

SULIT



**KEMENTERIAN PENDIDIKAN TINGGI
JABATAN PENDIDIKAN POLITEKNIK DAN KOLEJ KOMUNITI**

**BAHAGIAN PEPERIKSAAN DAN PENILAIAN
JABATAN PENDIDIKAN POLITEKNIK DAN KOLEJ KOMUNITI
KEMENTERIAN PENDIDIKAN TINGGI**

JABATAN MATEMATIK, SAINS & KOMPUTER

PEPERIKSAAN AKHIR

SESI II : 2024/2025

**BBM30073: ADVANCED CALCULUS FOR ENGINEERING
TECHNOLOGY**

TARIKH : 16 JUN 2025

MASA : 9.00 PAGI – 12.00 T/HARI (3 JAM)

Kertas ini mengandungi **LAPAN (8)** halaman bercetak.

Struktur (4 soalan)

Dokumen sokongan yang disertakan : Formula

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIARAHKAN

(CLO yang tertera hanya sebagai rujukan)

SULIT

INSTRUCTION:

This section consists of **FOUR (4)** structured questions. Answer **ALL** questions.

ARAHAN :

*Bahagian ini mengandungi **EMPAT (4)** soalan struktur. Jawab **SEMUA** soalan.*

QUESTION 1**SOALAN 1**

- CLO1 (a) Show that $y = Ae^{4x} + 2$ is a general solution of the differential equation for
Tunjukkan $y = Ae^{4x} + 2$ adalah penyelesaian am bagi persamaan perbezaan bagi
- $$\frac{dy}{dx} + 8 = 4y$$
- [5 marks]
[5 markah]
- CLO1 (b) Construct a differential equation for the equation, $y = Bxe^{2x}$.
Bentukkan persamaan perbezaan bagi persamaan, $y = Bxe^{2x}$.
- [7 marks]
[7 markah]
- CLO2 (c) Solve the following first order differential equations:
Selesaikan persamaan perbezaan peringkat pertama berikut:
- i) $\frac{dy}{dx} = \frac{xy}{y+1}$
- [6 marks]
[6 markah]
- ii) $\frac{dy}{dx} + \frac{4}{x}y = x^3y^2$
- [7 marks]
[7 markah]

QUESTION 2**SOALAN 2**

CLO2

(a) Given that the differential equation of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 0$.

Diberi bahawa persamaan perbezaan $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 0$.

i) Solve the general solution.

Selesaikan penyelesaian am.

[3 marks]

[3 markah]

ii) Solve the particular solution given $y(0) = 6$, $y'(0) = 2$.

Selesaikan penyelesaian khusus bagi $y(0) = 6$, $y'(0) = 2$.

[7 marks]

[7 markah]

CLO2

- (b) Given that the equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 12y = 24e^{2x} - 6$. Using the method of undetermined coefficient:

Diberi bahawa persamaan $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 12y = 24e^{2x} - 6$. Gunakan kaedah koefisien tak tentu:

- i) Solve the complementary function, y_c .

Selesaikan fungsi pelengkap, y_c .

[3 marks]

[3 markah]

- ii) Solve the particular solution, y_p and the general solution.

Selesaikan penyelesaian khusus, y_p dan penyelesaian amnya.

[4 marks]

[4 markah]

- iii) Solve the general solution when the boundary conditions are given as

$$y(0) = \frac{3}{2} \text{ and } y'(0) = 14.$$

Selesaikan penyelesaian am apabila syarat sempadan adalah $y(0) = \frac{3}{2}$ dan $y'(0) = 14$.

[8 marks]

[8 markah]

QUESTION 3**SOALAN 3**

CLO2

- (a) Solve the following second order partial differential equation by using direct partial integration.

Selesaikan persamaan perbezaan separa peringkat kedua berikut dengan menggunakan pengamiran separa langsung.

$$\frac{\partial^2 u}{\partial x \partial y} = 2y \ln x + 6xy^2$$

[5 marks]

[5 markah]

CLO2

- (b) Solve the complementary function for all following partial differential equation into its canonical form.

Selesaikan fungsi pelengkap bagi semua persamaan perbezaan separa berikut dalam bentuk kanoniknya.

i) $U_{xx} + 4U_{xy} + 4U_{yy} = 0$

[3 marks]

[3 markah]

ii) $5 \frac{\partial^2 u}{\partial x^2} - 6 \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$

[3 marks]

[3 markah]

iii) $\frac{\partial^2 u}{\partial x^2} + 2 \frac{\partial^2 u}{\partial x \partial y} + 5 \frac{\partial^2 u}{\partial y^2} = 0$

[4 marks]

[4 markah]

CLO2

- (c) Given that the function of second order partial differential equation with constant coefficients is $(D_x^2 - 2D_x D_y + D_y^2)Z = e^{x+3y}$.

