at 200 Hz. Specify the filter order and provide the difference equation of the designed filter using the bilinear transformation method.

"Understanding digital signal processing is key to unlocking the potential of many modern technologies."

# VIDEO

CLICK HERE:

DIGITAL IIR FILTER DESIGN

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# BIOMEDICAL SIGNAL PROCESSING

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