Diberi bahawa fungsi berikut bagi persamaan pembezaan separa terbitan kedua dengan pekali malar ialah $(D_x^2 - 2D_x D_y + D_y^2)Z = e^{x+3y}$.

- i) Solve the complimentary function (C.F)

Selesaikan fungsi pelengkap (F.P)

[4 marks]

[4 markah]

- ii) Solve a particular integral (P.I) and its general solution

Selesaikan penyelesaian kursus (P.K) dan penyelesaian amnya

[6 marks]

[6 markah]

QUESTION 4**SOALAN 4**

CLO1

(a) Based on the Laplace transform table, solve the Laplace transform for:

Berdasarkan jadual jelmaan Laplace, selesaikan jelmaan Laplace bagi:

i) $\mathcal{L}(e^{-3t} + 2t - 4)$

[3 marks]

[3 markah]

ii) $\mathcal{L}\{(t^5 e^{7t} + t \sin t)\}$

[4 marks]

[4 markah]

CLO2

(b) Solve the inverse Laplace transform for the following functions:

Selesaikan songsangan jelmaan Laplace bagi fungsi yang berikut:

i) $\mathcal{L}^{-1}\left\{\frac{s}{s^2+16} + \frac{6}{s^3}\right\}$

[3 marks]

[3 markah]

ii) $\mathcal{L}^{-1}\left\{\frac{3s+4}{(s-2)^2}\right\}$

[5 marks]

[5 markah]

CLO 2

(c) Consider the following differential equation:

Pertimbangkan persamaan perbezaan berikut:

$$\frac{d^2y}{dt^2} + 5 \frac{dy}{dt} + 6y = 4$$

- i) Solve $Y(s)$ by applying the Laplace transform and using partial fraction decomposition to simplify the expression, given the initial conditions $y(0) = 0$ and $y'(0) = 1$.

Selesaikan $Y(s)$ dengan menggunakan transformasi Laplace dan penguraian pecahan separa untuk memudahkan ungkapan tersebut, berdasarkan syarat awal $y(0) = 0$ dan $y'(0) = 1$.

[7 marks]

[7 markah]

- ii) Solve $y(t)$ by using the inverse Laplace transform of $Y(s)$.

Selesaikan $y(t)$ dengan menggunakan transformasi songsang Laplace ke atas $Y(s)$.

[3 marks]

[3 markah]

SOALAN TAMAT

FORMULA

Basic Differentiation		Basic Integration											
$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$		$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$											
$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$		$\int u dv = uv - \int v du$											
$\frac{d}{dx}(e^{ax}) = ae^{ax}$		$\int e^{ax} du = \frac{1}{a}e^{ax} + C$											
$\frac{d}{dx}(\ln x) = \frac{1}{x}$		$\int \frac{1}{x} dx = \ln x + C$											
$\frac{d}{dx}[\sin(ax)] = a \cos(ax)$		$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + C$											
$\frac{d}{dx}[\cos(ax)] = -a \sin(ax)$		$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + C$											
$\frac{d}{dx}[\tan(ax)] = a \sec^2(ax)$		$\int \sec^2(ax) dx = \frac{1}{a}\tan(ax) + C$											
First Order Differential Equation													
HOMOGENOUS $y = vx$ $\frac{dy}{dx} = v + x \frac{dy}{dx}$		LINEAR $\frac{dy}{dx} + P(x)y = Q(x)$ $ye^{\int P(x) dx} = \int Q(x) \cdot e^{\int P(x) dx} dx + C$											
EXACT $P(x,y)dx + Q(x,y)dy = 0$ $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$		BERNOULLI $\frac{dy}{dx} + P(x)y = Q(x)y^n$ $y^{1-n}(e^{\int (1-n)P(x)dx}) = \int (1-n)Q(x)(e^{\int (1-n)P(x)dx})dx + C$											
Second Order Differential Equation													
<table><tr><th colspan="2">General form : $a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = G(x)$</th></tr><tr><th>Roots</th><th>Form of y_c</th></tr><tr><td>$m_1 \neq m_2$</td><td>$y_c = Ae^{m_1x} + Be^{m_2x}$</td></tr><tr><td>$m = m_1 = m_2$</td><td>$y_c = e^{mx}(A + Bx)$</td></tr><tr><td>$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $m = \alpha \pm \beta i$</td><td>$y_c = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$</td></tr></table>				General form : $a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = G(x)$		Roots	Form of y_c	$m_1 \neq m_2$	$y_c = Ae^{m_1x} + Be^{m_2x}$	$m = m_1 = m_2$	$y_c = e^{mx}(A + Bx)$	$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $m = \alpha \pm \beta i$	$y_c = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$
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Roots	Form of y_c												
$m_1 \neq m_2$	$y_c = Ae^{m_1x} + Be^{m_2x}$												
$m = m_1 = m_2$	$y_c = e^{mx}(A + Bx)$												
$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $m = \alpha \pm \beta i$	$y_c = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$												

Second Order Differential Equation

Particular Integral:

$G(x)$	form of y_p
k (constant)	C
kx	$Cx + D$
kx^2	$Cx^2 + Dx + E$
$k \sin ax$ @ $k \cos ax$	$C \cos ax + D \sin ax$
$k \sinh ax$ @ $k \cosh ax$	$C \cosh ax + D \sinh ax$
e^{kx}	Ce^{kx}
xe^{kx}	$(Cx + D)e^{kx}$

WRONSKIAN DETERMINANT

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$$

PARTICULAR INTEGRAL

$$y_p = uy_1 + vy_2$$

$$u = - \int \frac{y_2 G(x)}{W} dx \quad \text{and} \quad v = \int \frac{y_1 G(x)}{W} dx$$

Partial Differential Equation

Complementary Function:

$$\text{General form : } A \frac{\partial^2 z}{\partial x^2} + B \frac{\partial^2 z}{\partial x \partial y} + C \frac{\partial^2 z}{\partial y^2} = F(x, y)$$

$$f(D_x, D_y) = F(x, y)$$

Canonical Form	Equation	Form of Z_c
$B^2 - 4AC > 0$	Hyperbolic	$Z_c = \phi_1(y + m_1x) + \phi_2(y + m_2x)$
$B^2 - 4AC = 0$	Parabolic	$Z_c = \phi_1(y + mx) + x\phi_2(y + mx)$
$B^2 - 4AC < 0$	Elliptic	$Z_c = \phi_1(y + m_1x) + \phi_2(y + m_2x)$ where $m = \alpha \pm \beta i$

Particular Integral:

$$Z_p = \frac{1}{f(D_x, D_y)} F(x, y)$$

If $F(x, y)$	Form of Z_p
e^{ax+by}	$Z_p = \frac{1}{f(a,b)} e^{ax+by}$ if $f(a, b) \neq 0$
$\sin(ax + by)$	$Z_p = \frac{1}{f(-a^2, -ab, -b^2)} \sin(ax + by)$ if $f(-a^2, -ab, -b^2) \neq 0$
$\cos(ax + by)$	$Z_p = \frac{1}{f(-a^2, -ab, -b^2)} \cos(ax + by)$ if $f(-a^2, -ab, -b^2) \neq 0$

Laplace Transform Table					
No.	$f(t)$	$F(s)$	No.	$f(t)$	$F(s)$
1.	a	$\frac{a}{s}$	13.	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
2.	at	$\frac{a}{s^2}$	14.	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
3.	t^n $n = 1, 2, 3, \dots$	$\frac{n!}{s^{n+1}}$	15.	$\sinh \omega t$	$\frac{\omega}{s^2 - \omega^2}$
4.	$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}$	16.	$\cosh \omega t$	$\frac{s}{s^2 - \omega^2}$
5.	e^{-at}	$\frac{1}{s+a}$	17.	$e^{at} \sinh \omega t$	$\frac{\omega}{(s-a)^2 - \omega^2}$
6.	te^{-at}	$\frac{1}{(s+a)^2}$	18.	$e^{-at} \sinh \omega t$	$\frac{\omega}{(s+a)^2 - \omega^2}$
7.	$t^n \cdot e^{at}$ $n = 1, 2, 3, \dots$	$\frac{n!}{(s-a)^{n+1}}$	19.	$e^{-at} \cosh \omega t$	$\frac{s+a}{(s+a)^2 - \omega^2}$
8.	$t^n \cdot f(t)$	$(-1)^n \frac{d^n}{ds^n} [F(s)]$	20.	$f_1(t) + f_2(t)$	$F_1(s) + F_2(s)$
9.	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$	21.	$\int_0^1 f(u) du$	$\frac{F(s)}{s}$
10.	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$	22.	$f(t-a)u(t-a)$	$e^{-as}F(s)$
11.	$t \sin \omega t$	$\frac{2\omega s}{(s^2 + \omega^2)^2}$	23.	First derivative $\frac{dy}{dt}, y'(t)$	$sY(s) - y(0)$
12.	$t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$	24.	Second derivative $\frac{d^2y}{dt^2}, y''(t)$	$s^2Y(s) - sy(0) - y'(0)$